ROBUST ELIMINATION LEMMA FOR POLYTOPIC SYSTEMS

Vojtech Veselý, Danica Rosinová and Vladimír Kučera

Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Ilkovicova 3, 81219 Bratislava, Slovak Republic;
Faculty of Electrical Engineering Czech Technical University in Prague, Czech Republic
danica.rosinova@stuba.sk ; vojtech.vesely@stuba.sk; kucera@fel.cvut.cz

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A linear algebra result, known as Elimination Lemma, has been used to solve a large number of filtering and control problems. In this paper we present a robust version of this result which simplifies, among other problems, the design of a robustly stabilizing static output feedback for linear polytopic systems.

The following set of inequalities:

$$G_i + U_i X V_i + V_i^T X U_i^T < 0, \quad \text{i} = 1, 2...\text{N} \tag{1}$$

appear in the study of robust stability conditions for linear systems with polytopic uncertainties. The matrices G_i, U_i, V_i and X all may depend on the control system parameters to be designed. In some cases the matrix X prevents an immediate application of control design procedures and should be eliminated from inequality (1), thus eliminating some parameters and simplifying the design inequalities. The linear algebra result, known as Elimination Lemma can be used to eliminate the matrix X from the inequalities (1).

Elimination Lemma

Let matrices $G = G^T$, U, V of appropriate dimensions be given. Then the following statements are equivalent.

(i) there exists a matrix X satisfying

$$G + UXV + V^T X^T U^T < 0 \tag{2}$$

(ii) $U^{\perp} G U^{\perp T} < 0$, or $UU^T > 0$

$$V^{\perp T} G V^{\perp} < 0, \text{ or } V^T V > 0. \tag{3}$$

where $U^{\perp}$ is any full row rank matrix orthogonal to U and $V^{\perp}$ is any full column rank matrix orthogonal to V

It has been shown that the Elimination Lemma has several limitations which prevent its application to inequalities (1) when N > 1, a typical situation in robust control. The challenge here is the fact that a single X should simultaneously satisfy all N inequalities. In this case, X cannot be eliminated from (1) using Elimination Lemma directly, since though (3) still implies (2) in such a case, the converse is not true in general.

This note is motivated by a specific case of inequalities (1) that arise in the robust static output feedback control design for linear polytopic systems. This paper presents a robust version of the

Elimination Lemma, namely sufficient conditions for the elimination of X from the set of inequalities (1). The application of the developed robust elimination lemma to static output feedback stabilization of polytopic uncertain systems is briefly outlined.