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EXPERIMENTAL PHASE CONTROL OF A FORCED CHUA'S CIRCUIT

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Abstract
In this paper the suitability of the phase control technique to control the forced Chua's circuit is demonstrated. According to this scheme a further sinusoidal term is introduced in the forced Chua's circuit at the same frequency of the original driving signal, but with a different phase. The experimental results discussed in the paper demonstrate that the phase difference between the two sinusoidal terms can act as a control parameter for the circuit.

Key words
Chaos, chaos control, Chua's circuit, phase control.

1 Introduction

The techniques to control chaos can be classified in feedback and open loop methods [1]. Feedback methods usually allow the system to be stabilized in any of the unstable periodic orbits lying in the chaotic attractor, but require fast and accurate response to work properly. On the other hand, open loop techniques usually exploit the effect of some (small) perturbations added to the system to modify the final state of the controlled dynamics.

Non-feedback methods have been mainly used to suppress chaos in periodically driven dynamical systems:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \lambda) + \mathbf{F}\cos\omega t \quad (1)$$

where $\mathbf{x}$, $\mathbf{f}$ and $\mathbf{F}$ are vectors of the m-dimensional phase space, and λ is a parameter of the system. The main idea of these non-feedback methods is to apply a harmonic perturbation either to some of the parameters of the system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \lambda(1 + \varepsilon\cos(r\omega t + \varphi))) + \mathbf{F}\cos\omega t \quad (2)$$

or as an additional forcing

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \lambda) + \mathbf{F}\cos\omega t + \mathbf{u}\cos(r\omega t + \varphi) \quad (3)$$

where $\mathbf{u}$ is a conveniently chosen unitary vector.

The effectiveness of this type of methods has been tested experimentally in different works [2; 3]. In the first where these non-feedback method was explored, the numerical and experimental explorations were essentially focused on the role played by the perturbation amplitude and the resonance condition r, but the role of the phase difference φ was hardly explored. However, in the Ref. [3], it was observed that the phase difference φ between the periodic forcing and the perturbation had certain influence on the dynamical behavior of the system. Furthermore, in the Ref. [4], the authors have shown that φ plays a crucial role on the global dynamics of the system. Thus, it was clear that the role of the phase difference is important in the global dynamics of the system. The type of control based on varying the phase difference φ in search of a desired dynamical behavior is known as the *phase control* technique [5]. In many systems a correct choice of the phase allows to suppress chaos with a very small amplitude harmonic perturbation. Phase control has been applied to the Duffing system [4; 6; 7], to control intermittency in a CO_2 laser [8] and to avoid escapes of a Helmholtz oscillator [9].

Both non-feedback and feedback techniques have been applied to the Chua's circuit [10]. In this paper we investigate the suitability of the phase control technique to suppress chaos in a driven Chua's circuit.

2 The Chua's circuit

In this paper, phase control is applied to the Chua's circuit, which can be described by the following dimensionless equations [10]:

$$
\begin{aligned}
\dot{x} &= \alpha[y - h(x)] \\
\dot{y} &= x - y + z \\
\dot{z} &= -\beta y - \gamma z
\end{aligned} \tag{4}
$$

with $h(x) = m_1 x + 0.5(m_0 - m_1)(|x+1| - |x-1|)$. From Eqs. (4) the classical double scroll attractor shown by the Chua's circuit is obtained for the following parameters: $\alpha = 9$, $\beta = 14.286$, $\gamma = 0$, $m_0 = -1/7$ and $m_1 = 2/7$. The double scroll attractor shown by this circuit is reported in Fig. 1.

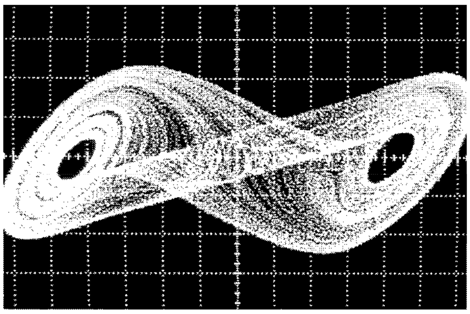

Figure 1. Projection on the plane $x - y$ of the double scroll Chua's attractor. Horizontal axis: $500mV/div$; vertical axis $200mV/div$.

Many different implementations have been proposed in literature to realize the Chua's circuit [10]. We focused on the so-called implementation based on State Controlled Cellular Nonlinear Network SC-CNN [11]. Without discussing the details of the SC-CNN implementation of Chuas circuit, we wish to briefly recall here the main ideas underlying this circuit. The SC-CNN implementation of Chuas circuit is essentially a compact and robust implementation of Chuas circuit based on operational amplifiers. For each state variables of equations (4) an algebraic summer operational amplifier and an RC filter are designed. The nonlinearity is implemented by exploiting the natural saturation of a further operational amplifier so that four operational amplifiers are needed to realize the whole circuit. In such a way an implementation where all the state variables can be easily accessible is obtained.

Although the Chua's circuit has been originally designed as an autonomous circuit, several studies have dealt with the case in which a forcing term is added to this circuit. For instance, in [12] the sinusoidal forcing is introduced by adding a new branch in the Chua's circuit (originally consisting of only two capacitors, an inductor, a passive resistor and a nonlinear resistor). The experimental investigation of this circuit carried on in [12] revealed a great variety of bifurcation sequences. In particular, period-adding bifurcations, quasi-periodicity, hysteresis and intermittent behaviors have been observed.

Another interesting result is the possibility of controlling many of these phenomena by adding a further sinusoidal generator in series with the previous one. In [13], for example, a second sinusoidal forcing with a different frequency is used. Experiments carried on increasing the amplitude of the second forcing demonstrate that a small amplitude is sufficient to induce drastic changes in the behavior of the system. In particular, starting from a chaotic behavior in absence of the second forcing, Murali and Lakshmanan demonstrate that periodic orbits (for instance of period-3) can be stabilized adding a sinusoidal term with a small amplitude. In the following we demonstrate that similar behavior can be obtained by using two sinusoidal terms at the same frequency but with different phase frequencies.

3 Phase control of the Chua's circuit

In the forced Chua's circuit considered in this paper the sinusoidal term is added to the second equation of Eqs. (4). A further sinusoidal term is then added to this equation. This "control" signal has the same frequency of the driving signal, but has a different phase. The Chua's circuit driven by a periodic forcing can be modelled by the following dimensionless equations:

$$
\begin{aligned}
\dot{x} &= \alpha[y - h(x)] \\
\dot{y} &= x - y + z + A_m sin(2\pi f_m t) - A_c sin(2\pi f_c t + \phi) \\
\dot{z} &= -\beta y
\end{aligned} \tag{5}
$$

with $h(x) = m_1 x + 0.5(m_0 - m_1)(|x+1| - |x-1|)$ and where $A_m sin(2\pi f_m t)$ represents the main driving signal and $A_c sin(2\pi f_c t + \phi)$ with $f_c = f_m$ the further harmonic perturbation used to control chaos. Our analysis has been carried out experimentally. As introduced above, the Chua's circuit has been implemented using the state variable approach described in [10], which has the advantage of an easy and flexible implementation of the forcing terms. The experimental results are discussed in the next section.

4 Experimental results

In our experiments, the amplitude of this signal is kept constant to a small value $A_c = 20mV$ (compared to $A_m = 240mV$) and the phase difference ϕ is varied. The other parameters of the system has been fixed to: $f_m = 4088Hz$, $\alpha = 7.85$, $\beta = 12.195$, $m_0 = -1/7$, and $m_1 = 2/7$. For such choice of parameter, in the absence of the main driving signal (i.e., $A_m = 0$) the system shows a chaotic behavior with a single scroll attractor. When the driving signal is applied (i.e., $A_m = 240mV$), the chaotic attractor of the system is a double scroll attractor. The addition of the small periodic control signal with zero phase does not significantly affect the circuit dynamics. However, tuning the phase difference parameter ϕ influences the dynamical behavior of the system, and there exist suit-

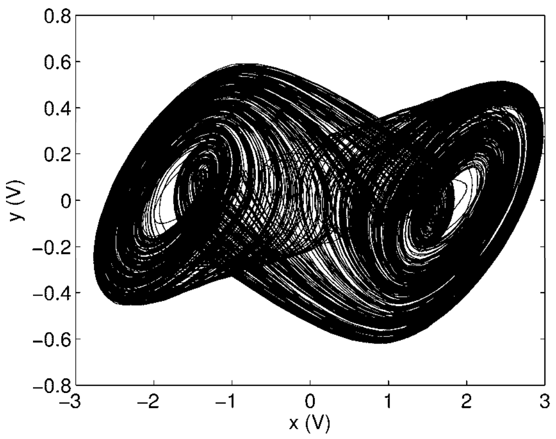

Figure 2. Projection of the chaotic attractor obtained for $\phi = 0°$ on the phase plane $x - -y$.

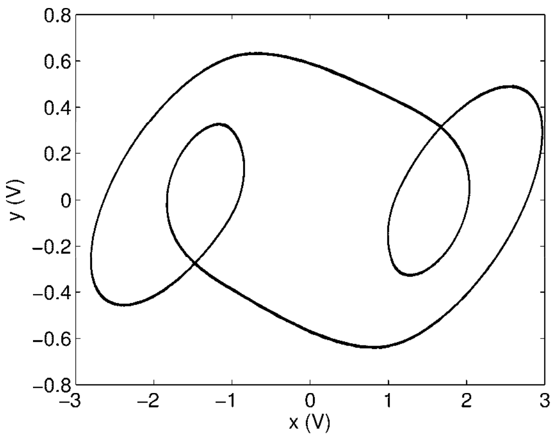

Figure 3. Projection of the chaotic attractor obtained for $\phi = 85°$ on the phase plane $x - -y$.

able values for which a stable limit cycle behavior is obtained.

Fig. 2 shows the double scroll chaotic attractor exhibited by the driven Chua's circuit (5) when the phase difference is $\phi = 0°$. Fig. 3 shows the effect of a control signal with a phase difference equal to $\phi = 85°$: a period behavior is obtained (in particular, a stable period-3 limit cycle). Finally, Fig. 4 shows the bifurcation diagram experimentally obtained. The bifurcation diagram shows the local minima of the state variable y with respect to ϕ. It can be observed that, in general, the phase parameter modulates the maximum amplitude of the state variable y, and that there exist several windows of periodic behavior, thus confirming the suitability of the phase control method to suppress chaos in a driven Chua's circuit.

5 Conclusions

We have shown the robustness and the general nature of phase control technique in the sense that the experimental implementation in the Chua's circuit, i.e., one of most investigated circuits in nonlinear dynamics, confirms the results.

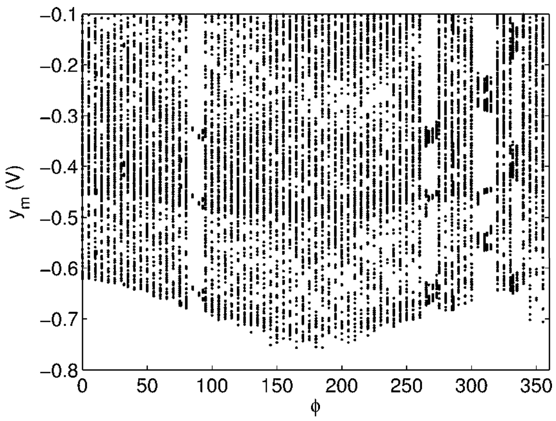

Figure 4. Experimental bifurcation diagram with respect to the parameter ϕ.

One important advantage of this scheme is its non-feedback nature. Furthermore, the key role of the phase in selecting the final dynamical state is very useful from a control point of view, since there is a large variety of situations in which the modulation of the accessible parameters might be limited, and φ is an additional degree of freedom that may be very useful.

In summary, we have shown that the phase control scheme is very versatile and useful in a wide variety of dynamical situations: to suppress local chaos, to control global dynamics as in the case of intermittency in chaotic systems close to a crisis or to avoid escapes in open dynamical systems.

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