BOUNDED ADAPTIVE STABILIZATION

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Abstract: The control is designed from the condition of decreasing Lyapunov function on the trajectories of the closed loop system. This control may be chosen a priori bounded. As example we consider the rotational system with nonlinear dissipative force and disturbances which periodically depends on the angle. We consider also the problem of evaluating unknown external periodic torque of this rotational system. The second example is adaptive stabilization of the uniform transition of the pendulum on a cart.

Keywords: nonlinear systems, parameter estimation, Lyapunov methods, stability, velocity control.

1. GENERAL SCHEME

Consider the following system

$$\dot{x} = f(x) + g(x)(u + h(x)\theta) \qquad (1)$$

where $x \in R^n$ is state vector, $u \in R^m$ is vector of control, $\theta \in R^l$ is vector of constant unknown parameters, the matrix $h(x)$ is known and bounded regressor. Suppose that there exists bounded globally stabilizing control $u = U_0(x)$ for the system $\dot{x} = f(x) + g(x)u$ and corresponding Lyapunov function $V(x)$. For globally stabilizing bounded control of eq. (1) the input $u = U_0(x) - h(x)F(\hat{\theta})$ and differential equation of adaptation

$$d\hat{\theta}/dt = \text{diag}\{\gamma_i \left(\frac{\partial F_i}{\partial \hat{\theta} i}\right)^{-1}\}h^{\top}(x)\frac{\partial V}{\partial x}g(x)$$

where the functions $F_i(\tau)$ are continuous, strictly increasing and θ_i belongs the range of F_i, $\gamma_i > 0$ are proposed. This adaptive scheme was synthesized by using Lyapunov function

$$V_a = V + \sum_1^n \frac{(F_i(\hat{\theta}_i) - \theta_i)^2}{2\gamma_i}$$

The novelty of our approach in comparison with Parks, 1966 and Fradkov e.a., 1999 is in second term of Lyapunov function V_a. This modification allows to synthesize a priori bounded control.

The proposed general scheme gives the solution of bounded stabilization of two specific systems: 1) combined alternator-starter 2) pendulum and cart

2. ADAPTIVE STABILIZATION WITH DISTURBANCE ESTIMATION

Consider the following system

$$J\dot{\omega} + K(\omega) = u + T_e, \qquad \dot{\varphi} = \omega \qquad (2)$$

where u is the control input, $J > 0$ is a moment of inertia, $K(\omega)$ is a dissipative force, φ is the angle of device, and T_e is the external periodic disturbance

$$T_e(\varphi) = T_0 + \sum_{i=1}^{N}[a_i \cos(i\varphi) + b_i \sin(i\varphi)] \qquad (3)$$

Here T_0 is a constant component of torque, N is a number of harmonics we account for. Assume that $K(\omega)$ is a continuous strictly increasing function vanishing at the origin. Typically $K(\omega) = k_v\omega + sign(\omega)c_x\omega^2/2$, where k_v is a coefficient of viscous damping and c_x is an aerodinamical coefficient. The problem arises in mechanical systems with eccentricity, in particular, in drives with magnetic bearings and gear boxes (Canudas and Praly, 2000). The goal of the controller is to track given speed reference $\omega_d = const$

$$\lim_{t\to\infty} \omega = \omega_d. \qquad (4)$$

and reject periodic disturbance $T_e(t)$.

Let us discuss in more details the motivation from engine control. The periodic fuel combustion processes and nonlinear engine geometry result in engine crankshaft speed oscillations. Usually a passive flywheel is used for reducing the pulsation of crankshaft speed. There exists an idea of using a reversible alternator for engine speed damping to improve vehicle driveline performance. Another potentiality of proposed system is active engine idle speed regulation.

The using of the supplemental torque source for control improvement of the engine idle speed and reducing of crankshaft speed pulsations have been investigated actively in recent years (Gusev e.a.,1996; Zaremba e.a., 1998; Burkov, 2001).

The dynamic model of the active flywheel system can be represented in the state-space form by eq. (2) and engine torque T_e which has the following form (Rizzoni, 1989)

$$T_e(\varphi) = T_0 + \sum_{i=1}^{N}[a_i \cos(mi\varphi) + b_i \sin(mi\varphi)] \quad (5)$$

Here T_0 is a dc component of engine torque, and m is a number of strokes (firings) during one crankshaft rotation ($m = 2$ for a four-stroke cycle four cylinder engine). The signal $\omega_d(t)$ can be a given engine speed (e.g., engine idle speed) . Consider the problem of idle speed regulation together with crankshaft speed pulsation attenuation.

In most previous work concerning periodic disturbances the authors considered the periodic disturbance as periodic function of time (Bodson e.a., 1994). But in many cases the assumption of dependance of T_e on angle is more realistic. In present paper we develop and generalize the results of Zaremba e.a.,1998; Burkov, 2001.

The model (2) describes also controlled rotation of a pendulum or a disbalanced horizontal rotor.

The following adaptive control is proposed

$$u = -\Phi(\omega - \omega_d)-$$

$$\sum_{i=1}^{N}[F_{a_i}(\hat{a}_i) \cos(i\varphi) + F_{b_i}(\hat{b}_i) \sin(i\varphi)] - F_{a_0}(\hat{a}_0) \quad (6)$$

$$\frac{d\hat{a}_0}{dt} = \alpha_0 (F'_{a_0})^{-1}(\omega - \omega_d),$$

$$\frac{d\hat{a}_i}{dt} = \alpha_i (F'_{a_i})^{-1} \cos(i\varphi)(\omega - \omega_d), \quad (7)$$

$$\frac{d\hat{b}_i}{dt} = \beta_i (F'_{b_i})^{-1} \sin(i\varphi)(\omega - \omega_d)$$

$$i = 1, \dots, N$$

where the functions F_{a_i}, F_{b_i} are differentiable and strictly increasing, the function Φ is continuous strictly increasing and vanishing at the zero, α_i, β_i are constant positive gain coefficients. Assume that real values a_i, $i = 1, ..., N$ belong to the ranges of functions F_{a_i}, b_i belongs to the ranges of F_{b_i} and $T_0 - K(\omega_d)$ belongs to the range of F_{a_0}. Note, that if these functions are chosen bounded (f.e., arctan) then the control u will be bounded. The differential equations (7) have to be solved during the process of regulation. The closed-loop system may be represent as

$$J\frac{\tilde{\omega}}{dt} + K(\omega) - K(\omega_d) =$$

$$-\Phi(\omega - \omega_d) - \sum_{i=1}^{N}[\tilde{a}_i \cos(i\varphi) + \tilde{b}_i \sin(i\varphi)] - \tilde{a}_0 \quad (8)$$

where $\tilde{\omega} = \omega - \omega_d, \tilde{a}_0 = F_{a_0}(\hat{a}_0) + K(\omega_d) - T_0, \tilde{a}_i = F_{a_i}(\hat{a}_i) - a_i, \tilde{b}_i = F_{b_i}(\hat{b}_i) - b_i, \quad i = 1, ..., N$

Proposition. For constant desired motion ω_d the closed-loop system (2),(3),(6),(7) is globally asymptotically stable with respect to $\tilde{\omega} = \omega - \omega_d, \tilde{\alpha}_i, \tilde{\beta}$.

Proof. Consider the following Lyapunov function

$$V = \frac{1}{2}[J\tilde{\omega}^2 + \tilde{a}_0^2/\alpha_0 + \sum_{i=1}^{N}(\tilde{a}_i^2/\alpha_i + \tilde{b}_i^2/\beta_i)] \quad (9)$$

Differentiating along the trajectories of the closed-loop system and substituting equations (2),(3),(6),(7) results in the following formula

$$\dot{V} = -\tilde{\omega}[\Phi(\tilde{\omega}) + K(\omega) - K(\omega_d)]$$

According to the Rumyantsev-Oziraner theorem (see paragraph 19 of their book let us analyze the closed-loop system under the condition $\dot{V} = 0$. As $\omega \to \omega_d$ then $\varphi \to \omega_d t + \varphi_0$. So there exist angles $\varphi_1, ..., \varphi_{2N+1}$ which are inequal in pairs. The condition $\tilde{\omega} = 0$ implies that the following equation holds

$$0 = \sum_{i=1}^{N}[\tilde{a}_i \cos(i\varphi) + \tilde{b}_i \sin(i\varphi)] + \tilde{a}_0 \quad (10)$$

From the theory of trigonometric polynoms we get $\tilde{a}_i = 0, \tilde{b}_i = 0$. The Lyapunov function V is not radially unbounded, nevertheless the surfaces of its levels are unbounded, so the attraction is global.

Modelling. In Fig. 1,2,3 there are the graphs of the angular velocity $\omega(t)$, and the compensation terms $F_{b2}(\hat{b}_2), F_{a0}(\hat{a}_0)$ for the following parameters $J = 1, k_x = 0.5, c_x = 0, T_0 = 1.2, \omega_d = 2$ $b_1 = 0, b_2 = 1, a_1 = 0, a_2 = 0$ $\Phi = F_{a0} = F_{b2} = \arctan$, and initial data $\varphi(0) = 0, \omega(0) = 1.5, \hat{a}_0(0) = 3, \hat{b}_2(0) = 1.3$.

ESTIMATION WITHOUT CONTROL

Let $u = 0$. In this section assume that each trajectory of uncontrolled system is nontrivial in the folowing sense: the angle φ become equal to $\varphi_1, ..., \varphi_{2N+1}$ which are inequal in pairs. Consider the following equations of estimation

$$J d\breve{\omega}/dt + K(\breve{\omega}) =$$

$$\breve{T}_0 + \sum_{i=1}^{N} [\breve{a}_i \cos(i\varphi) + \breve{b}_i \sin(i\varphi)] \qquad (11)$$

where $\breve{\omega}$ is the estimate of the angular velocity

$$d\breve{T}_0/dt = \alpha_0(\omega - \hat{\omega}),$$

$$d\breve{a}_i/dt = \alpha_i \cos(i\varphi)(\omega - \hat{\omega}), \qquad (12)$$

$$d\breve{b}_i/dt = \beta_i \sin(i\varphi)(\omega - \breve{\omega})$$

$$i = 1, \ldots, N$$

and $\breve{a}_i, \breve{b}_i, \breve{T}_0$ are estimates of Fourier coefficients in (2).

Proposition. In the closed loop system (2),(3), (11),(12) the estimates $\breve{\omega}, \breve{T}_0, \breve{a}_i, \breve{b}_i$ globally converge to its real values when time tends to infinity.

Proof. The closed-loop system may be represented as

$$J d\bar{\omega}/dt + K(\breve{\omega}) - K(\omega) =$$

$$\sum_{i=1}^{N} [\bar{a}_i \cos(i\varphi) + \bar{b}_i \sin(i\varphi)] + \bar{T}_0$$

where $\bar{\omega} = \breve{\omega} - \omega, \bar{a}_i = \breve{a}_i - a_i, \bar{b}_i = \breve{b}_i - b_i, \bar{T}_0 = \breve{T}_0 - T_0$

Consider the following Lyapunov function

$$V_e = \frac{1}{2}[J(\breve{\omega} - \omega)^2 + \bar{T}_0^2/\alpha_0 +$$

$$\sum_{i=1}^{N} (\bar{a}_i^2/\alpha_i + \bar{b}_i^2/\beta_i)]$$

Differentiating along the trajectories of the closed-loop system and substituting equations (7),(6), (11),(12) results in the following formula

$$\dot{V}_e = -(\breve{\omega} - \omega)[K(\breve{\omega}) - K(\omega)]$$

According to Rumyantsev-Oziraner theorem the trajectories converge to the set $\dot{V}_e = 0$ which is trivial one. This fact may be demonstrated similarly to the proof of first theorem.

PENDULUM ON CART

The planar pendulum on a cart is well-known physical device (see, f.e., Mazenc and Bowong, 2003).

Its dynamics obtained by Lagrange formulation are

$$(M + m)\dot{v} + ml\cos(\theta - \gamma)\dot{\omega} - ml\sin(\theta - \gamma)\omega^2 +$$

$$g\sin\gamma(M + m) = f, \quad \dot{z} = v,$$

$$(13)$$

$$\cos(\theta - \gamma)\dot{v} + l\dot{\omega} + g\sin\theta = 0, \quad \dot{\theta} = \omega$$

where (M, z) are mass and position of the cart moving along a strait line, which has an angle γ with horizontal line, (m, l, θ) are mass, length and angular deviation from the downward vertical position for the pendulum which is pivoting around a point fixed on the cart, f is a control force acting on the cart, g is a gravitational constant, see Fig.4. The system has the kinetic energy

$$K = \frac{1}{2}(M + m)v^2 + mlv\omega\cos(\theta - \gamma) + \frac{1}{2}ml^2\omega^2$$

the potential energy

$$\Pi = \Pi_z + \Pi_\theta,$$

with $\Pi_z = Mgz\sin\gamma + mgz\sin\gamma, \Pi_\theta = -gml\cos\theta$ and the kinetic moment

$$J = (M + m)v + ml\omega\cos(\theta - \gamma)$$

The pendulum on the cart is a simple model for overhead crane moving the load (d'Andrea-Novel and Coron, 2000; Burkov, 2005). If the angle $\gamma = \pi/2$ than our model describes lifting a load by the rope of a crane.

Let $v = v_d, \theta = 0, \omega = 0$ be the desired motion, the value $G = g\sin\gamma(M + m)$ in unknown .

By means of the Lyapunov function mentioned in sect. 1 we obtain the stabilizing control

$$f = F_a(\hat{G}) - F_v(v - v_d) \qquad (14)$$

$$d\hat{G}/dt = -\nu(\frac{\partial F_a}{\partial \hat{G}})^{-1}(v - v_d)$$

where $\hat{G}$ is tuning variable, $\nu > 0$ and the functions F_a, F_v have the same properties as the function F from sect. 1.

Proposition . The closed loop system (13), (14) is asymptotically stable with respect to the variables $v - v_d, \theta, \omega, F_a(\hat{G}) - G$.

Proof. Consider Lyapunov function

$$V = K + \Pi_\theta - v_d J + \frac{1}{2\nu}(F_a(\hat{G}) - G)$$

and investigate the set of trajectories under the condition $\dot{V} = 0$.

Modelling. In Fig. 5,6,7 there are the graphs of the velocity $v(t)$, the angle $\theta(t)$ and the compensation term $F_a(\hat{G})$ for the following parameters

$M = m = l = g = v_d = 1$, $F_a(\tau) = 3 \arctan \tau$, $F_v = \arctan, \gamma = \pi/2$ and initial data $z(0) = 0, v(0) = 1.5, \theta(0) = 1.3, \omega(0) = 0, \hat{G}(0) = 3$.

CONCLUSION

Our generalization of speed gradient adaptive control (Fradkov e.a., 1999) allows to design apriori bounded stabilization input. Some questions are open in considered problems; for example, robustness of stabilization with respect to noise in measurements and external deterministic or stochastic disturbances. It is necessary to investigate the domain of attractions. This is the theme for future research.

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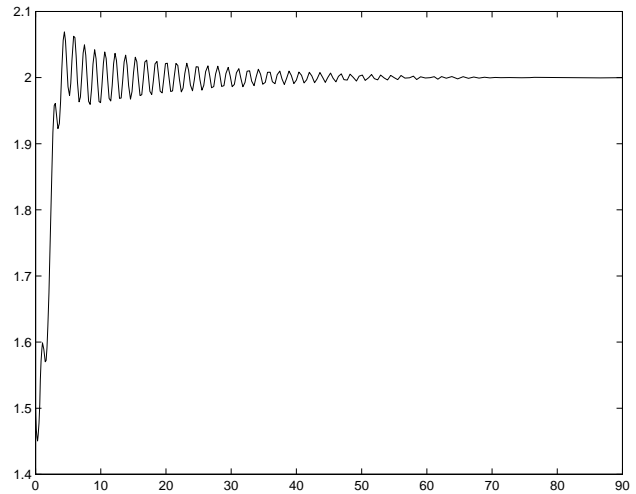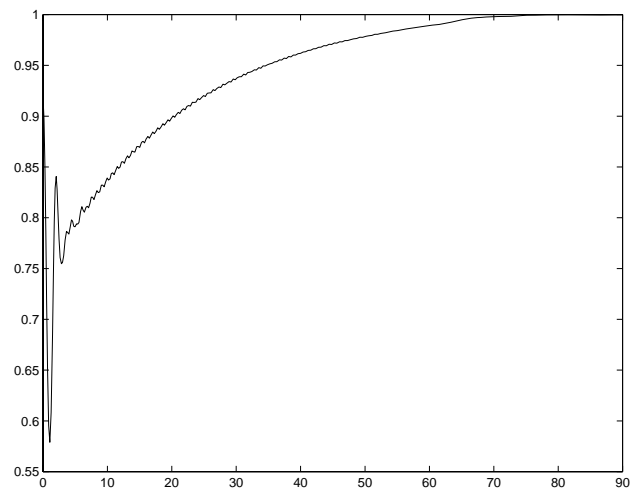

Figure 1: Angular velocity ω

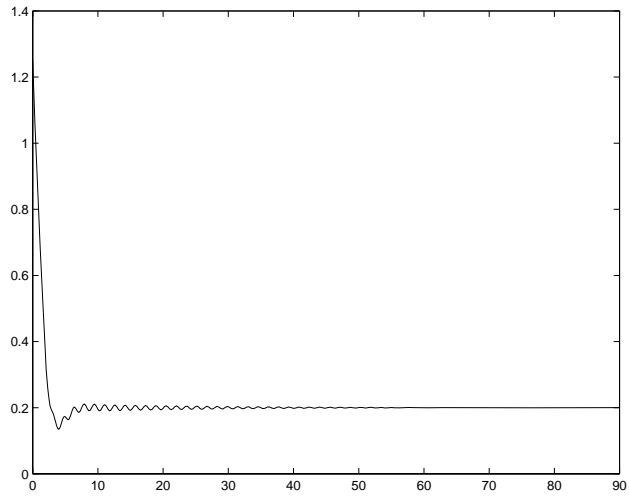Figure 2: Compensation term $F_{b2}(\hat{b})$

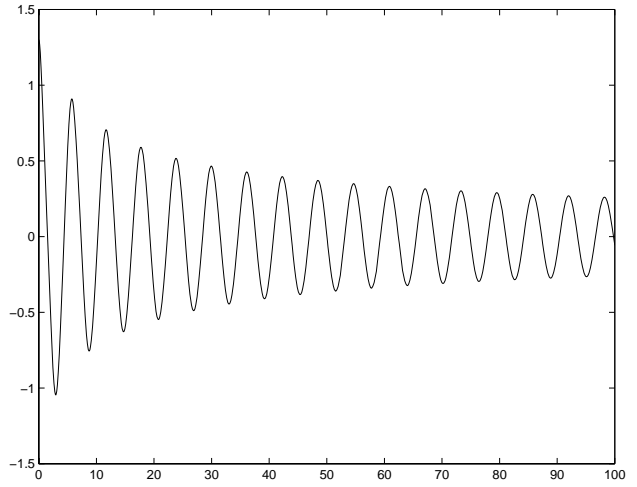Figure 3: Compensation term $F_{a0}(\hat{a})$

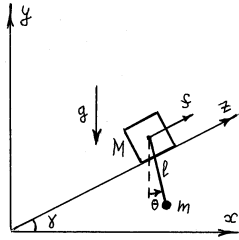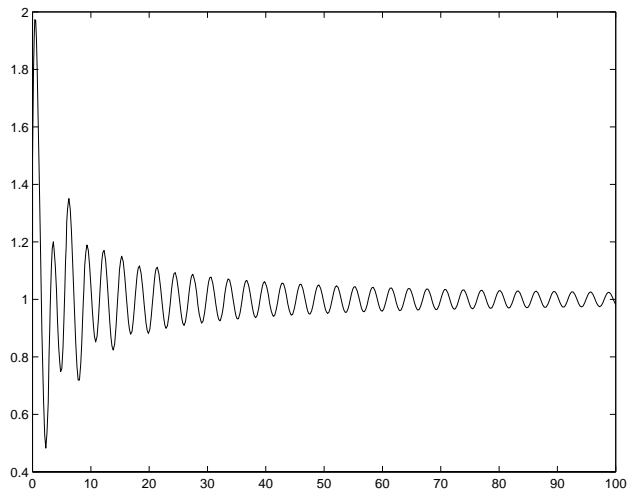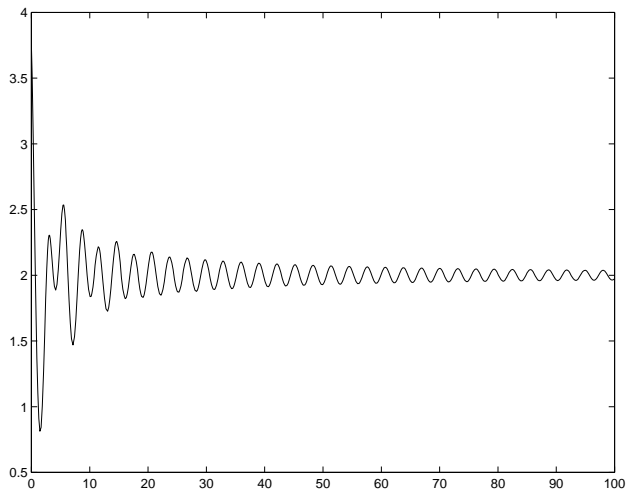Figure 6: Angle $\theta(t)$

Figure 4: Cart and pendulum

Figure 5: Velocity $v(t)$

Figure 7: Compensation term $F_a(\hat{G})$