A SEMI-EMPIRICAL FLUID FORCE MODEL FOR VORTEX-INDUCED VIBRATION OF AN ELASTIC STRUCTURE

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Abstract
A new semi-empirical model is presented for the vortex-induced vibration of structures. The lift force on a structure is assumed to consist of two components. The first component is a non-linear force that has a polynomial dependence on the velocity of the structure. The second component is a harmonic force with the Strouhal frequency. Only the crossflow motion of the structure is considered. The model predictions are compared with experimental results available in literature to show good qualitative agreement.

Key words
vortex-induced vibration.

1 Introduction

Many flexible engineering structures, such as marine cables towing instruments, flexible risers used in petroleum production and mooring lines are prone to Vortex Induced Vibration (VIV), see [Blevins 1977, Iwan 1981, Alexander 1981, Ramberg&Griffin 1976, Vandiver 1991, Newman&Karniadakis 1996]. The VIV can have considerable amplitude and significantly accelerate fatigue of the structures.
If the structure (or its segments) vibrates at nearly the forcing frequency, the structure and the wake will be in the state of "synchronization" or lock-in as termed in the classical work of Bishop and Hassan 1964. This state occurs in a narrow band of cylinder oscillation frequencies which includes the Strouhal frequency. About three decades ago, several investigators began employing nonlinear oscillator equations of the van der Pol type to represent the fluctuating lift force acting on the cylinder [Hartlen&Currie 1970, Skop&Grifffin 1975, Iwan&Blevins 1974]. The modeling was continued in [Skop&Balasubramanian 1997, Wang et al. 2003]. Their representation for the lift force was based on the similarity between the vortex-shedding process and the behavior of nonlinear oscillators rather than on the underlying fluid dynamics. For other types of the modeling of VIV process see a review of [Gabbai& Benaroya 2005].
Many fundamental features of VIV of an elastic cylinder are still not fully understood. For example, what is the maximum possible amplitude attainable for a cylinder undergoing VIV, for conditions of extremely small mass and damping? What modes of structural response exist, and how does the system jump between the different modes? The objective of this paper is to answer the above questions using an analytical approach and simple numerical computations with the help of a new semi-empirical model of the lift force. In this model, the lift force on the structure resulting from the shedding process consists of two components. The first component is a force that depends non-linearly on the velocity of the structure across the flow. The second component is a harmonic force with the vortex-shedding frequency. Only the cross-flow vibration of the structure is considered.

2 A fluid force model

Consider a flexible, circular cylindrical structure subjected to a uniform cross-flow of velocity V_0 and mass density ρ .

2.1. Formulation

For long structures where tension dominates bending, the PDE governing geometrically linear transversal vibrations can be written in the form:

$$\partial_t (M \partial_t u_y) + \varsigma \partial_t u_y = T_0 \, \partial_{xx} u_y + F, \qquad (1)$$

where $M = m_c + m_{add}$ is the mass per unit length of the structure including the added fluid mass, m_{add} is the added mass per unit length, m_c is the mass of the structure per unit length, u_y is the

transversal displacement, ς is the structural damping , T_0 is the tension in the structure (assumed to be constant), F is the fluid force.
It is proposed to represent the fluid force as follows:

$$F = 0.5c_n\rho dV_0^2(-b_1\partial_t u_y / V_0 + b_3(\partial_t u_y / V_0)^3 - b_5(\partial_t u_y / V_0)^5 + f_0 \sin(\omega_s t + \beta)), \quad (2)$$

where d is the diameter of the structure, c_n ,b_1, b_3, b_5 are coefficients to be found from experiments, f_0 is the amplitude of the periodic part of the force, ω_s is the Strouhal angular frequency; β is the phase of the periodic force**.** Eq. (1) should be supplemented with the boundary conditions. In the model, the force F acting on the structure in the cross flow direction as a result of the shedding process consists of two components. The first component is a force depending non-linearly on the velocity of the structural motion in the crossflow direction. The force is such, that at small velocities it adds to the hydrodynamic resistance force and is directed opposite to the velocity, this insures the structural stability. With the velocity rise the force becomes codirected with the velocity, which leads to the growth of the vibration amplitude. But with the further velocity rise the force becomes again directed opposite to the velocity, which limits the vibration amplitude. Such a nature of the force dependence on the cable velocity is proved experimentally for marine cables [Devnin 1975]. With the flow velocity rise the frequency of the force grows, and when it becomes close to the first natural frequency of the structural vibration, the intensive eigenmode vibration is exited. The non-linear force/velocity dependence leads to the limitation of the oscillation amplitude. The structure gets into a self-oscillation regime with the mode close to the eigenmode. With further increase of the flow velocity, the force frequency continues to grow, and when it is approaching to the second natural frequency, the intensive vibration is exited at the second eigenmode. The self-oscillation regime with the mode close to the first eigenmode ends and the structure gets into the self-oscillation regime with the mode close to the second eigenmode. With the further increase of the velocity the jump to the third natural frequency occurs and, correspondently, to the third eigenmode.

In Eq.(2) the non-linear component of the force differs from the model of classical forced Rayleigh-type oscillator by the additional term $(\partial_t u_y / V_0)^5$ and the signs in front of the linear and the first non-linear terms. The term $(\partial_t u_y / V_0)^5$ allows describing the stabilization of the structural vibration when the vibrating structure velocity rises.
The second component of the lift force is a harmonic force with the vortex-shedding frequency. It reflects the following experimental result: at small values of the oscillation amplitudes the lift coefficient varies according to the sinusoidal law [Devnin 1975]. Inserting Eq. (2) into Eq.(1), one obtains:

$$\partial_t((m_c + m_{add})\partial_t u_y) + \varsigma\partial_t u_y = T_0\partial_{xx}u_y + 0.5c_n\rho dV_0^2(-b_1\partial_t u_y / V_0 + b_3(\partial_t u_y / V_0)^3 - b_5(\partial_t u_y / V_0)^5 + f_0 \sin(\omega_s t + \beta)); \quad (3)$$

Eq. (3) allows to take into account the possible variation of the added mass of the fluid in time according to the variation of reduced velocity $V_r = \frac{V_0}{d\nu}$,where ν is natural frequency of the structure. The dependence of added mass on time and on the reduced velocity for small mass ratio cylinders was studied experimentally in [Sarpkaya 1979,Vikestad at al.2000]. The obtained results show that there are significant cycle-to cycle variations in added mass and vibration period . The added mass dependence on time was observed for $V_r = 4$. Once the lock-in commences, at approximately $V_r = 4$, then the natural frequency of the cylinder increases with increasing reduced velocity, enabling lock-in to persist, in this case up to approximately $V_r = 12$, because the mass ratio $\mu = m_c / \rho d^2 L$ was quite low ($\mu = 3$, L is the length of the cylinder).On the basis of results of [Vikestad at al.2000], in the present paper the behaviour of the system is described in the following way. The model of the system behaviour in the upper and lower branches of the lock-in range takes into account the dependence of the added mass on the reduced velocity and outside that regions will not. It was also shown by [Vikestad at al.2000] that the added mass is influenced by components of the cylinder displacement at frequencies which are different from the natural vortex-induced vibration response. This fact is not taken into account in this paper, but will be shortly discussed in the conclusion. Analytical research of the elastic structure dynamics described by Eq. (3) has not been performed in the past. Such a research can be carried out by perturbations methods and averaging techniques. The present paper presents preliminary results of such a research. Let us first consider the structural behaviour when the added mass is assumed to be constant. Then, Eq. (3) can be reduced to the equation for a single degree of freedom (SDOF) oscillator. SDOF models use a single ordinary differential equation to describe the behaviour of the structural oscillator (see for references [Gabbai& Benaroya 2005]. Such a model can be obtained for elastic systems that could be described by the one dimensional damped wave equation (e.g. wires, cables). The results obtained by [Skop&Grifffin 1975] indicate that for a particular pure response mode, the equations describing the system response reduced to those obtained for a rigid cylinder. Using Galerkin procedure, and seeking the solution of Eq.(4) in the form $u_y = \sum_{i=0}^{\infty} y_i(t) Z_i(x)$,one can come to the solution of the infinite system of ODES for determining time dependent coefficients $y_i(t)$. Neglecting interaction of modes, which is of course, the strong assumption, and dividing both parts of Eq. (3) by η, one has the following equation for i-th mode:

$$\frac{d^2 y_i}{dt^2} + \eta \frac{dy_i}{dt} + \Omega_i^2 y_i = F(\dot{y}_i, t) \qquad (4)$$

where η is the damping coefficient, y_i is the modal coefficient of the transverse displacement of the elastic system, $F(\dot{y}_i, t)$ is the hydroroelastic forcing function, Ω_i is the natural circular frequency of the i-th mode. In Eq. (3), the expression for the added mass is as follows: $m_{ad} = \chi \pi d^2 / 4$; χ is the coefficient depending on the structure design.For values of frequencies considered in the paper the added mass may be assumed independent on frequency[D.J.Crighton 1983].

2.2. Solution of the structure motion equation

Although cables and wires very often are made of synthetic fibers, for example Capron and Kevlar, some cables are made of stainless steel. Since the density of the fluid is smaller then the density of the structure material it is natural to introduce there ratio as a small parameter ε; then the right hand side of Eq. (4) can be represented as:

$$F(\dot{y}, t) = \varepsilon f(\dot{y}, t)$$

where ε is a small parameter. Since the lift coefficient frequency does not coincide with the natural frequency of the structure, the approximation of the solution of Eq. (4) will be constructed in the form:

$$y = A(\tau)\sin(\omega t + \psi(\tau)) \qquad (5)$$

where A is the displacement amplitude, ψ is the displacement phase, ω is the circular frequency of the structural vibration. In Eq.(5) the amplitude and the phase are time functions slowly changing. In Eq. (5) the slow time τ=εt is introduced. Then, the standard averaging procedure is used. One can get, up to the order of $O(\varepsilon^2)$:

$$\begin{cases} -\omega A \psi_\tau = \dfrac{1}{T}\displaystyle\int_0^T g(\dot{y}, t)\sin(\omega t + \psi)\,dt \\ \omega A_\tau = \dfrac{1}{T}\displaystyle\int_0^T g(\dot{y}, t)\cos(\omega t + \psi)\,dt \end{cases} \qquad (6)$$

Where :

$g(\dot{y}, t) = R(\dot{y}) + F_0$;

$R(\dot{y}) = -b_1 \dot{y} + b_3 \dot{y}^3 - b_5 \dot{y}^5$;

$F_0 = f_0 \sin(\omega_s t + \beta)$.

At $A > 0$ the phase ψ is changing so, that either $\psi \to \infty$ or, if the contribution of F_0 has roots $\psi_0, \psi_1, \ldots \psi_\infty$, then $\psi \to \psi_\infty (\tau \to \infty)$. It means that the equilibrium values of ψ_∞ are the roots of the contribution from F_0. These roots can be shown to satisfy the following equation:

$\frac{1}{T(\omega_s + \omega)}\cos[\frac{(\omega_s + \omega)T}{2} + \beta + \psi)]\sin\frac{(\omega_s + \omega)T}{2} + \frac{1}{\Delta\omega T}\sin\frac{\Delta\omega T}{2}\cos(\frac{\Delta\omega T}{2} + \beta - \psi)$	(7)

where $\Delta\omega = \omega_s - \omega$

Let us consider the case when $\omega = \omega_s + \varepsilon\alpha$, it is so called the lock-in region, its lower branch if α>0. Then the system of Eqs. (6) with the accuracy $O(\varepsilon^2)$ becomes:

$$\begin{cases} 2\bar{A}\theta_\tau = 2\bar{A}\alpha + \bar{f}_0 \cos\theta \\ -\bar{A}_\tau = R(\bar{A}) + \bar{f}_0 \sin\theta \end{cases} \qquad (8)$$

where $\theta = \beta - \psi + \alpha\varepsilon t = \beta - \psi + \alpha\tau$.

Let us now consider the behavior of the system when ω is closed to ω_s and detuning parameter α is small. For the steady state we have:

$$R(\bar{A}) \pm \bar{f}_0 \sqrt{1 - 4\alpha^2 \frac{\bar{A}^2}{\bar{f}_0^2}} = 0$$
$$\cos\theta = -2\alpha \frac{\bar{A}}{\bar{f}_0} \qquad (9)$$

Let us determine the types of the critical points which satisfy the steady state conditions (9). In the vicinity of $\theta_0 = \frac{\pi}{2}$ $R(\bar{A}_0) = \pm \bar{f}_0$ one can get after standard manipulations the following eigenvalues of Jacobian (when α→0):

$$\lambda_1 = \frac{(-1)^k f_0}{2\omega A_0} \qquad \lambda_2 = -\frac{1}{\omega} R'(A_0)$$

If $\lambda_1, \lambda_2 > 0$ then we have unstable node; if $\lambda_1, \lambda_2 < 0$, then the node is stable, and if $\lambda_1 < 0,\ \lambda_2 > 0$ or $\lambda_1 > 0,\ \lambda_2 < 0$ one has saddle. If α is not small but f_0 is small then the first equation of Eqs. (6) can be reduced to the equation: $\theta_\tau = \alpha$ and from this follows $\theta = \alpha\tau$ and $-\omega A_\tau = R(A) + f_0 \sin\alpha\tau$, then one has small oscillations near the equilibrium point. The solution of the equation for finding the dimensionless amplitude of the structure is shown on the fig1.

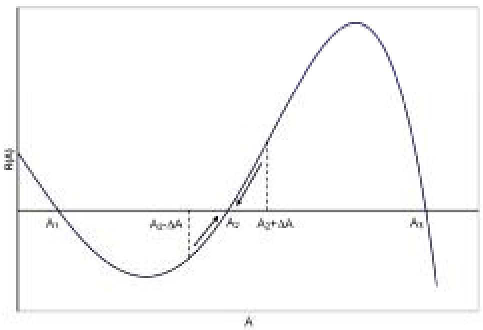

Figure 1. The solution of the equation for finding the dimensionless amplitude of the structure

It is seen that depending on the value of the force amplitude $\overline{f}_0$ it is possible to have one, two or three values of the resonant displacement amplitudes. The result of calculations is shown according the suggested in the present paper model is shown in Fig.2. As can be seen from Fig.2 the suggested model in the lower branch of the lock-in region has a satisfactory agreement with the experimental data from [Ramberg&Griffin 1976].

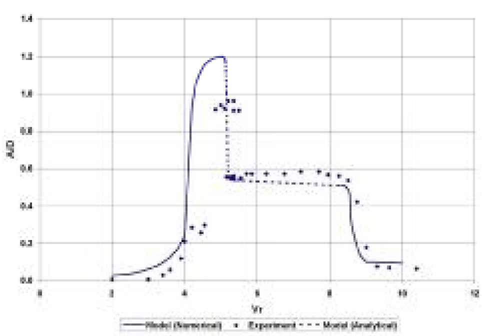

Figure 2. Comparison with experiment

2.3. Effect of variation of an added mass with the reduced velocity

The mean value of the added mass over one period of oscillations as a function of the reduced velocity was found in [Vikestad *at.al.* 2000]. Using that data it is possible to find the frequency of vibrations in the lock–in region for its lower branch more accurately. Variation of the mass as a function of the reduced velocity on the basis of data from [Vikestad *at.al.* 2000] can be approximated by the following expression:

$$m_{add} = m_0 e^{-aV_r} - 1 \qquad (10)$$

where m_0 = 14.135 and a=0.331.

If we substitute Eq.(9) into Eq.(3) then one can get that not only the natural frequency of the structure but also the structural damping is influenced by the variation of the added mass and the coefficients of the right hand side of the Eq.(4) as well. If we take into account the variation of the mass according to Eq. (10) then all formulas obtained in 2.2 are still valid.

4 An example

The results obtained in the presented work are compared in this section with experimental results available in literature [Devnin 1975]. Coefficients b_1, b_3, b_5 in the Eq. (3) can be determined by using the measured accelerations and the structure displacement amplitudes following the technique given in [Devnin 1975]. Measurements given in [Devnin 1975] have shown that for calculations of elastic cylinder oscillations or oscillations of some types of cables the same b_i coefficients can be used for particular parameters of the fluid flow and structures. The cylinders (straight and bent) used in that experiments were from 1 to 5m length and have diameters from 0.1-0.4 m. Maximum bend cylinder deflection was in the range of 0.23-0.66 m, Re=6000-50000 ; the Strouhal number corresponded $St = St \sin\alpha$, (where α is the angle of attack), was practically constant (for bent cylinders) when attack angles were smaller then 50°, and is equal to *St*=0.19 for $\mathrm{Re} < \mathrm{Re}_{critical}$ .Those experiments have been performed for flows with the values of velocities V_0=0.1-3m/s. For determination of the lift force parameters the following method was suggested. The lift force as was mentioned above consists of two non-correlated components. The first one can be estimated from the data obtained from the tests with fixed cylinder. The value of the second component, caused by the changing of the type of vortex shedding, can be found using the method suggested in [Devnin 1975] on the basis of knowledge of the dispersion of the cylinder oscillation amplitudes and spectrum, for different Reynolds numbers and for different ranges of the cylinder length and diameters. For the given range of the flow and structure parameters the above mentioned $b_i, \overline{f}_0, c_n$ coefficients can be taken as follows for an elastically mounted cylinder:

$$\begin{array}{lll} b_1 = 0.17 & b_3 = 1.4 & b_5 = 0.7 \\ \overline{f}_0 = 0.6 & c_n = 1.2 & St = 0.19 \end{array} \qquad (11)$$

The coefficient $\overline{f}_0$ depends on the type of structure (elastic cylinder, cable, wire) and its value is varying in the range 0.05-0.6. In the presented model the value of the mass-damping parameter $m\xi$ =0.013 and μ=8.72:

$$m = m_c / m_d, \quad m_d = \pi \rho_0 d^2 L/4, \quad \xi = \frac{\varsigma}{2(\sigma(m_c + m_{add}))^{0.5}}$$

where σ is the system stiffness.

Those values were taken close to those which were used in [A.Khalak& C.H.K.Williamson 1997]. For the lower branch of the lock-in region, when V_r=5.5-8.5, the dependence of the added mass on the reduced velocity was taken into consideration. As can be seen from Fig.2 the suggested model in the lower branch of the lock-in region has a satisfactory agreement with the experimental data from [R.H.J.Willden and J.M.R.Graham 2001]. The resonance value of amplitude is 20% higher then was found in the experiment. The computer simulation for the V_r=2-4.5 and V_r=8.5-10 was conducted according to Eq. (4) with the help of the Maple 9.5 and show a good agreement with the experimental data from [R.H.J.Willden and J.M.R.Graham 2001].

4 Conclusion and discussion

The proposed in this paper semi-empirical model of the fluid force allows highlighting some peculiarities of the VIV of the bluff bodies. The maximum amplitude value of the structure vibrating in the flow for the parameters range indicated in the part 3 has been estimated and found to be 20% higher then the experimental one. The amplitude of vibrations in the resonance range depends on the amplitude of the forcing term. When the amplitude of the forcing term reaches a particular value and the phase difference between the structure displacement and the periodic component of the lift force is equal to $\pi/2$ then the stable oscillations became impossible even if the condition $\frac{\omega}{\omega_s} = 1$ is fullfield. It is because the flow energy that the structure gets at some flow velocity cannot be accumulated on that one mode. As a result the system jumps between the different modes.

There are still however many open problems for future research which should be solved. Some of them are as follows. The case when the added mass is a varying with time function should be investigated, for example. The equation of the structural motion in this case is as follows:

$$(m + m_{add})\ddot{y} + \dot{m}_{add}\dot{y} + \varsigma\dot{y} + cy = F(\dot{y}, t) \qquad (12)$$

This equation is different from the equation which was used in [R.H.J.Willden and J.M.R.Graham 2001]. In Eq. (12) the additional term $\dot{m}\dot{y}$ is introduced. If the added mass is time-varying then in addition to the change of the oscillation frequency it can lead to the amplitude growth and instability of the structure. For the case when the forcing and structural damping terms vanish the exact solution of Eq. (12) can be obtained if the dependence of the added mass on time is a smooth function. In that case the solution of Eq. (12) can be found in the closed form [H.J.Holl,A.K.Belyaev,H.Irschik 1999].For example, for an exponential variation of the mass the authors of shows that when time increases, the oscillation amplitude grows exponentially. Then a solution for the nonhomogeneous equation can be obtained with the help of the method of variation of constants. That solution will be also unbounded. Then the only possible reasons for limitation of the amplitude can be a nonlinearity of the fluid force or a large structural damping. Using the equation of the form:

$(m + m_{add})\ddot{y} + \dot{m}_{add}\dot{y} + \varsigma\dot{y} + \Omega^2 y = F(\dot{y}, t)$ and dividing both parts of it by $m + m_{add}$, one can get the equation which will have time-varying coefficients. For the case when m_{add} vary with time according to the harmonic law or as a piece-wise-constant function the solutions of the linear equations with variable in time coefficients were obtained in [A.H.P. van der Burgh,Hartono,A.K.Abramian 2006]. Also the solution of the non-linear equation with variable in time coefficients and polynomial non-linearity with respect to the first derivative of the structure displacement of the power of three was found as well. The results obtained in [A.H.P. van der Burgh,Hartono,A.K.Abramian 2006] show that solutions and regions of the stability depend on the parameters of the system. In stead of the exponentially decaying mass in the mentioned above cases the stable oscillations are possible for the particular parameters of the system under consideration. The measurements given in [Devnin 1975 and K.Vikestad, J.K.Vandiver, C.M.Larsen 2000] show that the lift force consists of not one but several harmonics. Usually, only the first harmonic is taken into consideration. The presence of the higher harmonics leads to the fact that the structural oscillation frequency at resonance is smaller then the natural frequency of the structure. When the amplitude of vibrations is considered the influence of the higher harmonics on the amplitude is marginal but is significant on the vibration frequency. For simplicity assume that the structure is linear and has a small structural damping which can be neglected. The average values of the potential and kinetic energies of the structure over one period at resonance should be equal:

$$\frac{1}{T}\int_0^T M\dot{y}^2 dt = \frac{1}{T}\int_0^T cy^2 dt \qquad (13)$$

Then, one can seek a solution of Eq.(3) in the form:

$$\begin{aligned} y &= A_1 \sin\omega t + A_2 \sin(2\omega t + \varphi_2) + ... \\ \dot{y} &= A_1\omega \cos\omega t + A_2 2\omega\cos(2\omega t + \varphi_2) + ... \end{aligned} \qquad (14)$$

Then, substituting Eq. (13) into Eq. (14) and taking into consideration that the harmonics are orthogonal, one can get:

$$M\omega^2 \sum_1^\infty n^2 A_n^2 = c\sum_1^\infty A_n^2$$

where $\omega_0^2 = c/M$. So, finally for the frequency ω of the structural vibration one has:

$$\omega^2 = \omega_0^2 \frac{[A_1^2 + A_3^2 + ...A_n^2]}{[A_1^2 + 9A_3^2 + ... + n^2 A_n^2]} \qquad (15)$$

From Eq. (15) it follows that the vibration frequency at resonance became decreases as a result of the existence of odd harmonics in the lift force spectrum .When the amplitudes of vibration increase the frequency decreases, which coincide with experimental results obtained in [K.Vikestad, J.K.Vandiver, C.M.Larsen 2000]. So, the influence of the higher harmonics should be considered and there interaction as well. It can be done with a help of a more accurate technique provided by the averaging method and a computer simulation. The next approximations of the solution of the Eq. (4) where the terms of the order higher than $O(\varepsilon)$ are taking into account have to be found. The coupling of the different modes of the structural oscillations in the crossflow and inflow directions has to be considered as well. The coefficients $b_1, b_3, b_5, \overline{f}_0$ were found from the experimental data under the assumption that the lift force is uniformly distributed along the length of the tested cylinder. For such structural elements like cables and wires it is necessary to find the correction coefficients which will take into account the non-uniform distribution of the lift force along the length of the structure and its segments. The applicability of the suggested values of b_1, b_3, b_5, c_n for different types of structures has to be verified by a comparison with various experimental data. And, finally, the solution of the Eq. (1) should be found.

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