CONTROL OF SANDY BOTTOM PROFILE BY STRONGLY NONLINEAR SURFACE WAVES

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Abstract

In this report we present experimental results on sand bottom profile formation and particles segregation under strongly nonlinear surface (solitons) waves. We reveal the influence of bottom profile on dissipation rate of surface waves. Theoretical models for the interaction of solitons with sandy bottoms and the segregation of particles are proposed.

Key words: solitons, oscillating flow, particles segregation, modelling of theoretical trajectory**.**

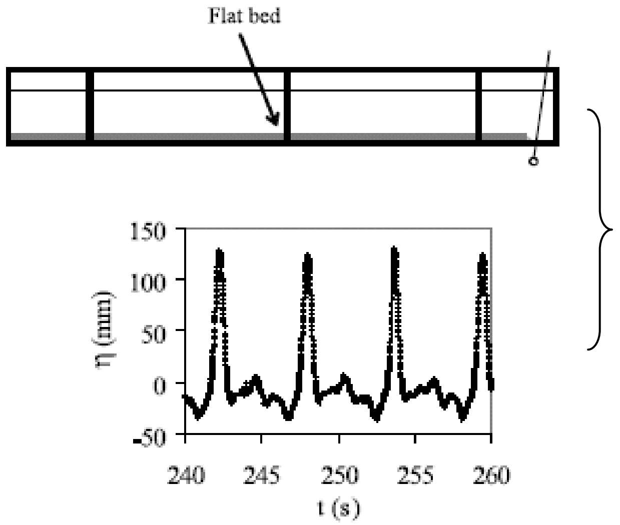
Introduction

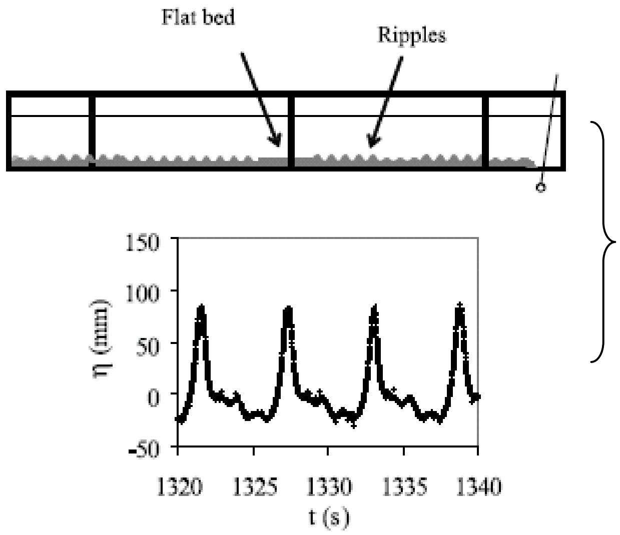
The formation of periodic structures on a sandy bottom caused by surface waves has been studied for a long time. The first significant paper had been published about sixty year ago by Bagnold [Bagnold, 1946], and it was devoted to the appearance of the so called sand ripples on the bottom. It was shown that ripples' generation was due to instability of border between the sand and the oscillating liquid. Afterwards, the main attention was paid to the harmonically oscillating flows for the investigation of patterns on the bottom (see for example [Blondeaux, 1990] and references therein). Meanwhile, in the natural conditions surface waves may be presented as one harmonic for small amplitude only. For waves of finite amplitude, the velocity field induced near the bottom may be presented as the superposition of a large number of harmonics, and forces acting on bottom particles have complex spectrum. The present work investigates this case of interaction between waves and a sandy bottom.

Experimental results

The tests have been carried out in a wave flume at Le Havre University (France). The mean water depth was H=0.26 m. Surface waves are produced by an oscillating paddle at one end of the flume; a near perfect reflection takes place at the other end (see Fig.1). The flume is used in resonant mode, without an absorbing beach. The frequency f of the oscillating paddle is chosen close to the resonant frequency f_r=0.166 Hz of the mode whose wavelength is equal to the flume length. In other words, the wavelength L_h of the standing harmonic wave equals the effective flume length L_e = 9.63 m. The ratio d/L_h =0.027 lies in the range where the theory of very long shallow water waves is valid, and the harmonic wave speed is $V_0=\sqrt{gH}$, where g is the acceleration due to gravity. For values of the amplitude of horizontal displacement of the oscillating paddle averaged over depth a_h lower than 2 cm, only standing harmonic waves are generated in the flume. For values of a_h greater than 2 cm, pulses propagating from one end of the channel to the other end are excited on the background of the standing harmonic wave [Ezersky et al, 2006]. The shape of these pulses was closed to the shape of soliton described by the classical Korteweg de Vries (KdV) equation: $\eta_s = A_s \operatorname{sech}^2[\sqrt{\frac{3A_s}{4H^3}}(x - V_s t)]$, where A_s is the soliton amplitude, and η_s is the displacement of the free surface due to soliton, $V_s = V_0(1 + A_s/2H)$ is the velocity of soliton. The generation of solitons in the flume results from the excitation of higher harmonics. The values of the frequency f and of the amplitude a_h are chosen such as one or two pulses (one or two solitons regime) propagate in each direction of the flume on the time period of the flow. The procedure of signal separation into two parts corresponding to the harmonics and nonlinear wave solitons has been discussed in [Ezersky et al, 2006; Marin and Ezersky, 2008].

We have obtained that the bottom profile and soliton amplitude depend on time. The amplitude of sand ripples on the bottom increases for increasing values of the time when the soliton amplitude decreases. The temporal evolution of the sandy bottom is presented in figure 1 for the regime of one soliton.





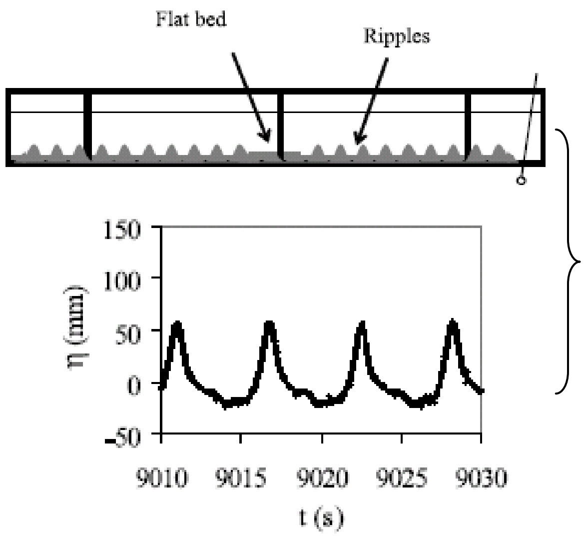
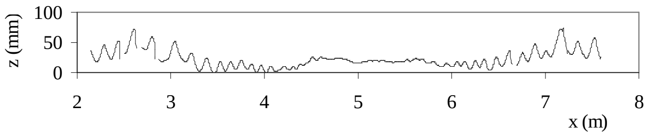
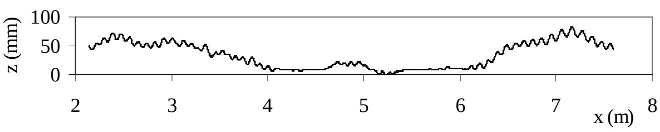


Figure 1. Evolution of profile of sandy bottom (above) and shape of surface waves (under) along time.

We have found that the bottom profile depends on the shape of non-linear waves. If one soliton is excited in the flume, the variation of the bed level with the distance *x* along the flume at the equilibrium state looks as depicted in figure 2 (a). The corresponding bed profile in the two solitons regime is shown in figure 2 (b). The generation of two solitons in comparison with the one soliton generation leads to a decrease of ripples period (small scale structure on the bottom) and an increase of dune (large scale structure on the bottom) amplitude.



(a)

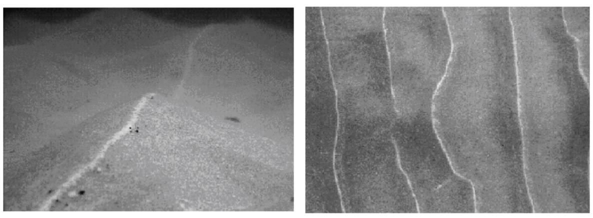


(b)

Figure 2. Profile of sandy bottom at the equilibrium state in the case of one soliton (a) and two solitons regime (b).

To investigate particle segregation due to nonlinear waves, we have used a mixture of sand and plastic particles. The sand is characterized by a median grain diameter D_S =

0.16mm and by a relative density $s = \frac{\rho_s}{\rho_w} = 2.65$, ρ_s ρ_w are correspondingly the densities of sand and water. One per cent (in mass) of plastic particles of relative density s = 1.35 were added to the sand. Two types of plastic particles were considered: particles with a mean diameter smaller (D_1=0.12 mm) and larger (D_2=0.20 mm) than the sand diameter. The experiments were carried out as follows. Before each test, the sand and plastic particles were carefully mixed and this mixture was distributed uniformly over the bottom of the flume. We switched on the wavemaker and attained a steady-state regime of soliton excitation in the channel. Soliton excitation induced the formation of sand ripples at the bottom. Ripple formation in the presence of solitons and the interaction of these ripples with solitons were discussed in detail in [Marin, Abcha, Brossard and Ezersky, 2005]. The oscillating boundary layer at the bottom was turbulent due to the high pulsation velocities. The turbulent vortices led to a layer of particles suspended over the sand ripples [Ezersky and Marin, 2008]. On reaching the steady-state in the “surface wave solitons – sand ripples” system we switched off the wavemaker. A short time (about several minutes) later, the wave motions in the channel damped. As the wave motions were damping, we observed appearance of regions where PVC particles accumulated. The particles were concentrated in a narrow region immediately at the ripple crest. This effect was excellently visualized because of the different colors of the PVC and sand grains. Images of such a particle distribution are given in figure 3.



(a) (b)

Figure 3. Segregation of sinking particles in the vicinity of sandy ripples crests: (a) side view, (b) top view.

Theoretical model

To characterize the solitons generated in the present system, it is possible to introduce the energy of soliton $E_s = \int_{-\infty}^{+\infty} \eta_s^2 dx \propto A_s^{3/2}$ and the phase shift between soliton and harmonic wave φ_s. We propose the following system of equations to describe the propagation of solitons on the background of a standing harmonic wave over a non flat bottom:

$$\frac{dE_s}{dt} = \frac{3}{2}\frac{\omega\eta_0}{H} E_s \cos\varphi_s - (\delta + \alpha_1 h)E_s \tag{1.1}$$

$$\frac{d\varphi_s}{dt} = \frac{\sqrt{gH}}{2}\frac{A_s k}{H} - \eta_0 k\sqrt{\frac{g}{H}}_s \sin\varphi_s - \Delta \tag{1.2}$$

$$\frac{dh}{dt} = \alpha_2 E_s - \alpha_3 h\,. \tag{1.3}$$

In these equations, *h* is the amplitude of sand ripples on the bottom, Δ is the detuning between the frequency of external forcing and the resonator mode $\Delta = f - f_r$, δ is the dissipation rate of soliton, α_1 α_2 α_3 are phenomenological coefficients describing the interaction between waves and the sandy bottom. A system of two equations for the case *h=0* has been obtained in book [Ostrovsky and Potapov, 1999] and discussed in papers [Ezersky et al, 2006; Marin and Ezersky, 2008; Marin et al, 2005]. The stable steady state of system (1) for this case has been analyzed in [Ezersky et al, 2006]. It describes the parametric generation of soliton by harmonic mode. Interaction with a rippled bottom leads to an additional dissipation of energy. It causes a decrease of soliton amplitude A_s and soliton phase φ_s. Such behaviour of soliton characteristics has been observed in experiments and qualitatively coincides with model equation predictions [Marin and Ezersky, 2008].

To describe segregation of particles we consider liquid flow in the vicinity of ripples crests. We neglect the curvature of the sand surface and consider a flat bottom. In this approximation, the stream function ψ may be presented as $\psi = -a(\alpha x + y)y$ [Ezersky and Marin, 2008]. The velocities in the vicinity of the crests are given by:

$$U_x = \frac{\partial \psi}{\partial y} = -a(\alpha x + 2y) \qquad (2.1)$$

$$U_y = -\frac{\partial \psi}{\partial x} = a\alpha y \qquad (2.2)$$

Where $\alpha = \alpha_0 \sin(2\pi f t)$, $a = a_0 \sin(2\pi f t)$; a is determined by the flow vorticity $\vec{\Omega} = rot\vec{U}$: $a = \Omega/2$, and α describes the flow geometry near the ripples crest [Ezersky and Marin, 2008].

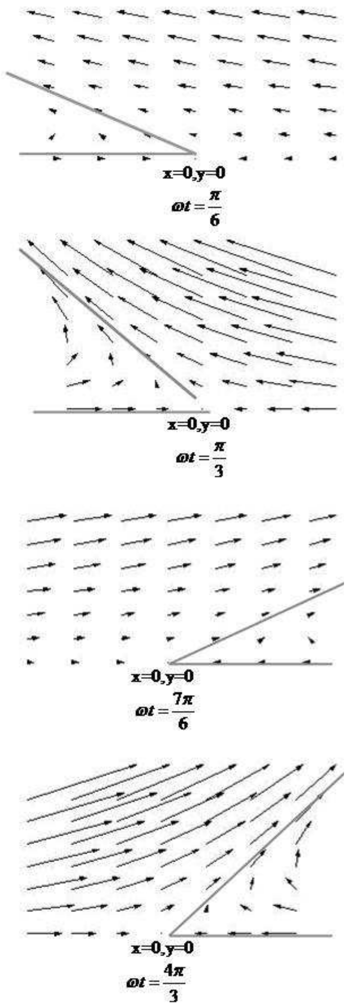


Figure 4. Velocity fields, $\vec{U} = (U_x, U_y)$ at different times. For ωt = π/6 and ωt = π/6 +π = 7π/6, the horizontal component of velocity field changes the sign.

Let us consider the equation describing the motion of grain in this flow field. We take into account only the Stokes force; we do not consider the turbulence drag and the viscosity terms.

$$\rho_{s,PVC}\frac{\pi d^3_{s,PVC}}{6}\frac{d\vec{V}}{dt} = 3\pi\rho_w \upsilon d_{s,PVC}(\vec{U} - \vec{V}) - \frac{\pi d^3_{s,PVC}}{6}\vec{\nabla}p + (\rho_{s,PVC} - \rho_w)\frac{\pi d^3_{s,PVC}}{6}\vec{g} \qquad (2.3)$$

In equation (2.3) $\vec{V}$ is the grain velocity, $\vec{U}$ the flow velocity, υ the fluid kinematic viscosity, p the pressure in the fluid, $\vec{g}$ the acceleration due to gravity, d the grain diameter, ρ is the density and the indices s, PVC and w correspond to sand, PVC and water, respectively. We can calculate the pressure gradient, assuming that it is possible to replace solid grain by liquid grain:

$$\rho_w \frac{d\vec{U}}{dt} = -\vec{\nabla}p \qquad (2.4)$$

Substituting equation (2.4) into equation (2.3), we get equation (2.5):

$$\vec{V} + \frac{St}{\omega}\frac{d\vec{V}}{dt} = \vec{U} + \frac{St}{\omega}\frac{\rho_w}{\rho_{s,PVC}}\frac{d\vec{U}}{dt} + \frac{(\rho_{s,PVC} - \rho_w)}{\rho_w}\left(\frac{d_{s,PVC}^{2}}{18\upsilon}\right)\vec{g} \qquad (2.5)$$

The flow velocity $\vec{U}$ is considered to change periodically in time with the pulsation ω. The grain velocity $\vec{V}$ does not coincide with the flow velocity $\vec{U}$. Because for our experiments, the Stokes number St is very small: $St = d^2_{s,PVC}\rho_{s,PVC}\omega/18\nu\rho_w \approx 10^{-3}$, it is possible to use St as an expansion parameter and write the grain velocity as:

$$\vec{V} = \vec{V}^{(0)} + St\vec{V}^{(1)} + St^2\vec{V}^{(2)} + \ldots \qquad (2.6)$$

From these equations (2.5) and (2.6), we can obtain a solution in the form:

$$\vec{V}^{(0)} = \vec{U} - \vec{e}_y \frac{d^2_{s,PVC}(\rho_{s,PVC} - \rho_w)}{18\nu\rho_w} \equiv \vec{U} - \vec{e}_y U_0$$

$$\vec{V}^{(1)} = -\frac{1}{\omega}\left(\frac{d(\sigma\vec{U} - \vec{e}_y U_0)}{dt}\right)$$

(2.7)

where $U_0 = \dfrac{d^2_{s,PVC}(\rho_{s,PVC} - \rho_w)}{18\nu\rho_w}$ is the Stokes sedimentation velocity and $\sigma = 1 - \rho_w / \rho_{s,PVC}$.

Neglecting the terms proportional to St^2 in the expansion (2.6), we substitute the expression (2.7) into the equation of grain motion (2.5). For the components of grain velocity field V_x and V_y near the ripple crest, we obtain the following expressions:

$$V_x = -\alpha a(\alpha x + 2y) + \frac{St}{\omega}[\sigma(\dot{a}\alpha + a\dot{\alpha})x + \dot{a}y -$$
$$-\sigma^2 a^2 \alpha^2 x - 2\sigma a U_0];$$
$$V_y = \sigma a \alpha y - U_0 - \frac{St}{\omega}[\sigma(\dot{a}\alpha + a\dot{\alpha})y + \sigma^2 a^2 \alpha^2 y -$$
$$-\sigma a \alpha U_0] \qquad (2.8)$$

When the oscillating paddle is stopped, the flow decreases with time. We consider that a and α decrease with time as: $a = a_0 \sin(\omega t) \times e^{-\gamma t}; \alpha = \alpha_0 \sin(\omega t) \times e^{-\gamma t}$ where γ is the rate of exponential decay of the surface waves.

Equation (2.8) becomes:

$$\dot{x} = V_x = -\sigma a_0 \sin(\omega t) e^{-\gamma t}[\alpha_0 \sin(\omega t) e^{-\gamma t} x(t) + 2y(t)] +$$
$$+\frac{St}{\omega}[2\sigma a_0 \alpha_0 x(t) \sin(\omega t) e^{-2\gamma t}(\omega\cos(\omega t) - \gamma\sin(\omega t)) +$$
$$+\alpha_0 y(t) e^{-\gamma t}(\omega\cos(\omega t) - \gamma\sin(\omega t)) -$$
$$-\sigma^2 a_0^2 \alpha_0^2 x(t) e^{-4\gamma t}(\sin(\omega t))^4 - 2\sigma a_0 U_0 e^{-\gamma t}\sin(\omega t)];$$
$$\dot{y} = V_y = \sigma a_0 \alpha_0 y(t)(\sin(\omega t))^2 e^{-2\gamma t} - U_0 -$$
$$-\frac{St}{\omega}[2\sigma a_0 \alpha_0 y(t) e^{-2\gamma t}\sin(\omega t)(\omega\cos(\omega t) - \gamma\sin(\omega t)) +$$
$$+\sigma^2 a_0^2 \alpha_0^2 y(t) e^{-4\gamma t}(\sin(\omega t))^4 - \sigma a_0 \alpha_0 U_0 e^{-2\gamma t}(\sin(\omega t))^2]$$

After that, we calculate the averaged velocity of particle $\langle V_x \rangle$ and $\langle V_y \rangle$ using equation (2.8) over time t = 1/ω, we have:

$$\langle V_x \rangle = -x(\frac{\sigma}{2} a_0 \alpha_0 e^{-\gamma t} + \frac{St}{4\omega} a_0^2 \alpha_0^2 e^{-2\gamma t})$$
$$\langle V_y \rangle = -(U_0 - y\sigma a_0 \alpha_0 e^{-\gamma t})(1 - \frac{St}{\omega}\sigma a_0 \alpha_0 e^{-\gamma t}) \qquad (2.9)$$

We carried out numerical simulations of the theoretical trajectories. Figure 5 depicts the trajectories obtained for several initial position of grains (X(0), Y(0)) with a_0 = 1.5 [s^{-1}]; ω = 1.96 [rad/s]; σ = 0.26; α_0 = 1 [rad]; St = 0.0092 [rad], U_0 = 0.012 [m/s]; γ = 0.0083 [s^{-1}].

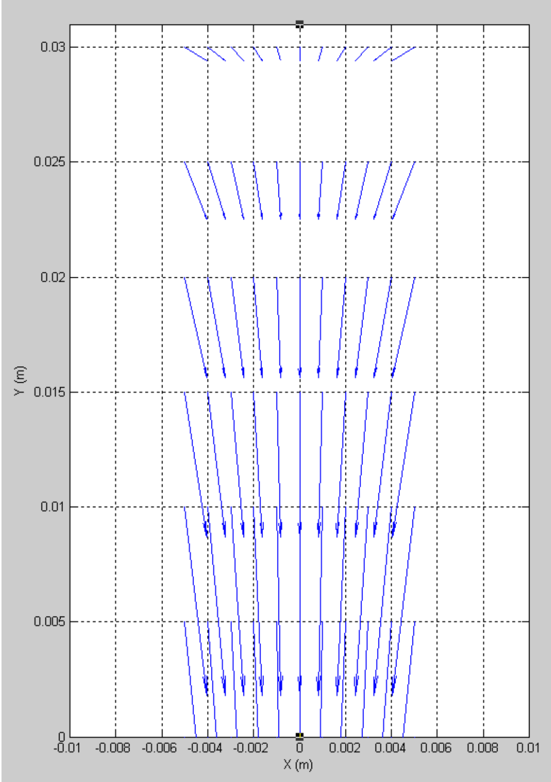


Figure 5. The velocity field of particles at the time: t = π (s) using equation (2.9)

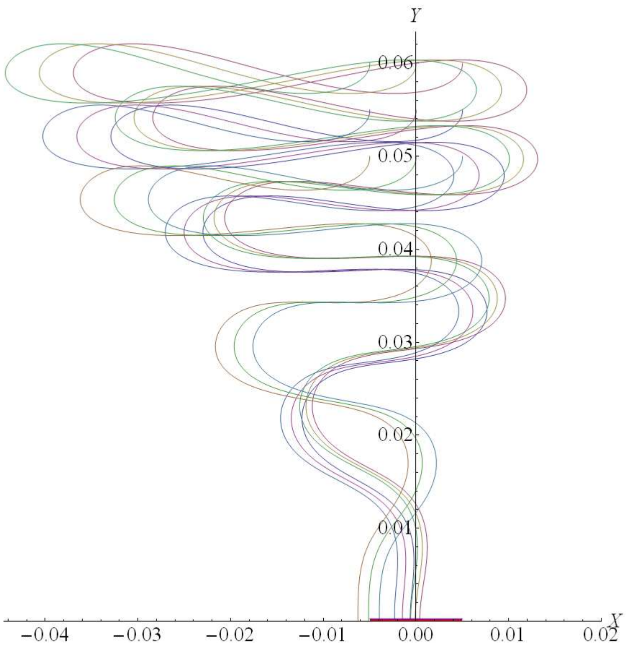


Figure 6. Theoretical trajectories of PVC grains The initial positions: t = 0s, X(0) = -0.005, 0, 0.005 (m); Y(0) = 0.050, 0.055, 0.060 (m). The different curves correspond to different positions of grains.

Figures 5 and 6 show that the particles tend to fall on the ripple crest (X=0, Y=0). The fluid velocity field leads to a mean drift towards the ripple crest (figure 4). After the stop of the wave maker, grains become to sink. For sand grains the time of sinking τ_s is less than for PVC grains. The concentration of the PVC grains should consequently increase everywhere at the bed surface; however, the PVC grains are observed only close to the ripple crests when the waves have damped. It may be explained as follows. After that the wave maker has been stopped and for $t < \tau_s$, both sand and PVC grains are concentrated near the ripple crests. For $t > \tau_s$, when most sand grains have settled on the bottom, only PVC grains begin to concentrate near the crests.

Conclusions

The generation of strongly nonlinear surface waves may significantly affect the bottom profile and lead to the: formation of sand ripples on the background of dunes as well as segregation of particles. Phenomena of this kind are very important for different applied problems like sediment transport in the coastal zone, formation and structure of bottom depositions, wave breaking.

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