NONLINEAR VIBRATION OF THE LAMINATED SHALLOW SHELLS WITH COMPLEX PLANFORM

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Abstract

Investigation method of free nonlinear vibration of laminated plates and shallow shells with an arbitrary plan form and different boundary conditions is proposed. The offered method is based on combined application of R-functions theory and variational methods. The passing to nonlinear system of the ordinary differential equations (NSODE) is connected with solving the sequence of the boundary problems in the domain of an arbitrary shape: linear vibration problem; sequence of problems of elasticity theory simulated by partial differential equations with special right part and corresponding boundary conditions. The variation method by Ritz together with R-functions theory is applied to solve foregoing boundary value problems. The final passing to NSODE is carried out by Galerkin procedure. The coefficients of the obtained NSODE are presented in explicit form and expressed through the double integrals of known functions for the cases of single- mode and multi-mode approximation. The following investigation of the obtained nonlinear ordinary differential equation or system is fulfilled by Rung-Kutt method. The proposed method is illustrated on specific examples and compared with another approaches.

1 Introduction

Nonlinear vibrations problems of the laminated shallow shells are very essential for practice because shells are important elements in many fields of lightweight construction. In spite of the practical importance of these problems as the most recent survey papers on nonlinear vibrations of the shallow shell with complex plan form fully attest that there are not available studies in the specialized literature addressing this topic. Due to mathematical complexity of the problem majority scientists consider only simply supported shallow shells with rectangular form of the plane.

The present study is devoted to solving problem for plates and shallow shells with complex form. Due to application of the R-functions theory the generalization of the classical idea allowing passing of the continual model to system with finite number of the freedom degrees simulated by nonlinear system of ordinary differential equations (NSODE) be found.

2 Mathematical statement

To demonstrate proposed method let us consider the geometrically nonlinear vibration problem of laminated shallow shells. To simplify discussion let us assume that shell consists from odd numbers layers. They are symmetrical relatively to the middle surface. The mathematical statement of this problem in framework of classical theory is based on hypothesis of strain less normal which is accepted for all package in whole. The governing system is nonlinear one of the differential equations with partial derivatives written below [1,2]

$$L_{11}u + L_{12}v + L_{13}w = -Nl_1(w) - \rho h \cdot \frac{\partial^2 u}{\partial t^2} \tag{2.1}$$

$$L_{21}u + L_{22}v + L_{23}w = -Nl_2(w) - \rho h \cdot \frac{\partial^2 v}{\partial t^2} \tag{2.2}$$

$$L_{31}u + L_{32}v + L_{33}w = -Nl_3(u,\ v,\ w) - \rho h \cdot \frac{\partial^2 w}{\partial t^2} \tag{2.3}$$

The linear differential operators L_{ij}, $i, j = 1,2,3$ in the equations (2.1)-(2.3) are determined as follows:

$$L_{11}(C_{ij}) = C_{11}\frac{\partial^2}{\partial x^2} + 2C_{16}\frac{\partial^2}{\partial x \partial y} + C_{66}\frac{\partial^2}{\partial y^2},$$

$$L_{22}(C_{ij}) = C_{66}\frac{\partial^2}{\partial x^2} + 2C_{26}\frac{\partial^2}{\partial x \partial y} + C_{22}\frac{\partial^2}{\partial y^2},$$

$$L_{12}(C_{ij}) = L_{21}(C_{ij}) = C_{16}\frac{\partial^2}{\partial x^2} + (C_{12} + C_{66})\frac{\partial^2}{\partial x \partial y} + C_{26}\frac{\partial^2}{\partial y^2},$$

$$L_{13}(C_{ij}) = L_{31}(C_{ij}) = (k_1 C_{11} + k_2 C_{12})\frac{\partial}{\partial x} + (k_1 C_{16} + k_2 C_{26})\frac{\partial}{\partial y},$$

$$L_{23}(C_{ij}) = L_{32}(C_{ij}) = (k_1 C_{12} + k_2 C_{22})\frac{\partial}{\partial y} + (k_1 C_{16} + k_2 C_{26})\frac{\partial}{\partial x}$$

$$L_{33}(C_{ij}, D_{ij}) = D_{11}\frac{\partial^4}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial y^2 \partial x^2} + 4D_{16}\frac{\partial^4}{\partial x^3 \partial y} + 4D_{26}\frac{\partial^4}{\partial y^3 \partial x} + D_{22}\frac{\partial^4}{\partial y^4} + \left(k_1^2 C_{11} + 2k_1 k_2 C_{12} + k_2^2 C_{22}\right). \tag{2.4}$$

Nonlinear operators $Nl_1(w), Nl_2(w), Nl_3(u,v,w)$ are defined as follows

$$Nl_1(w)=\frac{\partial}{\partial x}\left(\frac{1}{2}C_{11}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}C_{12}\left(\frac{\partial w}{\partial y}\right)^2+C_{16}\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)+$$

$$+\frac{\partial}{\partial y}\left(\frac{1}{2}C_{16}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}C_{26}\left(\frac{\partial w}{\partial y}\right)^2+C_{66}\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)=$$

$$=\frac{\partial w}{\partial x}\left(C_{11}\frac{\partial^2 w}{\partial x^2}+2C_{12}\frac{\partial^2 w}{\partial x\partial y}+C_{66}\frac{\partial^2 w}{\partial y^2}\right)+$$

$$+\frac{\partial w}{\partial y}\left((C_{12}+C_{66})\frac{\partial^2 w}{\partial x\partial y}+C_{16}\frac{\partial^2 w}{\partial x^2}+C_{26}\frac{\partial^2 w}{\partial y^2}\right), \qquad (2.5)$$

$$Nl_2(w)=\frac{\partial}{\partial x}\left(\frac{1}{2}C_{16}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}C_{26}\left(\frac{\partial w}{\partial y}\right)^2+C_{66}\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)+$$

$$+\frac{\partial}{\partial y}\left(\frac{1}{2}C_{12}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}C_{22}\left(\frac{\partial w}{\partial y}\right)^2+C_{26}\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right)=$$

$$=\frac{\partial w}{\partial x}\left(C_{16}\frac{\partial^2 w}{\partial x^2}+(C_{12}+C_{66})\frac{\partial^2 w}{\partial x\partial y}+C_{26}\frac{\partial^2 w}{\partial y^2}\right)+$$

$$+\frac{\partial w}{\partial y}\left(C_{66}\frac{\partial^2 w}{\partial x^2}+2C_{26}\frac{\partial^2 w}{\partial x\partial y}+C_{22}\frac{\partial^2 w}{\partial y^2}\right), \qquad (2.6)$$

$$Nl_3(u,v,w)=-\left(\frac{\partial^2 w}{\partial x^2}N_x+\frac{\partial^2 w}{\partial y^2}N_y+2\frac{\partial^2 w}{\partial x\partial y}S+\right.$$

$$+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2(C_{11}k_1+C_{12}k_2)+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2(C_{12}k_1+C_{22}k_2)+$$

$$+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}(C_{12}k_1+C_{22}k_2) \qquad (2.7)$$

Where C_{ij}, D_{ij} are rigid characteristics [2], which are defined with help elasticity constants B^i_{jk} for every $i-$th layers as

$$C_{jk}=2\left(B^{m+1}_{jk}h_{m+1}+\sum_{s=1}^{m}B^s_{jk}(h_s-h_{s+1})\right), \qquad (2.8)$$

$$D_{jk}=\frac{2}{3}\left(B^{m+1}_{jk}h^3_{m+1}+\sum_{s=1}^{m}B^s_{jk}(h_s-h_{s+1})^3\right), \qquad (2.9)$$

If we deals with laminated cross-ply, angle-ply shell then elasticity constants for $i-$layer are computed by the following formulas [2]

$$\begin{aligned}
\widetilde{B}_{11}&=B_{11}\cos^4\theta+2(B_{12}+2B_{66})\sin^2\theta\cos^2\theta+B_{22}\sin^4\theta,\\
\widetilde{B}_{22}&=B_{11}\sin^4\theta+2(B_{12}+2B_{66})\sin^2\theta\cos^2\theta+B_{22}\cos^4\theta,\\
\widetilde{B}_{12}&=B_{12}+[B_{11}+B_{22}-2(B_{12}+2B_{66})]\sin^2\theta\cos^2\theta,\\
\widetilde{B}_{66}&=B_{66}+[B_{11}+B_{22}-2(B_{12}+2B_{66})]\sin^2\theta\cos^2\theta,\\
\widetilde{B}_{16}&=\frac{1}{2}\left[B_{22}\sin^2\theta-B_{11}\cos^2\theta+(B_{12}+2B_{66})\cos 2\theta\right]\sin 2\theta,\\
\widetilde{B}_{26}&=\frac{1}{2}\left[B_{22}\cos^2\theta-B_{11}\sin^2\theta-(B_{12}+2B_{66})\cos 2\theta\right]\sin 2\theta,
\end{aligned} \qquad (2.10)$$

The differential equations (1.1) – (1.3) are supplemented by corresponding boundary conditions. Some of them we present below:

a) clamped edge $u=0,\ v=0,\ w=0,\ \frac{\partial w}{\partial n}=0$,

b) simply supported edge (immovable) $w=0,\ M_n=0,\ u=0,\ v=0$,

c) clamped movable edge $v_n=0,\ N_n=0,\ w=0,\ \frac{\partial w}{\partial n}=0$

here n is normal to domain boundary Ω, M_n and N_n is defined by known formulas [1, 2].

The initial conditions are taken as follows:

$$w=w_{\max},\frac{\partial w}{\partial t}=0$$

The given differential equations are nonlinear ones and enough complicated. The solving system (2.1)-(2.3) may be carried out by numerical method. Below one of such approach is proposed.

3 Method of solving

3.1 Solving the linear vibration problem

The first step of the proposed method is solving corresponding linear problem of free vibration of the shallow shells. It should be noted that it is the difficult problem in general case. This problem may be solved only with numerical methods. Distinctive feature of the proposed method is application of the variational-structure approach, based on theory R-functions and variational methods. Namely such approach allows finding natural frequencies and functions in analytical form for any domain and kind of the boundary conditions, what is very essentially for feature solving nonlinear problem.

The variational statement of the linear problem is reduced to finding minimum of the functional

$$J=U_{\max}-T_{\max}, \qquad (3.1)$$

where $U_{\max}$ is maxima potentional energy of the shells and $T_{\max}$ is maxima kinetic energy [1, 2].

To find minimum of the functional method by Ritz is applied. The consequence of basic functions satisfying given boundary conditions is generated by R-functions method [3, 4]. The main advantage of R-functions method is possibility to construct the basic functions in analytical form. The natural modes corresponding to linear vibration of the shells have been chosen as basic functions for representation of unknown functions.

3.2 Algorithm for solving geometrically nonlinear vibration problem

Essence of the proposed method are reduced to the following: the deflection function is expanded in Fourier series

$$w=\sum_{i=1}^{n}y_i(t)w_i(x,y), \qquad (3.2)$$

where $w_i(x, y)$ are the components of the eigenfunctions vector. Substituting this expressions into equations (2.1) – (2.2) and ignoring by the inertia terms one can obtain the following system

$$L_{11}u + L_{12}v = -\sum_{i=1}^{n} y_i(t) \cdot L_{13}w_i + Nl_1\left(\sum_{i=1}^{n} y_i(t)w_i\right), \qquad (3.3)$$

$$L_{21}u + L_{22}v = -\sum_{i=1}^{n} y_i(t) \cdot L_{23}w_i + Nl_2\left(\sum_{i=1}^{n} y_i(t)w_i\right), \qquad (3.4)$$

here

$$Nl_1\left(\sum_{i=1}^{n} y_i(t)w_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} y_i(t) \cdot y_j(t) \cdot Nl_1^{(2)}(w_i, w_j), \qquad (3.5)$$

$$Nl_2\left(\sum_{i=1}^{n} y_i(t)w_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} y_i(t) \cdot y_j(t) \cdot Nl_2^{(2)}(w_i, w_j), \qquad (3.6)$$

where

$$Nl_1^{(2)}(w_i, w_j) = \frac{\partial w_i}{\partial x}\left(C_{11}\frac{\partial^2 w_j}{\partial x^2} + C_{66}\frac{\partial^2 w_j}{\partial y^2} + 2C_{16}\frac{\partial^2 w_j}{\partial x \partial y}\right) + $$
$$+\frac{\partial w_i}{\partial y}\left((C_{12} + C_{66})\frac{\partial^2 w_j}{\partial x \partial y} + C_{16}\frac{\partial^2 w_j}{\partial x^2} + C_{26}\frac{\partial^2 w_j}{\partial y^2}\right)\Bigg), \qquad (3.7)$$

$$Nl_2^{(2)}(w_i, w_j) = \frac{\partial w_i}{\partial x}\left(C_{16}\frac{\partial^2 w_j}{\partial x^2} + \frac{\partial^2 w_j}{\partial x \partial y}(C_{66} + C_{12} + 2C_{26})\right)$$
$$+\frac{\partial w_i}{\partial y}\left(C_{26}\frac{\partial^2 w_k}{\partial x \partial y} + C_{66}\frac{\partial^2 w_k}{\partial x^2} + C_{22}\frac{\partial^2 w_k}{\partial y^2}\right)\Bigg) \qquad (3.8)$$

Due to kind of on the right part of the obtained system (3.3) - (3.4) we can determine the form of solution for tangent displacements $u(x, y, t)$ and $v(x, y, t)$. Obviously these functions may be presented as

$$u(x, y, t) = \sum_{i=1}^{n} y_i(t)u_i(x, y) + \sum_{i=1}^{n}\sum_{j=1}^{n} y_i(t)y_j(t)u_{ij}(x, y), \qquad (3.9)$$

$$v(x, y, t) = \sum_{i=1}^{n} y_i(t)v_i(x, y) + \sum_{i=1}^{n}\sum_{j=1}^{n} y_i(t)y_j(t)v_{ij}(x, y) \qquad (3.10)$$

Here (u_i, v_i) are components of the vector eigenfunctions corresponding to i-th linear frequency, previously defined and (u_{ij}, v_{ij}) is solution of the following system

$$L_{11}u_{ij} + L_{12}v_{ij} = -Nl_1^{(2)}(w_i, w_j), \qquad (3.11)$$

$$L_{21}u_{ij} + L_{22}v_{ij} = -Nl_2^{(2)}(w_i, w_j) \qquad (3.12)$$

The last system coincides with similar system for 2-dimensional elasticity problems for which the right parts play the role of mass forces imaginary. The boundary conditions depend on ways of shell fixing. To find these functions the RFM method is applied [4, 6].

Substituting the expressions (3.2), (3.9), (3.10) for $u(x, y, t), v(x, y, t), w(x, y, t)$ into movement equation (2.3) one obtains the nonlinear ordinary differential equation in unknown functions $y_i(t)$. Obtained equations may be reduced to nonlinear system of the ordinary differential equations in unknown functions $y_i(t)$ by Galerkin method. With this purpose the given equation is projected on eigenfunctions $w_i(x, y)$ step by step. So we obtain the following system

$$y_m^{//}(\tau) + \alpha_m y_m + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}^{(m)} y_i y_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{ijk}^{(m)} y_i y_j y_k = 0, \qquad (3.13)$$

where

$$\alpha_m = \frac{\omega_{Lm}^2}{\omega_L^2},$$

$$\beta_{ij}^{(m)} = \frac{1}{\lambda^2\|w_m\|^2}\left(\iint_\Omega \left(L_{31}u_{ij} + L_{32}v_{ij} - N_x^L(u_i, v_i, w_i)\frac{\partial^2 w_j}{\partial x^2} - \right.\right.$$
$$-N_y^L(u_i, v_i, w_i)\frac{\partial^2 w_j}{\partial y^2} - 2T^L(u_i, v_i)\frac{\partial^2 w_j}{\partial x \partial y} -$$
$$-\frac{1}{2}(C_{11}k_1 + C_{12}k_2)\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x} - \frac{1}{2}(C_{12}k_1 + C_{22}k_2)\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y} -$$
$$\left.\left.-(C_{16}k_1 + C_{26}k_2)\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}\right)w_m d\Omega\right) \qquad (3.14)$$

$$\gamma_{ijk}^{(m)} = -\frac{1}{\lambda^2\|w_m\|^2}\left(\iint_\Omega \left(N_{xp}^N(u_{ij}, v_{ij}w_{ij})\frac{\partial^2 w_k}{\partial x^2} + \right.\right.$$
$$\left.\left.+N_{yp}^N(u_{ij}, v_{ij}, w_{ij})\frac{\partial^2 w_k}{\partial y^2} + 2T_p^N(u_{ij}, v_{ij}, w_{ij})\left(\frac{\partial w_k}{\partial x \partial y}\right)\right)w_m\right)d\Omega, \qquad (3.15)$$

here

$$N_x^L(u_i, v_i, w_i) = C_{11}\left(\frac{\partial u_i}{\partial x} + k_1 w_i\right) + C_{12}\left(\frac{\partial v_i}{\partial y} + k_2 w_i\right) +$$
$$+C_{16}\left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}\right)$$

$$N_y^L(u_i, v_i, w_i) = C_{12}\left(\frac{\partial u_i}{\partial x} + k_1 w_i\right) + C_{22}\left(\frac{\partial v_i}{\partial y} + k_2 w_i\right) +$$
$$+C_{26}\left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}\right)$$

$$T^L(u_i, v_i) = C_{16}\left(\frac{\partial u_i}{\partial x} + k_1 w_i\right) + C_{26}\left(\frac{\partial v_i}{\partial y} + k_2 w_i\right) +$$
$$+C_{66}\left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}\right)$$

$$N_{xp}^N(u_{ij}, v_{ij}, w_{ij}) = C_{11}\left(\frac{\partial u_{ij}}{\partial x} + \frac{1}{2}\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\right) +$$

$$+C_{12}\left(\frac{\partial v_{ij}}{\partial y}+\frac{1}{2}\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\right)+C_{16}\left(\frac{\partial u_{ij}}{\partial y}+\frac{\partial v_{ij}}{\partial x}+\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}\right),$$

$$N_{yp}^{N}\left(u_{ij},v_{ij},w_{ij}\right)=C_{12}\left(\frac{\partial u_{ij}}{\partial x}+\frac{1}{2}\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\right)+$$

$$+C_{22}\left(\frac{\partial v_{ij}}{\partial y}+\frac{1}{2}\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\right)+C_{26}\left(\frac{\partial u_{ij}}{\partial y}+\frac{\partial v_{ij}}{\partial x}+\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}\right),$$

$$T_{p}^{N}\left(u_{ij},v_{ij},w_{ij}\right)=C_{16}\left(\frac{\partial u_{ij}}{\partial x}+\frac{1}{2}\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\right)+$$

$$+C_{26}\left(\frac{\partial v_{ij}}{\partial y}+\frac{1}{2}\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\right)+C_{66}\left(\frac{\partial u_{ij}}{\partial y}+\frac{\partial v_{ij}}{\partial x}+\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}\right).$$

In case plates the obtained system is essentially simplified because of the curvatures vanish and coefficients $\beta_{ij}^{(m)}=0$.

Let us note that the proposed method may be applied to laminated shells with an arbitrary numbers layers and in both case when we apply classical theory or theory by Timoshenko.

To investigate the obtained system of ordinary differential equations numerical methods may be used. In particular case when we use single mode it is possible to get the explicit dependence $v=\frac{\omega_N}{\omega_L}$ of the ratio of the nonlinear frequency to linear one on amplitude of vibration A [1]

$$\frac{\omega_N}{\omega_L}=\sqrt{1-\frac{8}{3\pi}\alpha A+\frac{3}{4}\beta A^2} \qquad (3.16)$$

4 Solving the sequence of auxiliary theory elasticity problems

As we noted the finding functions $\left(u_{ij},v_{ij}\right)$ is connected with solving the following system (3.11) - (3.12).

Obviously the system is supplemented by corresponding boundary conditions. It is possible to prove, that this problem is equivalent to the following variational problem

$$J\left(u_{ij},v_{ij}\right)=\iint_{\Omega}\left(C_{11}\left(\frac{\partial u_{ij}}{\partial x}\right)^2+C_{22}\left(\frac{\partial v_{ij}}{\partial y}\right)^2+C_{66}\left(\frac{\partial v_{ij}}{\partial x}+\frac{\partial u_{ij}}{\partial y}\right)^2\right.$$

$$+2C_{12}\frac{\partial u_{ij}}{\partial x}\frac{\partial v_{ij}}{\partial y}+2\left(C_{16}\frac{\partial u_{ij}}{\partial x}+C_{26}\frac{\partial v_{ij}}{\partial y}\right)\left(\frac{\partial v_{ij}}{\partial x}+\frac{\partial u_{ij}}{\partial y}\right)-$$

$$-2\left(Nl_1^{(2)}(w_i,w_j)u_{ij}+Nl_2^{(2)}(w_i,w_j)v_{ij}\right)dxdy-$$

$$-2\int_{\partial\Omega}\left(F_1^0\cdot U_n+F_1^0\cdot V_n\right)ds, \qquad (4.1)$$

where

$$U_n=u_{ij}l+v_{ij}m, V_n=-u_{ij}m+v_{ij}l,$$

$$F_1^0=-\left(\frac{1}{2}\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\left(C_{11}l^2-C_{12}m^2-2lmC_{16}\right)+\right.$$

$$+\frac{1}{2}\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\left(C_{12}l^2+C_{22}m^2+2lmC_{26}\right)+$$

$$\left.+\left(\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}+\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\right)\left(C_{16}l^2+C_{26}m^2-2lmC_{66}\right)\right),$$

$$F_2^0=-\left(\frac{1}{2}\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\left(C_{16}\left(l^2-m^2\right)+\left(C_{11}-C_{12}\right)lm\right)+\right.$$

$$+\frac{1}{2}\frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\left(C_{26}\left(l^2-m^2\right)+\left(C_{12}-C_{22}\right)lm\right)+$$

$$\left.+\left(\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial y}+\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x}\right)\left(C_{66}\left(l^2-m^2\right)+\left(C_{16}-C_{26}\right)lm\right)\right).$$

If $\omega=0$ is normalized equation of the domain boundary then $l=-\frac{\partial\omega}{\partial x}$, and $m=-\frac{\partial\omega}{\partial y}$..

The discretization of functional (3.1) and (4.1) are fulfilled by RFM and Ritz method.

According to method RFM to solve a boundary problem it is required to construct the solution structure, which is defined by the formula $\vec{U}=B(\Phi_i,\omega,\omega_i)$. This formula contains indefinite components Φ_i, and functions ω, ω_i with the help of which equations of a boundary domain or separate its part are described.

For example solution structure, that satisfies exactly the clamped boundary conditions may be presented as follows

$$w=\omega^2\Phi_1, u=\omega\Phi_2, v=\omega\Phi_3$$

Here $\omega=0$ is equation of the domain boundary, Φ_i are indefinite components. R-functions theory allows to construct equations in analytical form for any boundary with help only elementary functions.

To find the indefinite functions Φ_1,Φ_2,Φ_3 we will represent their as an expansion on series of some full system of functions (power polynomials, splines, Chebyshev's polynomials, etc).

$$\Phi_i=\sum_k a_k^{(i)}\Psi_k(x)$$

The expansion coefficients $a_k^{(i)}$ may be determined from the conditions of minimum of the corresponding functional.

The proposed method is numerically realized in framework of software "POLE-RL" and widely tested on many nonlinear vibration problems for plates and shallow shells at the large amplitudes.

5 Numerical results

Let us investigate the geometrically nonlinear free vibration of the cylinder panel shown in Fig 1. The shell consists of

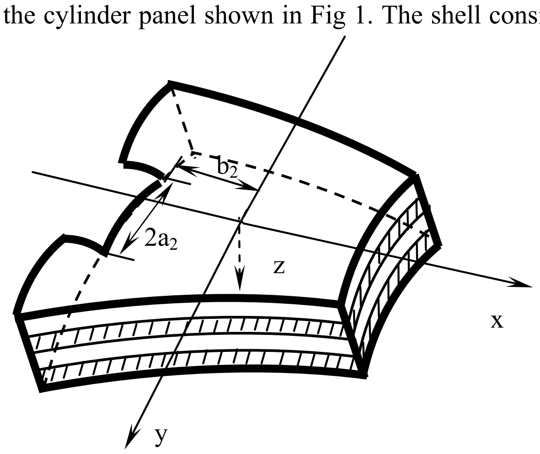

Fig.1 Shape of shell

four layer, ange-ply (0^0/-30^0/0^0/-30^0).

Geometric sizes are

$h/2a = 0.001, 2a/R_1 = 0.0, 2a/R_2 = 0.5$.

Suppose that the shell is carried out of the material with following phisycal rigid characteristics:

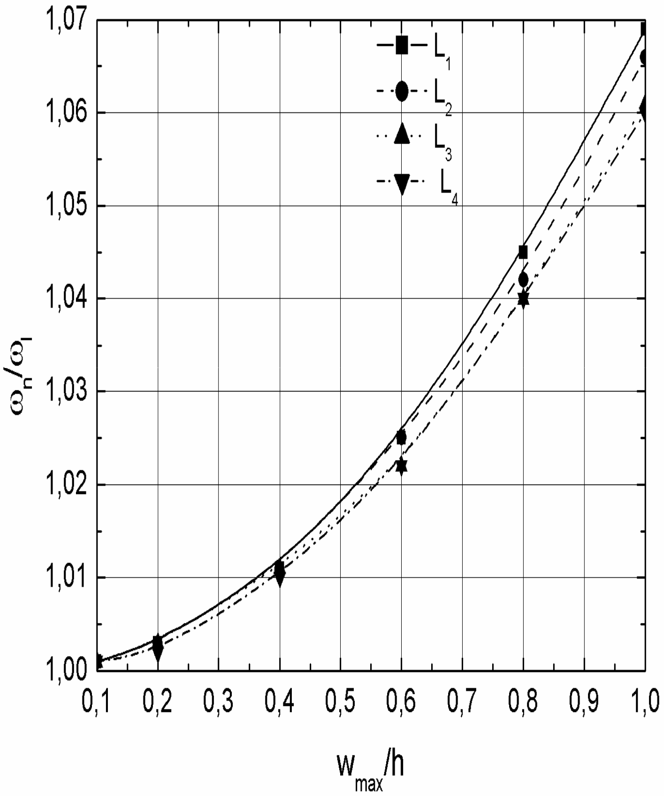

Fig2. Backbone curves for different values of cutout depth.

$E_{11} = 1.0, E_{22} = 0.4086, G_{12} = G_{13} = G_{23} = 0.1980, \nu_{12} = 0.23$.

$k_1^2 = k_2^2 = 5/6$.

The obtained backbone curves for different values of the parameters a_1, b_1 are presented in Fig.2: $a_1 = b_1 = 0.3$ (curve L_1), $a_1 = b_1 = 0.4$ (curve L_2), $a_1 = b_1 = 0.45$ (curve L_3), $a_1 = b_1 = 0.5$ (curve L_4). It is easy to see that curves tend to backbone curve L_4 corresponding to cylindrical panel with trapezoidal at decreasing depth of cutout. This fact and also comparison of the obtained results with available [5] confirm the verification of the proposed method.

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