NONLINEAR EMBEDDED CONTROL OF SPARK-IGNITED ENGINES USING THE DERIVATIVE-FREE NONLINEAR KALMAN FILTER

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Abstract
The performance of embedded control systems for the automotive industry depends on the efficiency of the associated nonlinear control and estimation methods. A nonlinear filtering and control method is proposed in this paper for spark ignited (SI) engines. The design of the SI engine's control loop is primarily based on differential flatness theory and on the use of a new nonlinear filtering approach, known as Derivative-free nonlinear Kalman Filtering. It is shown that the through the proposed approach, efficient control of engine parameters such as intake pressure and turn speed, can be succeeded. The followed methodology solves additional problems that arise in the design of the control loop, for example that (i) specific variables of the engine's state vector are not directly measurable (e.g. the ones associated with input pressure), (ii) the dynamic model of the SI engine is not always an accurate one while it is subjected to external perturbations and disturbances (such as friction torques). The proposed control scheme is evaluated through simulation experiments.

Key words
Spark-ignited engines, nonlinear control, differential flatness theory, dynamic feedback linearization, derivative-free nonlinear Kalman Filter.

1 Introduction

In the last years there has been significant research effort in the development of embedded control systems for the automotive industry, aiming at improving the performance of vehicles' engines in terms of produced power, at reducing fuel consumption and at eliminating the emission of exhaust gases. In particular, the problem of control of the rotation speed of SI-engines as well as the problem of control of the engine's pressure manifolds has been approached with different methods [Sugihira and Omori, 2008],[Zhang et al., 2010]. In [Ali and Blath, 2006] a nonlinear state-space controller for turn speed (and consequently for torque) of a spark-ignited engine is proposed. The controller's design is based on feedback linearization in combination with pole placement. In [Zhang and Sun, 2009] time-varying internal model-based design is applied to compensate for the time-varying but angle dependent pressure pulsations in the fuel injection system of SI-engines. In [Leroy et al., 2009] a control method for the air-path system of SI-engines is presented. The first part considers generation of the motion-planning trajectory of the intake manifold pressure from a torque set point. Then, feedforward and feedback control laws are presented. In [Moulin and Chauvin, 2011] a feedback linearization approach based on differential flatness theory is proposed for the control of the air system of a turbocharged gasoline engine. Finally, in [Flärdh et al., 2014] a model-based approach is pursued to maximize an SI-engine's torque through optimal control of the variable valve timing (VVT) and the variable gas turbine (VGT) model. Several other results on nonlinear control of spark ignited engines have been presented in [Alipi et al., 2003],[Ballachi et al., 2013],[Colin et al., 2007],[Nguyen et al., 2013],[Sepulveda, 2012],[Blake Vance et al., 2008].

In this paper a new nonlinear control and filtering scheme is proposed for SI-engines making use of differential flatness theory. By showing that the SI-engine model is a differentially flat one, it becomes possible to transform it to the linear canonical form. For the latter description of the system's dynamics the design of a state feedback controller becomes easier. Considering also that specific elements of the engine's state vector cannot be directly measured it is proposed to estimate them with the use of a nonlinear filtering method, which is the so-called Derivative-free nonlinear Kalman Filter. This filter consists of the Kalman Filter recursion on the linearized equivalent of the SI-engine and on the use of an inverse transformation (diffeomorphism) that is based on differential flatness theory and which provides the estimates of the state vector elements of the initial nonlinear system. By designing the nonlinear Kalman Filter as a disturbance observer it is shown that it is also possible to estimate in real-time disturbance terms affecting the engine's model and subsequently to compensate for them.

The structure of the paper is as follows: in Section 2 the dynamic model of the SI-engine is analyzed and its state-space description is given. In Section 3 feedback linearizing control of the SI-engine using Lie algebra is introduced. In Section 4 feedback linearizing control of the SI engine using differential flatness theory is analyzed. In Section 5 the Derivative-free nonlinear Kalman Filter is used for estimating and compensating for disturbance terms and modeling uncertainty affecting the SI-engine model. In Section 6 simulation experiments are carried out to assess the performance of the proposed nonlinear control and nonlinear filtering scheme for the SI-engine. Finally, in Section 7 concluding remarks are stated.

2 Dynamic Model of the SI Engine

2.1 State-space Description of the SI-engine

It is possible to control the intake pressure p_m and the rotational speed of the engine's shaft ω by adjusting the angle of the air throttle. It is considered that the associated control loop is independent from the loops of the fuel injection control and spark timing control (Fig. 1)

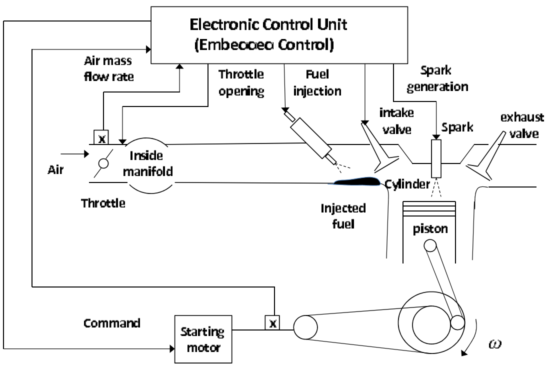


Figure 1. Diagram of the spark-ignited engine

The basic equations of the system are:

$$\begin{aligned}\dot{\omega} &= k_{\omega_1}p_m(t-\tau_d)+k_{\omega_2}+k_{\omega_3}T_{f_m}\\ \dot{p}_m &= k_{p_1}\omega p_m+k_{p_2}\omega+k_{p_3}u\\ y_1 &= \omega\end{aligned} \tag{1}$$

The variable of the intake pressure appears with time delay in the equation of the turn speed in the second row of the model of the SI engine. Using that $p_{md} = p_m(t-\tau_d)$ and

$$p_m(t-\tau_d)=\frac{1}{\tau s+1}p_m \tag{2}$$

while $\tau = a_d/\omega$ and a_d is a parameter that is measured in radians. Denoting $k_d = -1/a_d$ one has about the dynamics of the delayed intake pressure variable

$$p_{md} = k_d\omega(p_{m_d}-p_m) \tag{3}$$

Using the previous formulation, and defining the state variables $x_1=\omega$, $x_2=p_{m_d}$ and $x_3=p_m$, the dynamics of the SI engine is written as [Sugihira and Omori, 2008]

$$\begin{aligned}\dot{x}_1 &= k_{\omega_1}x_2+k_{\omega_2}+k_{\omega_3}T_{f_m}\\ \dot{x}_2 &= K_d x_1(x_2-x_3)\\ \dot{x}_3 &= k_{p_1}x_1x_3+k_{p_2}x_1+k_{p_3}u\end{aligned} \tag{4}$$

where T_{f_m} are friction torques, which can be also perceived as disturbances. In the above equations coefficients k_{p_i}, $i=1,2,3$, k_{ω_i}, $i=1,2,3$ and K_d are associated with the combustion cycle of the SI-engine and are defined in [Sugihira and Omori, 2008],[Zhang et al., 2010]. The model also takes the matrix form $\dot{x}=f(x)+g(x)u$ with

$$f(x)=\begin{pmatrix}k_{\omega_1}x_2+k_{\omega_2}+k_{\omega_3}T_{f_m}\\ k_d x_1(x_2-x_3)\\ k_{p_1}x_1x_3+k_{p_2}x_1\end{pmatrix} \quad g(x)=\begin{pmatrix}0\\0\\k_{p_3}\end{pmatrix} \tag{5}$$

3 Feedback Linearizing Control of the SI-engine Using Lie Algebra

Using Lie derivatives, the following state variables are defined for the SI-engine model of Eq. (4): $z_1 = h_1(x) = x_1$, $z_2 = L_f h_1(x)$ and $z_3 = L_f^2 h_1(x)$. It holds that

$$\begin{aligned}z_2 = L_f h_1(x) &= \tfrac{\partial h_1}{\partial x_1}f_1+\tfrac{\partial h_1}{\partial x_2}f_2+\tfrac{\partial h_1}{\partial x_3}f_3 \Rightarrow\\ z_2 = f_1 &\Rightarrow z_2 = k_{\omega_1}x_2+k_{\omega_2}+k_{\omega_3}T_{f_m}\end{aligned} \tag{6}$$

In a similar manner one obtains

$$\begin{aligned}z_3 = L_f^2 h_1(x) &= \tfrac{\partial z_2}{\partial x_1}f_1+\tfrac{\partial z_2}{\partial x_2}f_2+\tfrac{\partial z_2}{\partial x_3}f_3 \Rightarrow\\ z_3 = f_1 &\Rightarrow z_2 = k_{\omega_1}x_2+k_{\omega_2}+k_{\omega_3}T_{f_m} \Rightarrow\\ z_3 = k_{\omega_1}f_2 &\Rightarrow z_3 = k_{\omega_1}k_d x_1(x_2-x_3)\end{aligned} \tag{7}$$

Moreover, it holds that

$$\begin{array}{l} L_f^3 h_1(x) = \frac{\partial z_3}{\partial x_1} f_1 + \frac{\partial z_3}{\partial x_2} f_2 + \frac{\partial z_3}{\partial x_3} f_3 \Rightarrow \\ L_f^3 h_1(x) = k_{\omega_1} k_d (x_2 - x_3) f_1 + k_{\omega_1} k_d x_1 f_2 - k_{\omega_1} k_d x_1 f_3 \\ \Rightarrow L_f^3 h_1(x) = k_{\omega_1} k_d (x_2 - x_3)[k_{\omega_1} x_2 + k_{\omega_2} + k_{\omega_3} T_{f_m}] + \\ + k_{\omega_1} k_d x_1 [k_d x_1 (x_2 - x_3)] - k_{\omega_1} k_d x_1 [k_{p_1} x_1 x_3 + k_{p_2} x_1] \end{array} \tag{8}$$

Additionally, it holds that

$$\begin{array}{l} L_g L_f h_1(x) = L_g z_2 \Rightarrow \\ L_g L_f h_1(x) = \frac{\partial z_2}{\partial x_1} g_1 + \frac{\partial z_2}{\partial x_2} g_2 + \frac{\partial z_2}{\partial x_3} g_3 \Rightarrow \\ L_g L_f h_1(x) = \frac{\partial z_2}{\partial x_3} k_{p_3} \Rightarrow L_g L_f h_1(x) = \frac{\partial f_2}{\partial x_3} k_{p_3} \\ \Rightarrow L_g L_f h_1(x) = 0 \end{array} \tag{9}$$

while one also obtains

$$\begin{array}{l} L_g L_f^2 h_1(x) = L_g z_3 \Rightarrow \\ L_g L_f^2 h_1(x) = \frac{\partial z_3}{\partial x_1} g_1 + \frac{\partial z_3}{\partial x_2} g_2 + \frac{\partial z_3}{\partial x_3} g_3 \Rightarrow \\ L_g L_f^2 h_1(x) = -k_{\omega_1} k_d k_{p_1} x_1 \end{array} \tag{10}$$

which also shows that the relative degree of the SI-engine model is $n = 3$. It can be also confirmed that it holds

$$\begin{array}{c} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u \end{array} \tag{11}$$

which after defining the new control input $v = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u$ can be written in the linear canonical (Brunovsky) form

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \tag{12}$$

From the relation $z_1^{(3)} = v$ the state feedback control law for the SI-engine that assures asymptotic convergence of the state vector z to the desirable setpoint z_d is given by

$$\begin{array}{c} v = z_{1d}^{(3)} - k_1(\ddot{z}_1 - \ddot{z}_{1,d}) - \\ -k_2(\dot{z}_1 - \dot{z}_{1,d}) - k_3(z_1 - z_{1,d}) \end{array} \tag{13}$$

Using that the control input for the linearized model is $v = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u$ the control input that is finally applied to the SI-engine is

$$u = \frac{1}{L_g L_f^2 h_1(x)} [v - L_f^3 h_1(x)] \tag{14}$$

4 Feedback Linearizing Control of the SI-engine Using Differential Flatness Theory

4.1 Definition of Differentially Flat Systems

Differential flatness is a structural property of a class of nonlinear systems, denoting that all system variables (such as state vector elements and control inputs) can be written in terms of a set of specific variables (the so-called flat outputs) and their derivatives [Fliess and Mounier, 1999], [Lévine, 2011], [Menhour et al., 2013], [Sira-Ramirez, 2004], [Rudolph, 2003], [Rouchon, 2005]. The nonlinear system $\dot{x}(t) = f(x(t), u(t))$ is considered. The time is $t \in R$, the state vector is $x(t) \in R^n$ with initial conditions $x(0) = x_0$, and the input is $u(t) \in R^m$. Next, the properties of differentially flat systems are given [Bououen et al., 2011],[Laroche et al., 2007], [Martin and Rouchon, 1999], [Rigatos, 2011], [Villagra et al., 2007] :

The finite dimensional system $\dot{x}(t) = f(x(t), u(t))$ can be written in the general form of an ordinary differential equation (ODE), i.e. $S_i(w, \dot{w}, \ddot{w}, \cdots, w^{(i)})$, $i = 1, 2, \cdots, q$. The term w is a generic notation for the system variables (these variables are for instance the elements of the system's state vector $x(t)$ and the elements of the control input $u(t)$) while $w^{(i)}$, $i = 1, 2, \cdots, q$ are the associated derivatives. The definition of differentially flat systems is as follows:

Definition: The system $\dot{x} = f(x, u)$, $x \in R^n$, $u \in R^m$ is differentially flat if there exist relations: $h : R^n \times (R^m)^{r+1} \rightarrow R^m$, $\phi : (R^m)^r \rightarrow R^n$ and and $\psi : (R^m)^{r+1} \rightarrow R^m$, such that $y = h(x, u, \dot{u}, \cdots, u^{(r)})$, $x = \phi(y, \dot{y}, \cdots, y^{(r-1)})$, and and $u = \psi(y, \dot{y}, \cdots, y^{(r-1)}, y^{(r)})$. This means that all system dynamics can be expressed as a function of the flat output and its derivatives, therefore the state vector and the control input can be written as $x(t) = \phi(y(t), \dot{y}(t), \cdots, y^{(r)}(t))$, and $u(t) = \psi(y(t), \dot{y}(t), \cdots, y^{(r+1)}(t))$.

4.2 Flatness-based Control of the SI-engine

The state-space description of the SI-engine dynamics given in Eq. (4) is considered again

$$\dot{x}_1 = k_{\omega_1} x_2 + k_{\omega_2} + k_{\omega_3} T_{f_m} \tag{15}$$

$$\dot{x}_2 = K_d x_1 (x_2 - x_3) \tag{16}$$

$$\dot{x}_3 = k_{p_1} x_1 x_3 + k_{p_2} x_1 + k_{p_3} u \tag{17}$$

The flat output of the SI-engine model is taken to be $y = x_1$, which is the engine's turn speed. It will be shown that all state variables of the system and the control input can be written as functions of the flat output and its derivatives, and thus the SI-engine model is a differentially flat one.

Eq. (15) is solved with respect to x_2. Considering that the friction torque T_{f_m} is constant or piecewise constant this gives

$$x_2 = \frac{\dot{x}_1 - k_{\omega_2} - k_{\omega_3}T_{fm}}{k_{\omega_1}} \Rightarrow x_2 = \frac{\dot{y} - k_{\omega_2} - k_{\omega_3}T_{fm}}{k_{\omega_1}} \Rightarrow \\ x_2 = f_2(y, \dot{y}) \tag{18}$$

It is also noted that the friction torque can be considered as a disturbance term to the engine's model and this approach will be followed in Section 5. Next, Eq. (16) is solved with respect to x_3. This gives

$$x_3 = \frac{k_d x_1 x_2 - \dot{x}_2}{k_d x_1} \Rightarrow x_3 = \frac{k_d y f_2(y,\dot{y}) - \dot{f}_2(y,\dot{y})}{k_d y} \Rightarrow \\ x_3 = f_3(y, \dot{y}) \tag{19}$$

Moreover, from Eq. (17) it holds

$$u = \frac{\dot{x}_3 - k_{p_1}x_1x_3 - k_{p_2}x_1}{k_{p_3}} \Rightarrow \\ u = \frac{\dot{f}_3(y,\dot{y}) - k_{p_1} y f_3(y,\dot{y}) - k_{p_2} y}{k_{p_3}} \tag{20}$$

Therefore, all state variables and the control input in the model of the SI-engine are described as functions of the flat output and its derivatives. Consequently, the SI-engine model is a differentially flat one. It holds that

$$y = x_1 \\ \dot{y} = \dot{x}_1 \Rightarrow \dot{y} = k_{\omega_1}x_2 + k_{\omega_2} + k_{\omega_3}T_{f_m} \tag{21}$$

Differentiating once more with respect to time gives

$$\ddot{y} = k_{\omega_1}\dot{x}_2 + k_{\omega_3}\dot{T}_{f_m} \Rightarrow \\ \ddot{y} = k_{\omega_1}k_d x_1(x_2 - x_3) + k_{\omega_3}\dot{T}_{f_m} \tag{22}$$

By deriving once more with respect to time one gets

$$y^{(3)} = k_{\omega_1}k_d\dot{x}_1(x_2 - x_3) + k_{\omega_1}k_d x_1\dot{x}_2 - \\ -k_{\omega_1}k_d x_1\dot{x}_3 + k_{\omega_3}\ddot{T}_{f_m} \Rightarrow \\ y^{(3)} = k_{\omega_1}k_d(x_2 - x_3)[k_{\omega_1}x_2 + k_{\omega_2} + k_{\omega_3}T_{f_m}] + \\ +k_{\omega_1}k_d x_1[k_d x_1(x_2 - x_3)] - \\ -k_{\omega_1}k_d x_1[k_{p_1}x_1x_3 + k_{p_2}x_1] + \\ -k_{\omega_1}k_d k_{p_3}x_1 u + k_{\omega_3}\ddot{T}_{f_m} \tag{23}$$

Therefore one arrives at a description of the SI-engine dynamics which is equivalent to the one obtained from linearization with the use of Lie algebra

$$y^{(3)} = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u \tag{24}$$

where

$$L_f^3 h_1(x) = k_{\omega_1}k_d(x_2 - x_3)[k_{\omega_1}x_2 + k_{\omega_2} + k_{\omega_3}T_{f_m}] + \\ +k_{\omega_1}k_d x_1[k_d x_1(x_2 - x_3)] - \\ -k_{\omega_1}k_d x_1[k_{p_1}x_1x_3 + k_{p_2}x_1] + k_{\omega_3}\ddot{T}_{f_m} \tag{25}$$

and also

$$L_g L_f^2 h_1(x) = -k_{\omega_1}k_d k_{p_3}x_1 \tag{26}$$

For the previous description of the SI-engine dynamics, the following state variables are defined $z_1 = y$, $z_2 = \dot{y}$ and $z_3 = \ddot{y}$, the following state-space model is obtained

$$\dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = L_f^2 h_1(x) + L_g L_f^2 h_1(x) u \tag{27}$$

The linearized model of the SI-engine is finally written in the Brunovsky canonical form

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \tag{28}$$

From the relation $z_1^{(3)} = v$ the state feedback control law for the SI-engine that assures asymptotic convergence of the state vector z to the desirable setpoint z_d is given by

$$v = z_{1d}^{(3)} - k_1(\ddot{z}_1 - \ddot{z}_{1,d}) - \\ -k_2(\dot{z}_1 - \dot{z}_{1,d}) - k_3(z_1 - z_{1,d}) \tag{29}$$

Using that the control input for the linearized model is $v = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u$ the control input that is finally applied to the SI-engine is

$$u = \frac{1}{L_g L_f^2 h_1(x)}[v - L_f^3 h_1(x)] \tag{30}$$

5 Compensation of Disturbances Using the Derivative-free Nonlinear Kalman Filter

It was shown that following the linearization approach based on differential flatness theory, the model of the SI-engine is written in the canonical form

$$y^{(3)} = L_f^3 h_1(x) + L_g L_f^2 h_1(x) u + \tilde{d} \tag{31}$$

The additive disturbance term is given by $\tilde{d} = k_{\omega_1} k_d (x_2 - x_3) k_{\omega_3} T_{f_m} + k_{\omega_3} \ddot{T}_{f_m}$ and describes the effects of friction torque. This term can also incorporate the effects of model parametric uncertainty. It can be considered that the dynamics of the disturbance term is described by its n-th order derivative and the associated initial conditions. Without loss of generality, by setting $n = 3$ one has $\tilde{d}^{(3)} = f_d$. Next, as new state variables of the SI-engine model, the disturbance $\tilde{d}$ and its derivatives are introduced [Rigatos, Siano and Pessolano, 2012]. That is, $z_1 = y$, $z_2 = \dot{y}$, $z_3 = \ddot{y}$, $z_4 = \tilde{d}$, $z_5 = \dot{\tilde{d}}$ and $z_6 = \ddot{\tilde{d}}$. Thus one arrives at the extended state-space model

$$\begin{aligned} \dot{z} &= Az + B\tilde{v} \\ z^m &= Cz \end{aligned} \tag{32}$$

where the control input $\tilde{v} = [v, f_d]^T$ and matrices A, B and C are defined as

$$A = \begin{pmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \end{pmatrix} \quad B = \begin{pmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&0 \\ 0&0 \\ 0&1 \end{pmatrix} \quad C^T = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{33}$$

The associated state estimator is

$$\dot{\hat{z}} = A_o \hat{z} + B_o \tilde{v} + K_f (z^{meas} - C_o \hat{z}) \tag{34}$$

where $A_o = A$, $C_o = C$ and matrix B_o is given by

$$B_o^T = \begin{pmatrix} 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \tag{35}$$

The estimation of the extended state vector is performed using the discrete-time Kalman Filter. To this end, matrices A_o, B_o, C_o are discretized into A_d, B_d, C_D using common discretization methods. Then, the recursion of the Kalman Filter consists of the measurement update and time update stage [Basseville and Nikiforov, 1993],[Rigatos and Tzafestas, 2007], [Rigatos and Zhang, 2009], [Rigatos, 2012], [Rigatos et al., 2014]:

measurement update:

$$\begin{aligned} K(k) &= P^-(k) C_d^T [C_d P^-(k) C_d^T + R]^{-1} \\ \hat{z}(k) &= \hat{x}^-(k) + K(k)[z^m(k) - \hat{z}^m(k)] \\ P(k) &= P^-(k) - K(k) C_d P^-(k) \end{aligned} \tag{36}$$

time update:

$$\begin{aligned} P^-(k+1) &= A_d P(k) A_d^T + Q \\ \hat{z}(k+1) &= A_d \hat{z}(k) + B_d \tilde{V}(k) \end{aligned} \tag{37}$$

After the estimation of the state vector $\hat{z}$ the state feedback control law for the SI-engine given in Eq. (38) is modified with the inclusion of the estimated state variable $\hat{z}_4$. The term $\hat{z}_4$ will compensate for the disturbance input $\tilde{d}$

$$\begin{aligned} v = z_{1,d}^{(3)} - k_1(\ddot{z}_1 - \ddot{z}_{1,d}) - \\ -k_2(\dot{z}_1 - \dot{z}_{1,d}) - k_3(z_1 - z_{1,d}) - \hat{z}_4 \end{aligned} \tag{38}$$

Thus, it is assured that the tracking error for the state vector z will converge asymptotically to 0. That is

$$lim_{t \to \infty} e(t) = 0 \Rightarrow lim_{t \to \infty} z = z_d \tag{39}$$

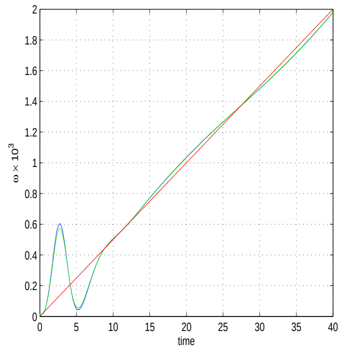
which also means that $lim_{t \to \infty} x_i = x_{i,d},\ i = 1, 2, 3$. It is also noted that through the previously analyzed filtering procedure one can obtain estimates of the input pressure of the SI-engine $\hat{x}_3 = \hat{p}_m$ using Eq (19) and (20) and by substituting in there the estimates of the flat output $\hat{y}$ and of its derivatives.

$$\hat{x}_2 = \frac{\dot{\hat{y}} - k_{\omega_2} + k_{\omega_3} \hat{T}_{f_m}}{k_{\omega_1}} \tag{40}$$

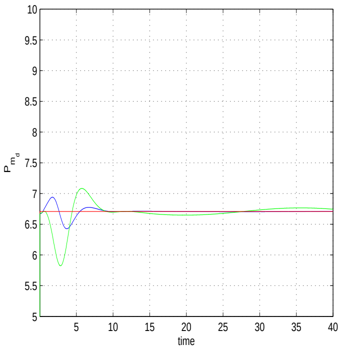
$$\hat{x}_3 = \frac{k_d \hat{y} \hat{x}_2 - \dot{\hat{x}}_2}{k_d \hat{y}} \tag{41}$$

6 Simulation Tests

Simulation tests have been carried out for the assessment of the performance of the proposed nonlinear control scheme based on differential flatness theory and on the Derivative-free nonlinear Kalman Filter. Indicative results where obtained for the tracking of two different setpoints. The time has been measured in sec. It can be observed that the SI engine's state vector elements (that is engine's turn speed ω, input pressure subjected to time delay P_{m_d}, and input pressure P_m) converged fast and smoothly to the associated setpoints. Moreover, the Derivative-free nonlinear Kalman Filter provided accurate estimates of the aggregate disturbance term T_m that is due to the friction torque T_{f_m} and to its derivatives. This enabled to compensate for the disturbances' effects, by including in the control input a new term that was based on the disturbances's estimates. The associated results in the case of tracking of setpoint 1 are depicted in Fig. 2, Fig. 3 and Fig. 4. Additional results for the case of tracking of setpoint 2 are shown in Fig. 5, Fig. 6 and Fig. 7.



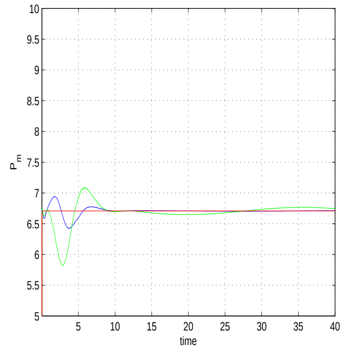
(a)



(b)

Figure 2. Tracking of setpoint 1 (a) Convergence of the SI engine's state variable $x_1 = \omega$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_1$ (green line) (b) Convergence of the SI engine's state variable $x_2 = P_{m_d}$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_2$ (green line)

Finally, the variations of the control input to the SI-engine (air-throttle angle) for the cases of the two aforementioned setpoints are given in Fig. 8. It has been confirmed, that with the application of the proposed nonlinear control scheme the control input exhibited smooth variations.



(a)

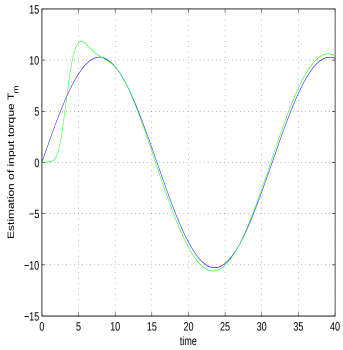


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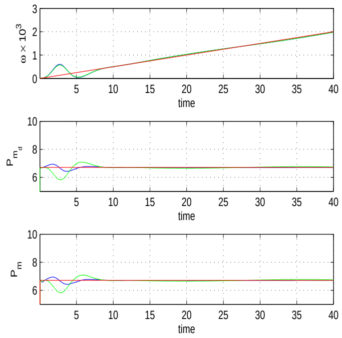
Figure 3. Tracking of setpoint 1 (a) Convergence of the SI engine's state variable $x_3 = P_m$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_3$ (green line) (b) Kalman Filter-based estimation (green line) of the aggregate disturbance torque T_m) (blue line)

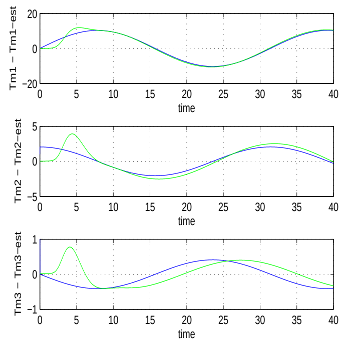
7 Conclusions

The article has proposed a nonlinear embedded control scheme for spark-ignited (SI) engines based on differential flatness theory and on the use of a new nonlinear filtering approach, known as Derivative-free nonlinear Kalman Filtering. It has been shown that the dynamic model of the SI-engine satisfies differential flatness properties, which means that all its state variables and the control input can be written as a function of a primary variable, called flat output, and also as functions of the flat output's derivatives. The flat output for the SI engine was shown to be its rotational speed. The application of differential flatness theory enables to transform the initial nonlinear model of the engine, to the canonical Brunovsky form. For the latter description of the system's dynamics it was made easy to design a stabilizing feedback control law.

Additional problems one has to handle in the design of the nonlinear feedback control scheme were that (i) specific variables of the engine's state vector were not directly measurable (e.g. the ones associated with

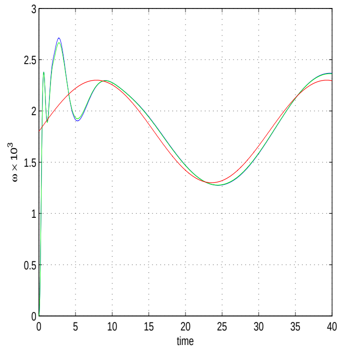


(a)



(b)

Figure 4. Tracking of setpoint 1 (a) Tracking of reference setpoints by the SI engine's state variable $x_1 = \omega$, $x_2 = P_{m_d}$ and $x_3 = P_m$ (b) Kalman Filter-based estimation of disturbance input torque T_m and of its derivatives



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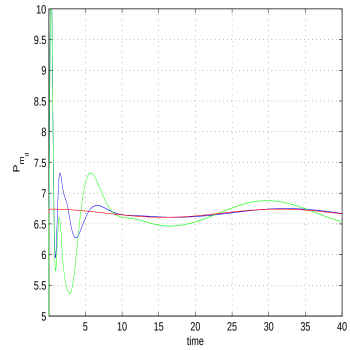

(b)

Figure 5. Tracking of setpoint 2 (a) Convergence of the SI engine's state variable $x_1 = \omega$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_1$ (green line) (b) Convergence of the SI engine's state variable $x_2 = P_{m_d}$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_2$ (green line)

input pressure), (ii) the dynamic model of the SI engine is not always an accurate one while it is subjected to external perturbations and disturbances (such as friction torques). To solve (i) the paper has proposed the use of Derivative-free nonlinear Kalman Filtering. This consists of the application of the Kalman Filter recursion to the linearized equivalent of the SI engine followed by an inverse transformation, in accordance to the diffeomorphism defined through differential flatness theory. This approach finally provides exact estimates for the state variables of the initial nonlinear system. To solve (ii), the Derivative-free nonlinear Kalman Filter is redesigned as a disturbance estimator and this permits to obtain simultaneously estimates of the non-measurable elements of the SI engine's state vector and of the disturbance or modeling uncertainty terms that affect the engine's model. The efficiency of the proposed nonlinear control method has been confirmed through simulation experiments.

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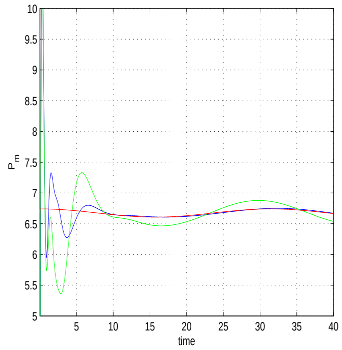

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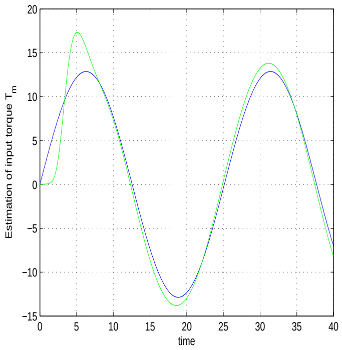
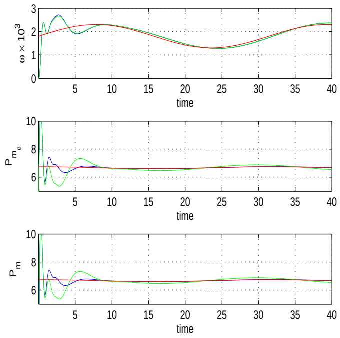


(b)

Figure 6. Tracking of setpoint 2 (a) Convergence of the SI engine's state variable $x_3 = P_m$ (blue line) to reference setpoint (red line) and associated state estimate $\hat{x}_3$ (green line) (b) Kalman Filter-based estimation (green line) of the aggregate disturbance torque T_m) (blue line)



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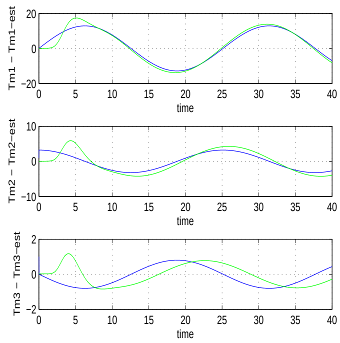

(b)

Figure 7. Tracking of setpoint 2 (a) Tracking of reference setpoints by the SI engine's state variable $x_1 = \omega, x_2 = P_{m_d}$ and $x_3 = P_m$ (b) Kalman Filter-based estimation of disturbance input torque T_m and of its derivatives

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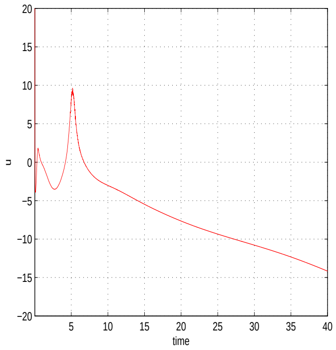

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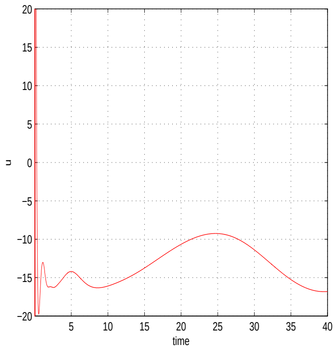

(b)

Figure 8. Variations of the control input of the SI engine (air-throttle angle) in case of (a) tracking of reference setpoint 1 (b) tracking of reference setpoint 2

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