The Optimization of Spacecraft Motion with Ion Engine in the Earth – Moon System

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Abstract: The control laws optimization and driving trajectories of spacecraft with the low-thrust engine fulfilling transport problems in a system the Earth - Moon are described. With the usage of formalism of the Pontryagin maximum principle the necessary requirements of control optimality are obtained from the minimum duration view point. The driving is considered within the framework of a flat circular restricted problem of three bodies. The solutions for problems optimum on response of reaching of inconvertible a libration points of a system and flyby of Moon on required angular distance are obtained. The approach for selection of initial parameters of boundary value problems is offered.

Keywords: Computer-aided method, nonlinear systems, modeling, spacecraft

1. INTRODUCTION

The spacecraft motion in the Earth–Moon system traditionally settled accounts within the framework of spheres of action theory. It is proved by the use of large thrust engines within the framework of the impulsive raising of ballistic tasks. However, while the motion calculation of in the Earth–Moon system with low thrust, the reactive acceleration from the motive system is comparable with the perturbation accelerations of the Earth and the Moon. In such terms it is inconsistent to suppose that an optimum control does not depend on gravitation influences of a second attracting center. The problem of optimal control from point of fast-acting and trajectories of motion appropriate to it within the framework of the flat circular limited task of three bodies is the main theme research is connected with.

Indignation from disturbance of the gravitation field of the Earth attraction and other celestial bodies is not taken into account. Indignations, related to the work of a motive setting, for example, the falling of power under the influence of radiation belts of the Earth or limits on the control, related to the features of spacecraft angular motion, in this research are not taken into account.

2. MATHEMATICAL MODEL FOR MOTION AND OPTIMUM FAST-ACTING CONTROL

The spacecraft motion is described in the inertial polar barycentric coordinates system. The basic plane is combined with the middle plane of the Moon motion, in this plane the polar axis is directed toward the point of the vernal equinox, a center of coordinates is located in Earth–Moon system barycenter. The position of the vehicles masses center is regard to barycenter is determined by the vector $\bar{r}$ and the polar angle φ. The distance from spacecraft to the Earth and the Moon accordingly is set by vectors $\overline{r_E}$ and $\overline{r_M}$. The engines thrust reactive acceleration vector lies in the basic plane, it's size depends only on the spacecraft mass. Direction of the acceleration is set by the control angle $\lambda(t)$, which is counted off from radius-vector in counter clockwise direction. Positions of the Earth and the Moon are given by setting vectors R_E, R_M sizes constant and by the angle ϑ, having a permanent rate of change ω equal to the average angular rate of the Moon motion on the orbit.

All phase coordinates of the system are brought to the dimensionless form. For this purpose all linear distances refer to the middle distance from the Moon to the barycenter, time to the middle period of the Moon circulation, acceleration to the gravity acceleration at the Moon's orbit. In this case equations of motion are:

$$\frac{d\overline{m}}{dt} = \frac{a_0\delta}{c}, \qquad \frac{dr}{dt} = V_r, \qquad \frac{d(\Delta\varphi)}{dt} = \frac{V_\varphi}{r} - \omega,$$

$$\frac{dV_r}{dt} = \frac{V_\varphi^2}{r} - (1-\mu_M)\frac{r + R_E\cos(\Delta\varphi)}{r_E^3} - \frac{\mu_M\left(r - R_M\cos(\Delta\varphi)\right)}{r_M^3} + a_r, \tag{1}$$

$$\frac{dV_\varphi}{dt} = -\frac{V_r V_\varphi}{r} + (1-\mu_M)\frac{R_E\sin(\Delta\varphi)}{r_E^3} - \frac{\mu_M R_M\sin(\Delta\varphi)}{r_M^3} + a_\varphi,$$

$$a_r = \frac{a_0\delta}{(1-\overline{m})}\cos\lambda, \qquad a_\varphi = \frac{a_0\delta}{(1-\overline{m})} sin\,\lambda.$$

Here, V_r, V_φ are components of dimensionless velocity vector of spacecraft relative to barycenter; $\mu_M \approx 0.0123$ – the ratio of the Moon mass toward the total mass of the Earth-Moon system; a_0 is the nominal dimensionless acceleration from engines thrust; c – dimensionless exhaust velocity of propellant; δ - is a switch on and off function engines; $\overline{m} = m_T/m_0$ - is a percentage consumption of propellant $\Delta\varphi = \varphi - \vartheta$ is angle Moon – barycenter – spacecraft. These distance denotations from the spacecraft to the Earth and the Moon are equal accordingly:

$$r_E = \sqrt{r^2 + R_E^2 + 2rR_E \cos(\Delta\varphi)},\quad r_M = \sqrt{r^2 + R_M^2 - 2rR_M \cos(\Delta\varphi)} \tag{2}$$

We use the minimum performance duration of a special purpose objective as a criterion:

$$T = \int_0^T dt \to \min \ . \tag{3}$$

We will enter the vector of phase coordinates system $\overline{X} = (r,\ \Delta\varphi,\ V_r,\ V_\varphi,\ \overline{m})^T$. Formally the optimization task is described as follows: it is required to define the vector of control functions $\overline{u}(t) = (\lambda(t), \delta(t))^T$ from the possible set of U, satisfying boundary conditions $\overline{X}(t_0) = \overline{X_0}$, $\overline{X}(T) = \overline{X_K}$, and delivering minimum to the optimal criterion of T at the fixed project parameters vector

$$\overline{u}_{opt}(t) = \arg\min_{\overline{u}(t)} T\left(\overline{u} \,\middle|\, \overline{p} = fixe, \overline{X}_0 = fixe, \overline{X}_K = fixe\right). \tag{4}$$

Under the formalism of Pontryagin's maximum principle will enter the vector of conjugate variables $\overline{P} = \left(P_r,\ P_{\Delta\varphi},\ P_{V_r},\ P_{V_\varphi},\ P_{\overline{m}}\right)^T$ and Gamiltonian $H = (d\overline{X}/dt)^T \cdot \overline{P}$. According to a Gamiltonian maximum we define optimum direction of acceleration vector $\lambda_{opt}(t)$ and the function of switch on and off engines:

$$\sin\lambda_{opt} = \frac{P_{V_\varphi}}{\sqrt{{P_{V_r}}^2 + {P_{V_\varphi}}^2}}, \tag{5}$$

$$\delta = \begin{cases} 0, & \Delta < 0 \\ 1, & \Delta > 0 \end{cases}, \quad \Delta = \frac{P_m}{c} + \frac{\sqrt{{P_{V_r}}^2 + {P_{V_\varphi}}^2}}{1 - \overline{m}}. \tag{6}$$

Thus, a task about optimum on a fast-acting flat motion in the Earth–Moon system is taken to the following point-to-point four-parametric to the boundary problem. It is required to find such initial values conjugate variables, that on the ends of optimum trajectory initial and eventual conditions were executed $\overline{X}(t_0) = \overline{X}_0$, $\overline{X}(T) = \overline{X}_T$.

Let starting orbit be a circle, with r_0 radius, and the spacecraft position in relation to the Moon is $\Delta\varphi_0$ in initial moment, in this case the initial vector of phase coordinates is set by values

$$\overline{X}_0 = \left(\frac{r_0}{R_M},\ \ \Delta\varphi_0,\ \ V_{r0} = 0,\ \ V_{\varphi 0} = \sqrt{\frac{R_M}{r_0}}\right)^T. \tag{7}$$

Depending on flight objective the eventual values of vector of phase co-ordinates are set as follows:

1) for the task of achievement of steady points of the libration system

$$\overline{X}_T = \left(1,\ \ \Delta\varphi_k = \pm\frac{\pi}{3},\ \ V_{rk} = 0,\ \ V_{\varphi_k} = 1\right)^T; \tag{8}$$

2) for flight of the Moon on the set angular distance $\Delta\varphi_k$ with the not fixed velocity vector, taking into account the transversal terms for the velocity vector

$$\overline{X}_T = \left(1,\ \ \Delta\varphi_k,\ \ P_{V_r} = 0,\ \ P_{V_\varphi} = 0\right)^T. \tag{9}$$

Because of $\dot{P}_{\overline{m}} \le 0$, at all $t \in [t_0,\ T]$, and for the task on an optimum fast-acting the expense of working body is not fixed in the eventual moment of time, $P_{\overline{m}}(T) = 0$ in obedience to the transversal terms. At $t \in [0, T)$ $P_m(t) > 0$, therefore for a task on an optimum fast-acting function of switch on-switch off engines, according to (6), $\delta(t) \equiv 1$ that an engine works without shutdowns.

3. NUMERICAL METHODS, APPLIED AT A DECISION

The simulation of optimum movement spacecraft is reduced to the decision of a Koshi problem for the system of the differential equations of movement and the interfaced multipliers (1-2), (7) and optimum control (5-6). The numerical decision the Runge-Kutta method with constant step was used.

The introduction of the augmented phase coordinates vector $\overline{z} = \left(r,\ \Delta\varphi,\ V_r,\ V_\varphi,\ \overline{m},\ P_r,\ P_{\Delta\varphi},\ P_{V_r},\ P_{V_\varphi},\ P_{\overline{m}}\right)^T$ allows to bring a decision over the point-to-point regional task to the decision of the system of nonlinear equalizations: $f(\overline{z}(t_0)) = 0$ is the vector function of it is disparity in eventual moment of time depending on the initial values of phase coordinates and attended multipliers in initial moment of time $\overline{z}(t_0)$. For the purpose to achieve of steady points of libration:

$$f(\overline{z}(t_0)) = \left(r(T) - 1,\ \ \Delta\varphi(T) - \Delta\varphi_K,\ \ V_r(T),\ \ V_\varphi(T) - 1\right)^T, \tag{10}$$

and for flight of the Moon on the set distance

$$f(\overline{z}(t_0)) = \left(r(T) - 1,\ \ \Delta\varphi(T) - \Delta\varphi_K,\ \ P_{V_r}(T),\ \ P_{V_\varphi}(T)\right)^T. \tag{11}$$

If the initial value of vector is $\overline{z}(t_0)$, satisfying terms (10 or 11) with the set error at the differential equalizations (1-6), it is certain, that the required task of optimization is decided. The disparity function of the system $f(\overline{z}(t_0)) = 0$ is very sensible to the initial values of the picked up parameters and have multi-extreme, «gullied» character.

For the determination of the system of nonlinear equalizations the modified Newton method was used with the automatic estimation of convergence and change of step of calculation of derivatives and limits on increases. In order to receive decisions with the different values of project parameters vehicles and the scope terms of flights the method of continuation on a parameter essence was used the whole point of this method is described below.

Let the decision of optimization task for some fixed vector of design-ballistic parameters of flight $\overline{b_0}$ is known, i.e. the value of the extended vector of phase coordinates $\bar{z}(t_0)$ is known delivering minimum to the criterion of optimality $T(\overline{b_0})$. It is required to find the decision of task in the raising, but for other parameters of flight $\overline{b_*}$.

If the statement of the problem is correct, at a small difference $\overline{b_0}$ from $\overline{b_*}$, the decision of task of optimization (4) and criterion of optimality (3) will differ insignificantly. Let break up a segment from $\overline{b_0}$ to $\overline{b_*}$ on k to parts and will build a sequence

$$\overline{b_i} = \overline{b_0} + \frac{\overline{b_*} - \overline{b_0}}{k}.$$

If to consider that at the insignificant change of vector of design-ballistic parameters of value of vector $\bar{z}$ change linearly, will get

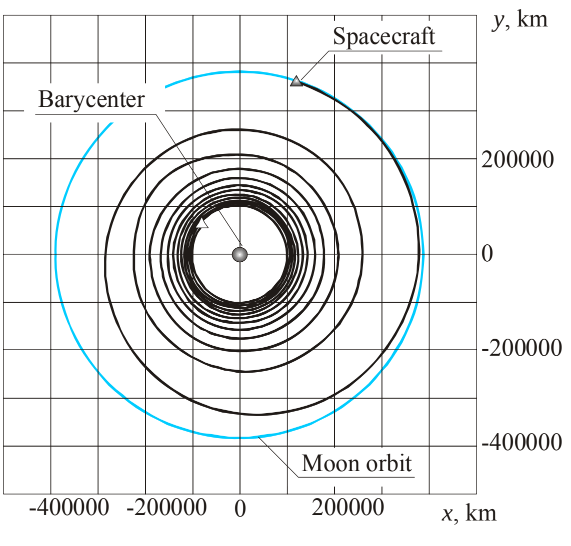
$$\bar{z}(\overline{b_{i+1}}) = \bar{z}(\overline{b_i}) + A_i(\overline{b_{i+1}} - \overline{b_i}). \quad (12)$$

The formula (12) looks simplier in the case when only one variable changes in a vector $\overline{b_i}$. In Konstantinov et al. (2001) the method of continuation on a parameter showed high efficiency for the decision of the task about the optimum on a fast-acting spacecraft motion in the Earth-Moon system in case of the use of averaged equalizations of motion. In this research the use of this method allowed to get the decision of the put task for the different values of design-ballistic parameters and scope terms of flight.

4. ACHIEVEMENT THE LIBRATION POINTS OF THE EARTH – MOON SYSTEM

The problem of achievement of the libration point L4 of the Earth-Moon systems (the spacecraft moves on a circular orbit in radius equal to average radius of Moon's orbit, advancing it on 60°) is considered. Design parameters of the device are chosen close to parameters research spacecraft of the European Space Agency "SMART-1": $m_0 = 400$ kg, $P = 0.5$ H, $c = 15$ km/s.

The gathering speed maneuvers and increase of large semi-axes orbit vehicles with the low thrust engines on low orbits near the Earth are well studied, including indignations from the Moon gravitation and other celestial bodies; not centrality of the gravitation field and atmosphere of the Earth. From point of influence of the Moon gravitation on the optimum laws of control and trajectory of motion the guidance of vehicle on high orbits is interesting, therefore the circular orbit is examined with large semi-axis $a_0 = 100000$ km as starting. Differential equalizations, which describe the conduct of the system (1-7), allow to carry out the continuous change of parameter of the second gravitation body from zero to the required value. It enables to use the results of the task about optimum a fast-acting flight as an initial approximating for the picked up values of the attended multipliers between circular complanar orbits at the not fixed angular distance of flight in eventual moment of time. In this case, according to the transversal condition there is $P_{\Delta\varphi}(T) = 0$. If in equations (1-7) of put $\mu_M = 0$ (not to take into account the Moon's gravitation), at any moment of time $P_{\Delta\varphi}(t) \equiv 0$ and the order of boundary problem goes down to three. The decision of this task and approximate formulas for the initial values of the attended multipliers were got in this research (Starinova, 2005).



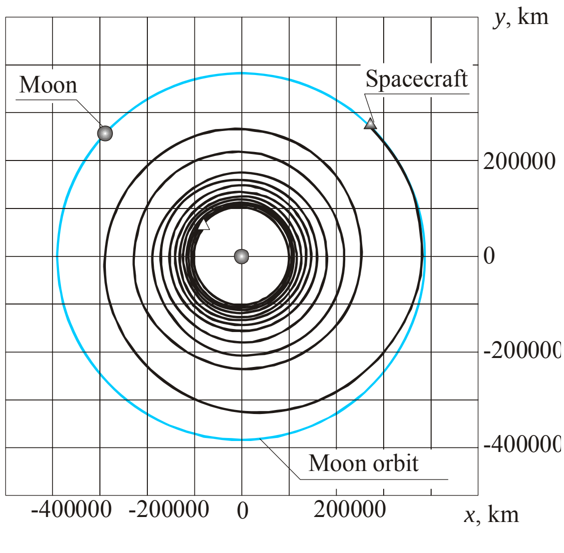


Fig. 1. a) Trajectories of Moon orbit achievement without the gravitation account; b) Optimum on a fast-acting achievement of libration point L4 of the Earh-Moon system

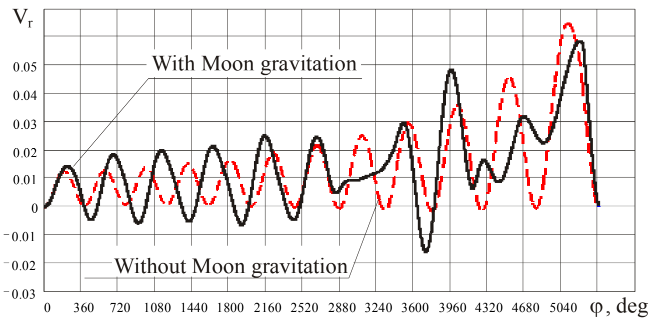


Fig. 2. Dependence of dimensionless radial making speed on angular distance of flight for the task of achievement of point of libration L4

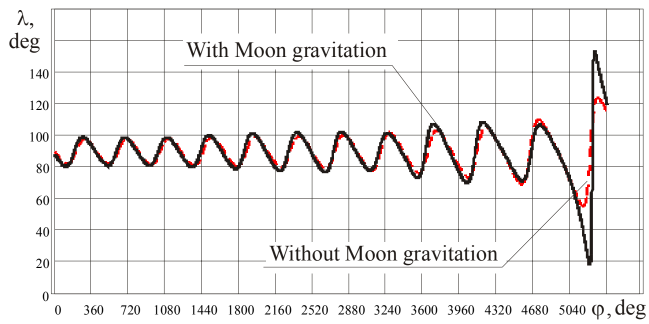


Fig. 3. Optimum program of management, proper achievement of libration point L4 of the Earth-Moon system

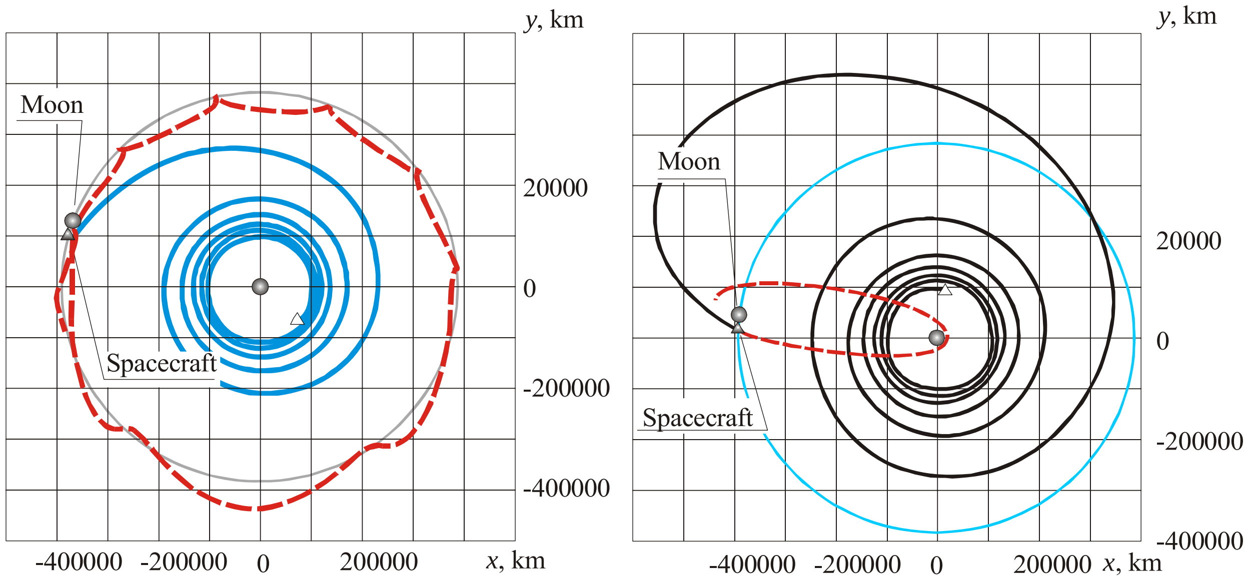


Fig. 4. Optimum on a fast-acting trajectories of flight of the Moon: a) the monotone change of the orbit radius (T = 39.31 day); b) the current orbit radius is exceeded by the radius of the Moon's orbit (T = 68.13 day).

The changed vector $\bar{z}(t_0)$ and proper control and trajectory of motion taking into account the Moon's gravitation is determined by using the method of moving on the gravitation parameter of the second attracting body from 0 to μ_M with some not fixed eventual angular distance of spacecraft-Moon $\Delta\varphi^*(T)$. Then the transversal condition in eventual moment of time $P_{\Delta\varphi}(T)=0$ is substituted by the condition of achievement the required angular distance of spacecraft-Moon $\Delta\varphi(T)=\Delta\varphi_K$.

Now, it is possible to get the changed vector $\bar{z}(t_0)$, optimum control and trajectory at the required scope terms, carrying out procedure of moving on a variable $\Delta\varphi(T)$ from $\Delta\varphi^*$ to $\Delta\varphi_K = 60°$.

The use of such method allows not to test substantial difficulties in the choice of initial values of the attended multipliers for the decision of regional task and to get decisions for different scope terms and project parameters spacecraft.

In the Fig. 1a) a trajectory, used as an initial approaching, is demonstrated (achievement of the Moon orbit with ambulatory by angular distance of the Earth-Moon without the account of the Moon's gravitation).

In the Fig. 1b) the final optimum trajectory of achievement the point of libration L4 is represented, taking into account the Moon's gravitation and with the fixed angular distance in eventual moment of time $\Delta\varphi_K = 60°$. In a Fig. 2 there are dependences of the spacecraft speed component on an optimum trajectory from angular distance of flight.

For comparison analogical dependences are resulted for the task of flight of the Moon's orbit nonregistering the Moon gravitation used as an initial approximating (stroke lines). In Fig. 3 there is the optimum program of control, proper achievement of libration point L4.

Moon renders most influence on an optimum control and change of phase coordinates the last coils of orbit in moments of the maximal rapprochement with spacecraft. Especially this influence is noticeable on the components of speed vector. However, the whole duration and angular distance of flight, an optimum control and trajectories of motion appeared close to the decision, used as an initial approximating.

5. TASK OF MOON FLIGHT ON THE SET ANGULAR DISTANCE

The task of Moon flight is examined at the fixed angular distance of spacecraft-Moon in eventual moment of time. The initial values of project parameters (except for engines thrust) and starting orbit are chosen the same, as for the task of achievement of points of libration. The nominal thrust of the motive setting is $P = 1$ N.

As an initial approximating for the decision of task the decision of task is used about achievement of circular orbit with a radius equal to the middle radius of the Moon's orbit, described in Starinova (2005). The use of method of moving on a parameter allows to get the decisions of task of optimization for different scope terms and project parameters of spacecraft.

It is shown that the Moon gravitation renders substantial influence on the optimum law of control and proper motion trajectory. For example, in Fig. 4 there are optimum trajectories for flight of Moon on angular distance from it at $5°$, and a start from a circular orbit with $a_0 = 100000$ km. A continuous line shows the motion with the included engine and optimum law of thrust direction. By the dotted line trajectories of passive spacecraft motion after completion of having a special purpose task and disconnecting of engines are shown.

The task of optimization has multi-extreme character. At first, there are different classes of decisions, shown in Figs. 4a and 4b: trajectories with the monotonous change of radius of spacecraft orbit and trajectories with the increase of radius of trajectory there is a substantially more radius of Moon's orbit. In addition, within the limits of one class trajectories angular distance of which differs on the whole amount of coils are possible. Naturally, the value of the optimality criterion, for all these trajectories is different.

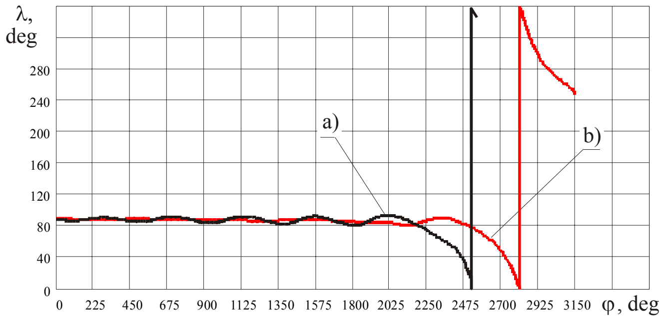


Fig. 5. The Optimum program of management, proper the task of flight of the Moon

On trajectories with the monotonous change of the spacecraft orbit radius (Fig. 4a), appears to be taken by gravitation field of the Moon and goes out on the reserved Moon-centric trajectory after the engine cut off.

These trajectories can be used for forming the orbit round the Moon with set-up parameters. Trajectories with the non monotone change of the orbit radius (Fig. 4b) are characterized by high values of spacecraft – Moon relative speed at the moment of engines cut-off and can be used for forming the trajectory of the returning to the Earth. In Fig. 5 there is the program of optimum control for monotonous and the non monotone of optimum trajectories.

6. CONCLUSIONS

Thus, the offered method showed the efficiency for optimization of difficult multiple-turn trajectories of spacecraft motion with the low thrust engines in the attraction field of two bodies. The obtained optimum laws of control and appropriate to him trajectories of motion can be used for the decision of tasks of forming of the set selen-centric orbits.

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