TIME–DELAYED CONTROL OF CHAOS IN THE IKEDA SYSTEM

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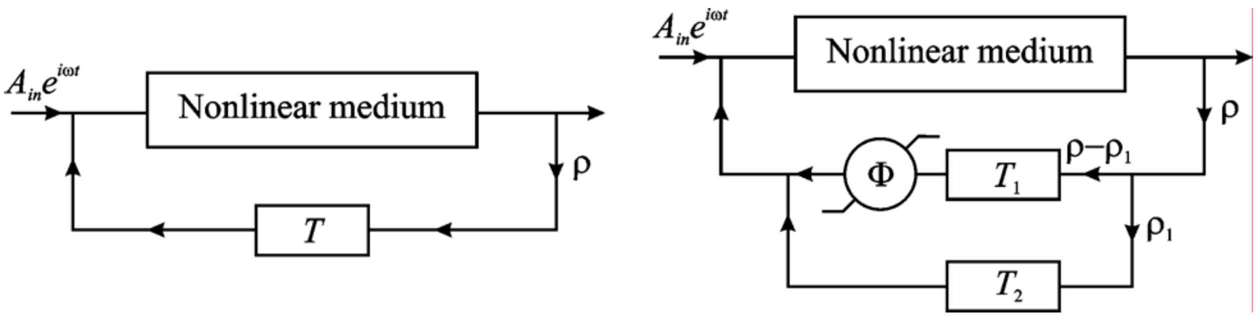
The problem of controlling chaos usually means stabilization of unstable periodic orbits of a dynamical system by simple controlling impacts [1]. Using controlling chaos techniques allows to reach steady periodic behavior at that parameters where without control the dynamics is chaotic. Among the methods of chaos control the most popular is the time-delayed autosynchronization suggested by Pyragas [2] when an additional feedback path (FP) is applied with delay time approximately equal to the period of motion to be stabilized. In the paper [3] this method has been extended on a case of stabilization of unstable fixed points near the Hopf bifurcation threshold. In this case delay time should be close to the period of unstable perturbations. In this work, we develop a generalization of the Pyragas method for distributed systems which itself possess time-delayed terms. The proposed method is based on introduction of an additional feedback loop with parameters chosen so that the fundamental frequency components after passing through different FP appear in phase, while the most unstable sidebands appear in antiphase, thus suppressing each other. As an example we consider the well-known Ikeda system [4] that is a ring-loop resonator partly filled with a nonlinear dispersive media and forced with an external harmonic signal. Fig. 1 shows the schematics of the Ikeda system (a) and its modification with an additional FP (b).



a b

Fig. 1. Schematics of a ring-loop resonator partly filled with a nonlinear dispersive media (a) and its modification with an additional feedback serving for chaos control. $T_{1,2}$ are delay times, ρ and ρ_1 are amounts of feedback, Φ is a phase shifter required to adjust the phase difference of the two signals passing through different loops.

Suppose that the nearly single-frequency plane wave propagates in the in the nonlinear dispersive medium. In that case one can write down the nonlinear Schrödinger equation (NSE)

$$i\left(\frac{\partial A}{\partial t}+V_g\frac{\partial A}{\partial x}\right)+\frac{\omega_0''}{2}\frac{\partial^2 A}{\partial x^2}+\beta\left|A\right|^2 A=0\,. \tag{1}$$

describing the slowly varying complex amplitude of the wave, $A(x,t)$ [5]. Here V_g is the group velocity, ω_0'' is the group velocity dispersion parameter, β is the nonlinearity parameter. Taking into account the external forcing and two delayed feedback loops we obtain the following delayed boundary condition

$$A(0,t)=A_{in}e^{i\omega t}+\rho_1 e^{i\psi_1}A(l,t-T_1)+(\rho-\rho_1)e^{i\psi_2}A(l,t-T_2)\,, \tag{2}$$

where $\psi_{1,2}$ and $T_{1,2}$ are phase shifts and delay times of the two FPs respectively, l is the length of the nonlinear medium. Nonlinear dynamics of this system with only one feedback loop

($\rho_1 = 0$, see Fig. 1(a)) has been investigated in detail in [6]. It has been shown, that there are two basic types of behavior. In case of weak dispersion increasing of either input power or amount of feedback leads to the Ikeda instability followed by period doubling transition to chaos. This picture is quite similar to the widely investigated Ikeda system [4,7]. When the dispersion is strong the instability of a single-frequency regime is caused by the modulation instability, and quasiperiodic route to chaos is usually observed.

First we examine a simplified model in the case of zero dispersion ($\omega_0'' = 0$) when the equations (1) and (2) could be reduced to the discrete map

$$A_{n+1} = A_{in} + \rho_1 A_n e^{i\left(\varphi + |A_n|^2\right)} + \left(\rho - \rho_1\right) A_{n-1} e^{i\left(\varphi + |A_{n-1}|^2\right)}, \tag{3}$$

where A_n — complex amplitude of a signal at n-th moment of discrete time, $\varphi = \psi_{1,2} + \omega l / V_g$ is a total phase shift of the signal passing around the resonator. Here we suppose that $\psi_1 = \psi_2$ and $T_2 = 2T_1$. When $\rho_1 = 0$ (3) becomes the classical Ikeda map which is well investigated [4,7].

We analyze theoretically the stability of fixed points for the map (3) and find that threshold of the Ikeda instability grows with ρ_1 and tends to infinity when $r \equiv \rho_1/\rho = 0.5$. Thus application of the second FP leads to stabilization of the steady-state regime. However, at $r > 1/3$ a domain of quasiperiodic motion appears restricting the capability of control. The theoretical results are in good agreement with numerical calculations. Fig. 2 presents the bifurcation maps on A_{in}, φ plane for $\rho = 0.5$ without (a) and with (b) the control. Theoretical lines of tangent, period-doubling and Neimark-Sacker bifurcations are shown in Fig. 2 with dashed, solid and dotted curves respectively. One can see that the domain of stability of the steady-state regime enlarges with the increase of control feedback. The domain of stability is maximal at $r = 1/3$.

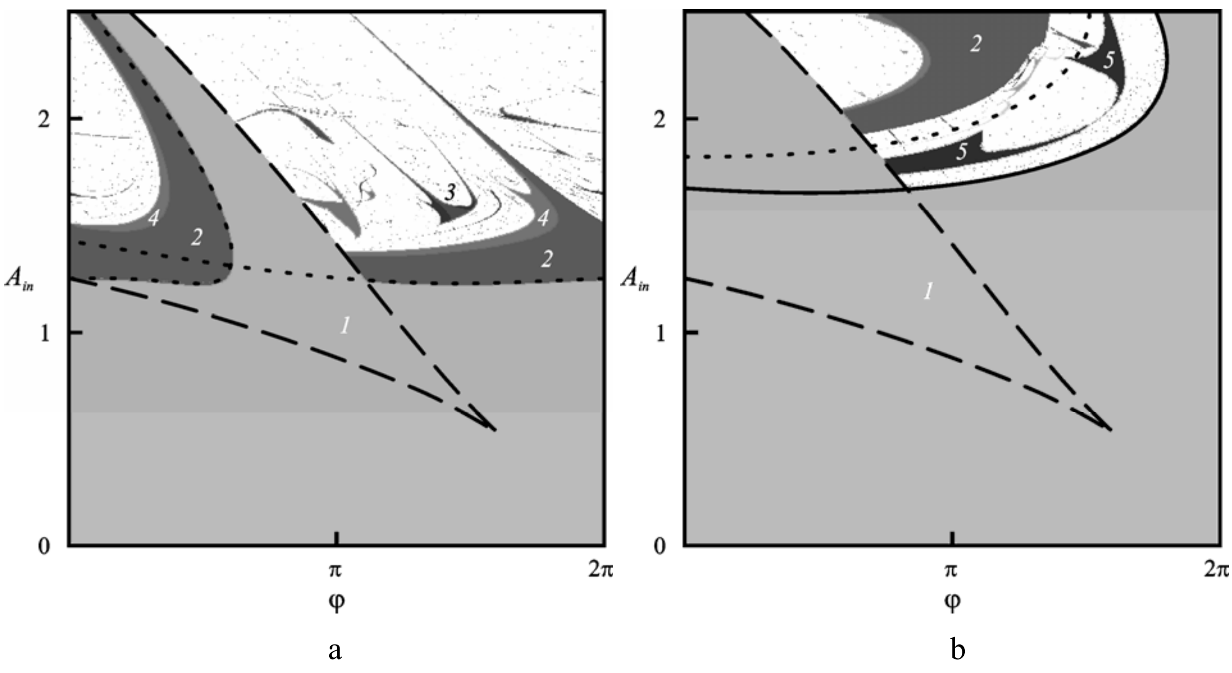


Fig. 2. Bifurcation maps for the modified Ikeda map (3) for $r = 0$, i.e. there is no control feedback (a) and for $r = 0.36$ (b). Theoretical lines of a tangent bifurcation (dashed), period-doubling bifurcation (solid) and Neimark-Sacker bifurcation (dotted) are plotted above the bifurcation maps. Numbers on the maps mark the corresponding periods. Domains of quasiperiodic and chaotic motion are colored in white.

Second, we consider the dynamics of the distributed system (1), (2) when the dispersion should be taken into account. Analysis shows that suppression of instability is provided with the following choice of control parameters:

$$\psi_1 = 2\pi n + \psi_2 + \omega\left(T_1 - T_2\right),$$
$$\Omega\left(T_1 - T_2\right) = 2\pi m + \pi,$$

where Ω is the self-modulation frequency which can be obtained from the results of numerical simulations [6]. Numerical modeling show that proper selection of delay times $T_{1,2}$ and phases $\psi_{1,2}$ provide stable periodic motion even at those parameters for which the behavior of the system without control is chaotic. In Fig. 3 waveforms of output signal intensities $I = \left|A(l)\right|$ are presented confirming that adding of the control loop with rather small amount of feedback completely suppresses chaos and results in single-frequency operation with constant amplitude.

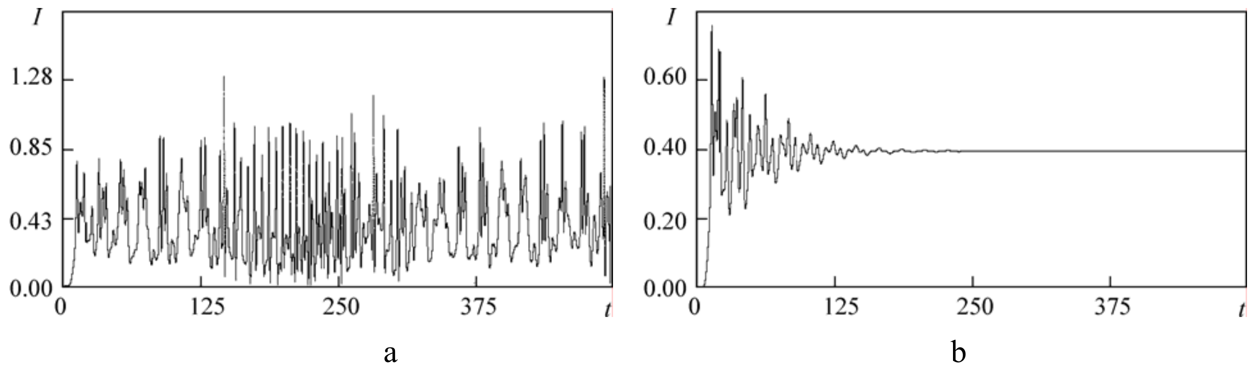


Fig. 3. Output signal waveforms of for the system (1), (2) at $\rho = 0.5$, $\beta = 1$, $V = 2$, $\omega_0'' = 1.0$ $T_2 = 9$, $\omega = 0$, $A_{in} = 0.5$ and $\rho_1 = 0$ (a) and $\rho_1 = 0.2$ (b).

In this work we proposed the modification of time-delayed controlling chaos technique for suppress of self-modulation instability in delayed-feedback distributed systems. Efficiency of the method was demonstrated for the example of a ring-loop resonator partly filled with a nonlinear dispersive media. It was shown that application of the additional external feedback with properly chosen delay and phase shift can stabilize steady-state single-frequency oscillations even when the dynamics of the system without control is chaotic.

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