ADAPTIVE CONTROL FOR THE SYSTEMS WITH HYSTERESIS AND UNCERTAINTIES

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Abstract: This paper discusses the adaptive control for the uncertain discrete time linear systems preceded with hysteresis and disturbances. The contribution of the paper is the fusion of the hysteresis model with the adaptive control techniques without constructing the inverse hysteresis nonlinearity. The proposed scheme eliminates the traditional over-parameterization by only adapting the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the sliding mode controller. The stability in the sense that all signals in the loop remain bounded can be guaranteed. Furthermore, if the disturbance and reference signal are slow varying with respect to the sampling frequency, the output tracking error can be controlled to be as small as required by choosing the design parameters. Simulation results show the effectiveness of the proposed algorithm.

Keywords: Hysteresis, uncertainty, discrete-time system, adaptive control, sliding mode control.

1. INTRODUCTION

The hysteresis phenomenon can be found in diverse disciplines ranging from, e.g., smart materials (Banks and Smith, 2000; Moheimani and Goodwin, 2001; Webb, *et al*, 1998), to ferromagnetism and superconductivity (Mayergoyz, 1991), to economics (Cross, *et al*, 2001), to geosciences (Guyer, *et al*, 1994). When a plant is preceded by the hysteresis, the system usually exhibits undesirable inaccuracies or oscillations and even instability (Tao and Kokotovic, 1995). The development of control techniques to mitigate the effects of hysteresis has been studied for decades and has recently re-attracted significant attention, e.g. Moheimani and Goodwin (2001) and the references therein. Much of the interest is a direct consequence of the importance of hysteresis in numerous new applications. Interest in studying dynamic systems with actuator hysteresis is also motivated by the fact that they are nonlinear system with nonsmooth nonlinearities for which traditional control methods are insufficient and thus require development of alternate effective approaches (Tan and Baras, 2004). Development of a general frame for control of an uncertain dynamical system in the presence of unknown hysteresis is quite a challenging task.

To deal with the control problem of a dynamical system preceded by hysteresis, the thorough characterization of the hysteresis nonlinearities forms the foremost task. Appropriate hysteresis models may then be applied to the formulation of control algorithms. Hysteresis model can be roughly classified into physics-based models and phenomenological models. Physics-based models are built on first principles of physics (Jiles and Atherton, 1986). Phenomenological models are used to produce behaviors similar to those of the physical systems without necessarily providing physical insight into the problems. The basic idea consists of the modeling of the real complex hysteresis nonlinearities by the weighted aggregate effect of all possible so-called elementary hysteresis operators. Elementary hysteresis operators are noncomplex hysteretic nonlinearities with a simple mathematical structure. The popular phenomenological models are Preisach model (Adly, *et al*, 1991; Croft, *et al*, 2001; Natale, *et al*, 2001; Mayergoyz, 1991), Prandtl-Ishlinskii model (Brokate and Sprekels, 1996; Visintin, 1994), and Krasnosel'skii-Pokrovskii model (Krasnosel'skii-Pokrovskii, 1989; Visintin, 1994). The Preisach model and Krasnosel'skii- Pokrovskii (KP) model are parameterized by a pair of threshold variables (Mayergoyz, 1991), whereas the Prandtl-Ishlinskii (PI) model is a superposition of elementary stop operators which are parameterized by a single threshold variable (Visintin, 1994).

With the developments in various hysteresis models, it is by nature to seek means to fuse these hysteresis models with the available control techniques to mitigate the effects of hysteresis, especially when the hysteresis is unknown, which is a typical case in many practical applications. However, the results on the fusion of the available hysteresis models with the available control techniques is surprisingly spare in the literature (Chen, *et al*, 2006; Su, *et al*, 2000, 2005; Tao and Kokotovic, 1995). The most common approach in coping with hysteresis in the literature is to construct an inverse operator, which is pioneered by Tao and Kokotovic (1995), and the reader may refer to, for instance, Krejci and Kuhnen (1999) and Tan and Baras (2004) and the references therein. Essentially, the inversion problem depends on the phenomenological modeling methods. Due to multi-valued and non-smooth features of hysteresis, the inversion always generates certain errors and possesses strong sensitivity to the model parameters. These errors directly make the stability analysis of the closed-loop system very difficult except for certain special cases (Tao and Kokotovic, 1995), where over-parameterization is used to handle the bilinearly coupled hysteresis parameters and linear controller parameters.

This paper proposes a new approach for fusion of the adaptive control techniques with the PI hysteresis model for uncertain linear discrete time dynamical systems preceded with hysteresis and disturbances. The adaptive control technique used in this paper is based on the first author's previous work (Chen, 2006) for discrete time dynamical systems with unknown parameters and disturbance, where the disturbance is composed of model uncertainties, nonlinearities, external disturbances, etc. Only the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated online using adaptive algorithms with dead-zone. The adaptive sliding mode controller, in which linear feedbacks of the variable $s(k)$ and the estimation error are added respectively to adjust the system response and to compensate the disturbance, is synthesized by using the estimated parameters. The stability in the sense that all signals in the loop remain bounded can be guaranteed. Furthermore, if the disturbance and reference signal are slow varying with respect to the sampling frequency, the output tracking error can be controlled to be as small as required by choosing the design parameters.

The remainder of this paper is organized as follows. Section 2 describes the problem and the PI-type hysteresis model. In Section 3, first, the parameters (which are generated from the parameters of the linear system and the density function of the hysteresis) directly needed in the formulation of the controller are adaptively estimated. Then, the adaptive sliding mode controller is formulated and the stability of the closed system is analyzed. In Section 4, the simulation results are presented. Section 5 concludes this paper.

2. PROBLEM STATEMENT

2.1 System Description

Consider a system composed of an uncertain linear plant preceded by hysteresis described by

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})w(k) + \sigma(k)\,, \qquad (1)$$

$$w(k) = H[v](k)\,, \qquad (2)$$

where $y(k)$ and $v(k)$ are the output and input, respectively; $\sigma(k)$ is an unknown signal composed of model uncertainties, nonlinearities, external disturbances, etc. For simplicity, $\sigma(k)$ is called "disturbance" in this paper. $H[v]$ is the hysteresis operator which will be given later; q^{-1} is the delay operator. d is the time delay; $A(q^{-1})$ and $B(q^{-1})$ are polynomials defined by

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n}\,, \qquad (3)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}\,,\ b_0 \neq 0\,,\ m < n\,, \qquad (4)$$

and $A(q^{-1})$ and $B(q^{-1})$ are assumed to be relatively prime. $w(k)$ is the hysteresis output described in (2).

The control purpose is to drive the output $y(k)$ to track a uniformly bounded signal $y_d(k)$ for the uncertain system (1) preceded by hysteresis.

For the system (1), we make the following assumptions.

Assumption 1. The time delay d and the plant order n are known.

Assumption 2. The plant is in minimum phase.

Assumption 3. The parameters $\{a_i, b_i\}$ in the polynomials $A(q^{-1})$ and $B(q^{-1})$ are unknown. Furthermore, the sign of $b_0 \neq 0$ is known. Without loss of generality, it is assumed $b_0 > 0$.

2.2 Hysteresis Model

In this note, we adopt the Prandtl-Ishlinskii (PI) model in discrete time. The hysteresis is denoted by the operator $w(k) = H[v](k)$, where $v(k)$ is the input, $w(k)$ is the output of the hysteresis. The basic element of the PI operator is the so-called stop operator $\omega(k) = E_r[v](k)$ with threshold r. For arbitrary piece-wise monotone function $v(k)$, define $e_r : R \to R$ as

$$e_r(v) = \min(r, \max(-r, v))\,. \qquad (5)$$

For any initial value $w_{-1} \in R$ and $r \geq 0$, the stop operator $E_r[\cdot\ ;\ w_{-1}](k)$ is defined as

$$E_r[v;\ w_{-1}](0) = e_r(v(0) - w_{-1}), \quad (6)$$

$$E_r[v;\ w_{-1}](k) = e_r(v(k) - v(k_i) + E_r[v; w_{-1}](k_i)), \quad (7)$$

for $k_i < k \leq k_{i+1}$, where the function $v(k)$ is monotone for $k_i \leq k \leq k_{i+1}$ (Brokate and Sprekels, 1996). The stop operator is mainly characterized by the threshold parameter $r \geq 0$ which determines the height of the hysteresis region in the $(v,\ w)$ plane. For simplicity, denote $E_r[v;\ w_{-1}](k)$ by $E_r[v](k)$ in the following of this note. It should be noted that the stop operator $E_r[v](k)$ is rate-independent. The PI hysteresis model is defined by

$$w(k) = \int_0^\infty p(r) E_r[v](k)dr, \quad (8)$$

where $p(r)$ is the density function which is usually unknown, satisfying $p(r) \geq 0$ with $\int_0^\infty rp(r)dr < \infty$ (Su, *et al*, 2005; Visintin, 1994; Webb, *et al*, 1998). Since the density function $p(r)$ vanishes for large values of r, it is reasonable to assume that there exists a constant R such that $p(r)$ =0 for $r > R$ (Brokate and Sprekels, 1996; Visintin, 1994). Thus, model (8) gives

$$w(k) = \int_0^R p(r) E_r[v](k)dr. \quad (9)$$

3. ADAPTIVE CONTROL FOR THE DISCRETE TIME SYSTEMS PRECEDED BY HYSTERESIS

3.1 Some Preliminaries

Define the variable

$$s(k+d) = C(q^{-1})\big(y(k+d) - y_d(k+d)\big), \quad (10)$$

where $C(q^{-1})$ is a Schur polynomial defined by

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_n q^{-n}. \quad (11)$$

Let the sliding surface be defined by

$$s(k+d) = 0. \quad (12)$$

It is obvious that $\lim_{k\to\infty} s(k) = 0$ means $\lim_{k\to\infty}\big(y(k) - y_d(k)\big) = 0$. Now, consider the polynomial equation

$$C(q^{-1}) = A(q^{-1})\Omega(q^{-1}) + q^{-d}F(q^{-1}), \quad (13)$$

where $\Omega(q^{-1})$ and $F(q^{-1})$ are in the following form

$$\Omega(q^{-1}) = h_0 + h_1 q^{-1} + \cdots + h_{d-1}q^{-d+1}, \quad (14)$$

$$F(q^{-1}) = f_0 + f_1 q^{-1} + \cdots + f_{n-1}q^{-n+1}. \quad (15)$$

Thus, the parameters in $\Omega(q^{-1})$ and $F(q^{-1})$ can be determined uniquely and $h_0 = 1$ (Astrom and Wittenmark, 1989).

Assumption 4. The disturbance $\sigma(k)$ is of the following form

$$\sigma(k) = \sigma^{(1)}(k) + \sigma^{(2)}(k), \quad (16)$$

$$\left|\sigma^{(1)}(k)\right| \leq \alpha\left(\left\|\phi(k-d)\right\|_2^2 + \sum_{i=0}^{m-1}\int_0^R \left(E_r[v](k-d-i)\right)^2 dr\right)^{\frac{1}{2}} \quad (17a)$$

$$\left|\sigma^{(2)}(k)\right| \leq \bar{\rho}, \quad (17b)$$

where α is a very small unknown nonnegative constant; $\bar{\rho}$ is an unknown nonnegative constant; the regressor $\phi(k)$ and the norm $\left\|\phi(k)\right\|_2$ are respectively defined as

$$\phi(k) = \left[\, y(k),\ y(k-1), \cdots,\ y(k-n+1)\right]^T \quad (18)$$

and

$$\left\|\phi(k)\right\|_2 = \left\{\phi^T(k)\phi(k)\right\}^{\frac{1}{2}}. \quad (19)$$

If m is unknown, m should be replaced by n in the proposed formulation.

Now, by multiplying equation (13) with $y(k)$ and employing equation (1), it yields

$$C(q^{-1})y(k+d) = G(q^{-1})w(k) + F(q^{-1})y(k) + \omega(k+d), \quad (20)$$

where $G(q^{-1})$ and $\omega(k)$ are respectively defined by

$$G(q^{-1}) = \Omega(q^{-1})B(q^{-1}) = g_0 + g_1 q^{-1} + \cdots + g_{m+d-1}q^{-m-d+1}, \quad (21a)$$

$$\omega(k) = \Omega(q^{-1})\sigma(k). \quad (21b)$$

Substituting (2) into (21) yields

$$C(q^{-1})y(k+d) = \phi^T(k)\theta + \int_0^R g_0 p(r)E_r[v](k)dr + \cdots + \int_0^R g_{m+d-1}p(r)E_r[v](k-m-d+1)dr + \omega(k+d), \quad (22)$$

with

$$\theta = \left[f_0, f_1, \cdots,\ f_{n-1}\right]^T. \quad (23)$$

For the uncertainty $\omega(k)$, let

$$\omega(k) = \omega^{(1)}(k) + \omega^{(2)}(k), \quad (24)$$

where $\omega^{(1)}(k)$ and $\omega^{(2)}(k)$ are respectively defined by

$$\omega^{(1)}(k) = \Omega(q^{-1})\sigma^{(1)}(k),\ \omega^{(2)}(k) = \Omega(q^{-1})\sigma^{(2)}(k). \quad (25)$$

From (17) and (25), it yields

$$\left\|\omega^{(1)}(k)\right\|_2 \leq \alpha\cdot\beta T(k-d),\ \left\|\omega^{(2)}(k)\right\|_2 \leq \rho, \quad (26)$$

where $T(k-d)$ is defined by

$$T(k-d) = \max_{0\leq\tau\leq d-1}\left(\left\|\phi(k-d-\tau)\right\|_2^2 + \sum_{i=0}^{m-1}\int_0^R \left(E_r[v](k-d-\tau-i)\right)^2 dr\right)^{\frac{1}{2}} \quad (27)$$

β and ρ are unknown constants defined by

$$\beta = \sum_{i=0}^{d-1}\left|h_i\right|,\ \rho = \bar{\rho}\left(\sum_{i=0}^{d-1}\left|h_i\right|\right). \quad (28)$$

3.2 Parameter Estimation

Since the parameters $\{a_i, b_i\}$ of the polynomials $A(q^{-1})$ and $B(q^{-1})$ are unknown, the parameters in $G(q^{-1})$ and $F(q^{-1})$ can not be obtained. In the following, we will try to estimate these parameters together with the hysteresis density function by using adaptive algorithms with dead-zone. Let

$$\hat{\theta}(k)=\left[\hat{f}_0(k),\hat{f}_1(k),\cdots,\hat{f}_{n-1}(k)\right]^T \quad (29)$$

denote the estimate of the unknown parameter θ at the k -th step and let $\hat{p}_i(r,k)$ be the estimate of $g_i p(r)$ at the k -th step for a fixed r. Define the estimation error as

$$e(k)=C(q^{-1})y(k)-\phi^T(k-d)\hat{\theta}(k-1) - \sum_{i=0}^{m+d-1}\int_0^R \hat{p}_i(r,k-1)E_r[v](k-d-i)dr . \quad (30)$$

Then, by substituting (22) into (30), $e(k)$ can also be expressed as

$$e(k)=\phi^T(k-d)\{\theta-\hat{\theta}(k-1)\} +\omega(k) + \sum_{i=0}^{m+d-1}\int_0^R \left(g_i p(r)-\hat{p}_i(r,k-1)\right)E_r[v](k-d-i)dr . \quad (31)$$

Because β and ρ are unknown, their corresponding estimates $\hat{\beta}(k-1)$ and $\hat{\rho}(k-1)$ are used to construct the dead-zone function.

Now, define

$$\hat{\eta}(k-1)=\gamma\cdot\hat{\beta}(k-1)T(k-d) +\hat{\rho}(k-1) , \quad (32)$$

where $\gamma>0$ is a design parameter which is very small, the initial values of $u(k)$ (for $k=3-2d-m,\cdots,0$) and $y(k)$ (for $k=3-2d-n,\cdots,0$) can be assigned to be any small values. The dead-zone function is defined as

$$\lambda(k)=\begin{cases}1-\dfrac{\hat{\eta}(k-1)}{|e(k)|} & if \ |e(k)|>\hat{\eta}(k-1)\\ 0 & otherwise\end{cases} . \quad (33)$$

For simplicity, define

$$D(k-d)=\left(\gamma\cdot T(k-d)\right)^2 +\phi^T(k-d)\phi(k-d)+\sum_{i=0}^{m+d-1}\int_0^R\left(E_r[v](k-d-i)\right)^2dr . \quad (34)$$

The estimates $\hat{\theta}(k)$ and $\hat{p}_i(r,k)$ are updated by the following adaptation laws with constraint

$$\hat{\theta}(k) = \hat{\theta}(k-1)+\chi\frac{\lambda(k)e(k)\phi(k-d)}{1+D(k-d)} , \quad (35)$$

$$\hat{p}_i'(r,k)=\hat{p}_i(r,k-1) +\chi\frac{\lambda(k)e(k)E_r[v](k-d-i)}{1+D(k-d)} , \quad (36)$$

$$\hat{p}_0(r,k)=\begin{cases}\hat{p}_0'(r,k) & if \ \hat{p}_0'(r,k)\geq 0\\ 0 & otherwise\end{cases} , \quad (37a)$$

$$\hat{p}_i(r,k)=\hat{p}_i'(r,k) ,\ i=1,2,\ldots,m+d-1 . \quad (37b)$$

The initial condition $\hat{p}_i(r,0)$ should be chosen such that $\hat{p}_i(r,0)\geq 0$ and $\int_0^R r\hat{p}_i(r,0)dr<\infty$. The estimates $\hat{\beta}(k)$ and $\hat{\rho}(k)$ are respectively updated by the following adaptation laws

$$\hat{\beta}(k)=\hat{\beta}(k-1)+\frac{\chi\lambda(k)|e(k)|}{1+D(k-d)}\gamma T(k-d) , \quad (38)$$

$$\hat{\rho}(k)=\hat{\rho}(k-1)+\frac{\chi\lambda(k)\cdot|e(k)|}{1+D(k-d)} . \quad (39)$$

The initial values $\hat{\beta}(0)$ and $\hat{\rho}(0)$ can be any very small nonnegative constants.

Remark 1: The parameter adaptation gain χ ($0<\chi<1$) in (35)-(39) is introduced to adjust the adaptation speed.

Lemma 1. For the adaptation algorithm in (35)-(39), the following properties hold.

(P1). $\hat{\theta}(k)$, $\hat{\beta}(k)$, $\hat{\rho}(k)$ and $\int_0^R\left(\tilde{p}_i(r,k)\right)^2dr$ are bounded for all $k>0$.

(P2). $\sum_{k=1}^{\infty}\frac{\lambda^2(k)e^2(k)}{1+D(k-d)}<\infty$.

(P3). $\lim_{k\to\infty}\frac{\lambda^2(k)e^2(k)}{1+D(k-d)}=0$

(P4). $\sum_{k=\upsilon}^{\infty}\left\|\hat{\theta}(k)-\hat{\theta}(k-\upsilon)\right\|_2^2<\infty$,

$$\sum_{k=\upsilon}^{\infty}\left(\hat{\beta}(k)-\hat{\beta}(k-\upsilon)\right)^2<\infty ,$$

$$\sum_{k=\upsilon}^{\infty}\left(\hat{\rho}(k)-\hat{\rho}(k-\upsilon)\right)^2<\infty ,$$

$$\sum_{k=\upsilon}^{\infty}\int_0^R\left(\hat{p}_i(r,k)-\hat{p}_i(r,k-\upsilon)\right)^2dr<\infty$$

for any positive finite integer υ .

3.3 The Adaptive Sliding Mode Control Design

In this section, the control input is determined so that the sliding mode exists along the sliding surface $s(k)=0$. Define

$$W(k)=-\phi^T(k)\hat{\theta}(k) -\int_0^R\hat{p}_1(r,k)E_r[v](k-1)dr -\cdots-\int_0^R\hat{p}_{m+d-1}(r,k)E_r[v](k-m-d+1)dr +C(q^{-1})y_d(k+d)+\delta\cdot s(k) -e(k) , \quad (40)$$

where δ is the weighting factor in the range of $0<\delta<1$, $e(k)$ defined in (30) is added to compensate the disturbance.

For an assigned admissible error ε , we try to find the pseudo-inverse $V^*(k)$ such that

$$\left|\int_0^R\hat{p}_0(r,k)E_r[V^*](k)dr-W(k)\right|\leq\varepsilon . \quad (41)$$

Let $[v_{\min}, v_{\max}]$ be the practical input range to the hysteresis operator, which is a strict subset of $[-R, R]$. Let the saturation output of $\int_0^R\hat{p}_0(r,k)E_r[v](k)dr$ for $v_{\min}\leq v\leq v_{\max}$ be $\hat{W}_{sat}(k)$, i.e.

$$\left|\int_0^R\hat{p}_0(r,k)E_r[v](k)dr\right|\leq\hat{W}_{sat}(k) . \quad (42)$$

$V^*(k)$ should be derived as that stated in Chen, Su and Kano (2006).

In this paper, the adaptive sliding mode control input is considered as

$$v(k)=V^*(k) . \quad (44)$$

3.4 *Stability Analysis and Disturbance Rejection*

In the following, we suppose that

$$\left|W(k)\right| \le \hat{W}_{sat}(k) . \quad (45)$$

From (40) and (41), it has

$$\phi^T(k)\hat{\theta}(k) + \int_0^R \hat{p}_0(r,k)E_r[v](k)dr + \int_0^R \hat{p}_1(r,k)E_r[v](k-1)dr + \cdots + \int_0^R \hat{p}_{m+d-1}(r,k)E_r[v](k-m-d+1)dr = C(q^{-1})y_d(k+d) + \delta s(k) + \rho(k) - e(k), \quad (46)$$

where $\rho(k)$ is an unknown signal satisfying

$$\left|\rho(k)\right| \le \varepsilon . \quad (47)$$

By equations (31) and (46), we have

$$\phi^T(k)\theta + \int_0^R g_0 p(r)E_r[v](k)dr + \cdots + \int_0^R g_{m+d-1} p(r)E_r[v](k-m-d+1)dr = C(q^{-1})y_d(k+d) + \delta s(k) + e(k+d) + \rho(k) - e(k) - \omega(k+d) + \varpi(k), \quad (48)$$

where $\varpi(k)$ is defined by

$$\varpi(k) = \phi^T(k)\left(\hat{\theta}(k+d-1) - \hat{\theta}(k)\right) + \int_0^R \left(\hat{p}_0(r,k+d-1) - \hat{p}_0(r,k)\right)E_r[v](k)dr + \cdots + \int_0^R \left(\hat{p}_{m+d-1}(r,k+d-1) - \hat{p}_{m+d-1}(r,k)\right)E_r[v](k-m-d+1)dr \quad (49)$$

By substituting (48) into (21) and using (10), the variable $s(k+d)$ becomes

$$s(k+d) = \delta s(k) + e(k+d) + \rho(k) - e(k) + \varpi(k) . \quad (50)$$

Theorem 1. Consider the system (1) satisfying Assumptions (1-4) controlled by the adaptive controller (44). Then, there is a constant $\gamma^* > 0$ such that, for $0 < \gamma < \gamma^*$, the existence of a constant $\alpha^* > 0$ is guaranteed such that, for the class of disturbances described in (16-17) satisfying $0 < \alpha < \alpha^*$,

(i) $\lim_{k\to\infty} \lambda(k)\left\|e(k)\right\|_2 = 0$, i.e.

$\limsup_{k\to\infty} \{\left\|e(k)\right\|_2 - \hat{\eta}(k-1)\} \le 0$,

(ii) all the signals in the closed-loop remain bounded.

In the steady state, from (50), the dynamics of $s(k)$ is governed by

$$s(k+d) = \delta s(k) + e(k+d) + \rho(k) - e(k) + \varpi(k) . \quad (51)$$

Before investigating the disturbance rejection property of the proposed controller, an additional assumption is made.

Assumption 5. The disturbance $\sigma(k)$ described in (16-17) and the desired trajectory $y_d(k)$ are slowly varying with respect to the sampling frequency, i.e.

$$\sigma(k) \approx \sigma(k-1) \approx \cdots \approx \sigma(k-d), \quad (52a)$$

$$y_d(k) \approx y_d(k-1) \approx \cdots \approx y_d(k-d) . \quad (52b)$$

If Assumption 5 is satisfied, the controlled system behavior is governed by the external inputs in the steady state. Thus, it can be seen that $\phi(k)$ and $E_r[v](k)$ are also slowing varying with respect to the sampling frequency in the steady state. By observing the definition of $e(k)$ in (30) and applying the result (P4) of Lemma 1, it can be concluded that $e(k)$ is also slowing varying with respect to the sampling frequency in the steady state, i.e. $e(k+d) \approx e(k)$. Furthermore, by the definition of $\varpi(k)$ and the results of Lemma 1 and Theorem 1, it is obvious that $\varpi(k) \to 0$ as $k \to \infty$. Therefore, in the steady state, equation (51) yields

$$s(k+d) \approx \delta s(k) + \rho(k) , \quad (53)$$

By applying (47), (53) means that $\limsup_{k\to\infty} \left|s(k)\right| \le \dfrac{\varepsilon}{1-\delta}$. By the definition of $s(k)$, it can be seen that the output tracking error is determined by the design parameters ε, δ and the roots of $C(q^{-1})$.

Remark 2: If Assumption (A5) is satisfied, the output tracking error can be controlled to be as small as possible by choosing very small ε.

Remark 3: The parameter δ ($0 < \delta < 1$) in (32) is introduced in order to modify the system response and to control the size of the control signal.

4. SIMULATION RESULTS

Consider the system described by

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_0 w(k-1) + b_1 w(k-2) + \sigma(k),$$

$$w(k) = \int_0^R p(r)E_r[v](k)dr$$

with $a_1 = -1.3$, $a_2 = 0.42$, $b_0 = 1$, $b_1 = -0.65$, $p(r) = e^{-0.067(r-1)^2}$, the initial value of w_{-1} is 0, and

$$\sigma(k) = 0.04(\cos(0.05k))\left(y(k-1) + y(k-2) + 2\int_0^R p(r)E_r[v](k-1)dr\right) + 0.1(\sin(0.05k))\left(1 + \frac{\left\|\phi(k-1)\right\|_2 + \sqrt{\int_0^R \left(E_r[v](k-1)\right)^2 dr}}{1 + \left\|\phi(k-1)\right\|_2 + \sqrt{\int_0^R \left(E_r[v](k-1)\right)^2 dr}}\right)$$

with $\phi(k-1) = \left[y^T(k-1), y^T(k-2)\right]^T$.

The polynomial $C(q^{-1})$ is chosen as

$$C(q^{-1}) = 1 + q^{-1} + 0.25q^{-2} .$$

$\phi(0)$ is set to $\phi(0) = [0.15,\ 0]^T$. The initial condition $\hat{\theta}(0)$ is set to $\hat{\theta}(0) = [0,\ 0]^T$. The initial conditions of $\hat{p}_0(r,0)$ and $\hat{p}_1(r,0)$ are all set to 0.4. The initial value of w_{-1} is simply assumed as 0. The parameter γ in (32) is chosen as $\gamma = 0.1$.

The adaptation gain χ is chosen as $\chi = 0.2$. The weighting factor δ in $W(k)$ is chosen as $\delta = 0.1$. R is chosen as R=20, L is chosen as L=2000. The admissible error ε is set to $\varepsilon = 0.005$. The desired trajectory of the output is $y_d(k) = \sin(0.01k\pi)$. The convergence of $\hat{\theta}(k) = \left[\hat{f}_0(k), \hat{f}_1(k)\right]^T$, $\hat{p}_0(r,\ k)$ and $\hat{p}_1(r,\ k)$ is confirmed by simulation.

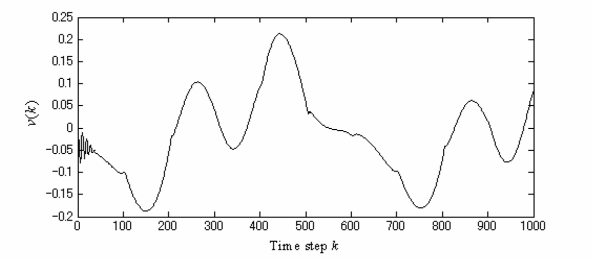

Fig.1 The control input.

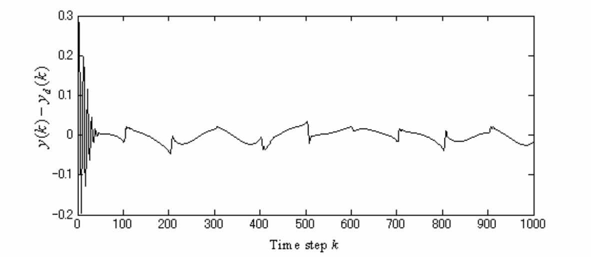

Fig. 2 The output tracking error.

Figure 1 shows the control input described in (44), Figure 2 shows the output tracking error. It can be seen that a very good result is obtained in a relatively short time.

5. CONCLUSION

This paper discusses the design of an adaptive sliding mode controller for a general class of uncertain discrete time linear systems preceded by hysteresis and disturbance, where the hysteresis is represented by Prandtl-Ishlinskii model, and the disturbance is composed of model uncertainties, nonlinearities, external disturbances, etc. The unknown parameters of the system together with the density function of the hysteresis are updated by adaptive algorithms with dead zone. The robust sliding mode tracking controller, in which linear feedbacks of the variable $s(k)$ and the estimation error are added respectively to adjust the system response and to compensate the disturbance, is synthesized by using the estimated parameters and the density function of the hysteresis. The stability of the controlled system is guaranteed in the sense that all signals remain bounded. If the disturbance and the reference signal are slowly varying with respect to the sampling frequency, the proposed sliding mode controller can reject the disturbance and output tracking can be controlled by the design parameters.

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