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OPTIMAL CONTROL OF COUPLED BUILDINGS UNDER SEISMIC EXCITATION

Polina Tkachenko
Scientific center for information technologies and artificial intelligence
Sirius University of Science and Technology
Sirius Federal Territory
PTkachen@gmail.com

Dmitry Balandin
Scientific center for information technologies and artificial intelligence
Sirius University of Science and Technology
Sirius Federal Territory
dbalandin@yandex.ru

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Abstract

The article explores the concept of managing coupled buildings under seismic excitation. Connecting two closely spaced structures enables the redistribution of energy and reduction of structural responses. To minimize the resulting damage, the use of smart control is proposed. Smart control operates using feedback, allowing the system to adapt in real-time. To solve the problem of finding an optimal control, the paper suggests considering it within the framework of multicriteria optimization problems, with the criteria being the maximum deformation of each building described using the generalized H_2-norm.

The article presents a method for solving multicriteria problems using linear matrix inequalities (LMIs) and Hermeyer convolution. Detailed examples are provided for a system of two coupled buildings. The results of computing the optimal smart control for systems of varying dimensions, as well as the outcomes of modeling using real earthquake data, are presented MATLAB software, utilizing the SDPT3 and YALMIP libraries, is employed for performing the calculations and visualizing the results.

Key words

Optimal control, multi-objective control problem, coupled strucures, seismic protection, linear matrix inequalities, generalized H_2-norm

1 Introduction

The structural control of seismic protection of structures has garnered significant attention worldwide [Spencer and Sain, 1997; Nishimura and Kojima, 1999; Doroudi and Lavassani, 2021; Shelenok, 2024]. The primary goal of seismic protection systems is to mitigate the response induced by earthquakes. Minimizing the damage to buildings from severe earthquakes is crucial for maintaining urban functionality, especially if buildings house critical facilities. Although passive isolation systems have been employed in civil structures, in the event of earthquakes of such magnitude that these systems prove ineffective, a smart seismic protection system may prove beneficial.

A smart seismic protection system consists of sensors, data processing units, acquisition units, and controller devices, including dampers and actuators, which generate the required control force (see Fig. 1)

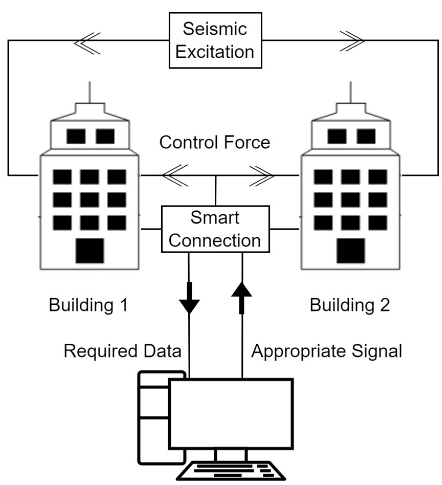

Figure 1. The model of a smart control

These components allow smart systems to monitor environmental changes and adapt. Among the various ideas and technical solutions for seismic protection of tall buildings, the idea of coupling closely spaced structures together to damp their vibrations during earthquakes is a key concept [Doroudi and Lavassani, 2021; Klein et al., 1972; Kunieda, 1976; Richardson et al., 2013; Xiang and Nishitani, 2015; Park and Ok, 2015]. Using coupled building control appears to be an effective way to reduce critical responses. This idea was first introduced by Klein [Klein et al., 1972], and four years later, it was further developed in Japan by Kunieda [Kunieda, 1976]. Miller's research on two adjacent single-degree-of-freedom (SDOF) structures subjected to harmonic ground motion has also contributed to this field. Over the last four decades, research into coupled buildings has continued to evolve based on structural control techniques.

Similar approaches are already practiced [Doroudi and Lavassani, 2021; Ali and Al-Kodmany, 2012; Hadi et al., 2018; Nishimura et al., 2011] and are often referred to as "Sky Bridge". The example is shown in Fig. 2 (this figure borrowed from [Doroudi and Lavassani, 2021])

Figure 2. Pinnacle@Duxton tall building

In the coupled building control strategy, two nearby structures are coupled through settlement joints, which are categorized into several types, including passive, semi-active, active, and hybrid control approaches to protect buildings from seismic excitation.

This paper focuses on the calculation of smart control for two coupled buildings based on a two-criterion statement, where the criteria determine the maximum deformations of each building under seismic excitation. A crucial feature of this formulation is the lack of information about seismic disturbance, with only the class of potential seismic disturbances being specified. The smart control problem is formulated as a state-feedback control problem for the most dangerous disturbance, i.e., as the worst-case disturbance. To solve this two-criterion problem, the concepts of generalized H_2-norm for linear invariant systems and Linear Matrix Inequalities (LMIs) techniques are utilized [Wilson, 1989; Balandin et al., 2018; Balandin and Kogan, 2018].

This article is structured as follows. The second section focuses on multi-criteria control problems. The mathematical formulation of the multi-criteria control problem is provided. Optimality criteria are defined using a generalized H_2-norm. An algorithm for calculating these criteria using LMIs is presented, along with an algorithm for solving the problem. The third section is dedicated to the examination of examples of applying the proposed algorithm to a system of two coupled buildings. The fourth section presents the results of transient modeling for a system of two coupled buildings based on real earthquake data.

2 Multi-criteria control problem

2.1 General control problem

As previously mentioned, this paper focuses on the development of a feedback controller by solving optimization problems. Let us consider the general formulation of the control problem.

Suppose we have a system represented by the differential equation

$$\begin{aligned} \dot{x} &= Ax + B_v v + B_u u \\ y &= C_y x + D_y v \\ z_i &= C_i x + D_i u \\ u &= \Theta x \\ x(0) &= 0, \quad i = 1...n \end{aligned} \tag{1}$$

where $x \in R^{n_x}$ is the state, $y \in R^{n_y}$ is the measured output, $z_i \in R^{n_{z_i}}$ are controlled outputs, u is the control input $v = v(t), v \in L_2$ is the disturbance input, $A, B_v, B_u, C_y, D_y, C_i, D_i$ are system parameters, Θ are the feedback gain matrix.

Our goal is to design a controller that minimizes in Pareto sense a set of predefined criteria

$$\Theta_P = \arg\min_{\Theta}\{J_k(\Theta), k = 1, ..., m\} \tag{2}$$

where J_k is the k-th criterion.

2.2 Description of criteria

To describe the criteria, we will employ the generalized H_2-norm of a linear controlled system proposed in [Wilson, 1989], with has the form

$$J_k = \sup_{v \in L_2} \frac{\sup\limits_{t \geq 0} |z_k(t)|_\infty}{||v||_2}, \quad k = 1, ..., m \tag{3}$$

where J_k is the criterion, z_k is the controlled output, v is the disturbance.

This approach is advantageous because it enables us to find solutions for the worst disturbance case v

The generalized H_2-norm can be understood as the maximum ratio between the maximum time-varying ∞-norm of the output and the L_2-norm of the input disturbance.

The generalized H_2-norm of internally stable system (1) without control ($u = 0$) can be determined using the formula

$$J_k = \sqrt{d_{\max}(C_k Y C_k^T)} \tag{4}$$

where $d_{max}(\Gamma)$ is the maximum diagonal element of matrix Γ, Y is solution to the Lyapunov equation [Balandin and Kogan, 2017; Tkachenko and Balandin, 2024],

$$\begin{aligned} AY + YA^T + B_v B_v^T = 0 \\ Y = Y^T \geq 0 \end{aligned} \tag{5}$$

and A, B_v, C_k are system parameter.

If the system has a small dimension, Lyapunov equation (5) can be solved analytically. However, for large systems, this can be challenging. In the general case, LMIs can be applied to find H_2-norm. By using Schur's lemma, we proceed to the problem of semidefinite programming in terms of the variables Y and γ^2 subject to constraints as following LMI

$$\begin{gathered} \begin{pmatrix} AY + YA^T & B \\ B^T & -I \end{pmatrix} < 0, \quad \begin{pmatrix} Y & YC_i^T \\ C_i Y & \gamma^2 I \end{pmatrix} \geq 0 \\ \gamma^2 \to \inf \\ i = 1,...,n \end{gathered} \tag{6}$$

After solving this semidefinite programming problem we can obtain the desired value of the generalized H_2-norm that will be equal to the minimal value of the variable γ^2 (for more details, see [Balandin and Kogan, 2017]).

2.3 Solution of multi-criteria control problem

Return to the optimal control problem described in section 2.1. To solve it, we need to move on to considering a closed-loop system

$$\begin{aligned} &\dot{x_c} = A_c(\Theta)x_c + B_c(\Theta)v \\ &z_i = C_i(\Theta)x_c \\ &x_c(0) = 0 \\ &A_c(\Theta) = A + B_u\Theta \\ &B_c(\Theta) = B_v \\ &C_k(\Theta) = C_k + D_k\Theta \end{aligned} \tag{7}$$

To solve multi-criteria introduice the Hermeyer convolution of criteria

$$J_\alpha = \max_{i \in [1,m]} \left\{ \frac{J_i(\Theta)}{\alpha_i} \right\}, \quad \alpha_i > 0 \tag{8}$$

where $J_i(\Theta)$ are values of the criterion, α are parameters of convolution.

Based on results [Balandin and Kogan, 2017], for a given set $\alpha_1, ...\alpha_n$ we have matrix inequalities with respect to the variables Y, Θ, γ^2

$$\begin{gathered} \begin{pmatrix} AY + YA^T + B_u\Theta Y + (\Theta Y)^T B_u^T & B_v \\ B_v^T & -I \end{pmatrix} < 0 \\ \begin{pmatrix} Y & YC_k^T + \Theta Y D_k^T \\ C_k Y + D_k \Theta Y & \gamma^2 \alpha_k^2 I \end{pmatrix} \geq 0 \\ \gamma^2 \to \inf, \quad k = 1...m \end{gathered} \tag{9}$$

However, applying LMIs (6) becomes impossible due to the appearance of the nonlinear component ΘY.

We introduce a new matrix variable Z of the form (10)

$$Z = \Theta Y \tag{10}$$

where Θ are the feedback gain matrix, Y is solution to the Lyapunov equation.

We obtain the semidefinite programming problem

$$\begin{gathered} \begin{pmatrix} AY + YA^T + B_u Z + Z^T B_u^T & B_v \\ B_v^T & -I \end{pmatrix} < 0 \\ \begin{pmatrix} Y & YC_k^T + ZD_k^T \\ C_k Y + D_k Z & \gamma^2 \alpha_k^2 I \end{pmatrix} \geq 0 \\ \gamma^2 \to \inf, \quad k = 1...m \end{gathered} \tag{11}$$

with respect variables Y, Z, γ^2 , where $\{\alpha_1, ...\alpha_m\}$ are parameters of convolution, A, B_u, B_z, C, D are parameters of system.

After determining the matrix variables Y and Z, for given $\alpha_1, ..., \alpha_m$ the feedback is calculated as follows

$$\Theta_\alpha = Z_\alpha Y_\alpha^{-1} \tag{12}$$

where Θ_α is the feedback gain matrix, Y_α is solution to the Lyapunov equation, Z_α is matrix variable of nonlinear component.

3 The coupled control buildings

3.1 Mathematical model and problem statement

The main advantage of the described approach is its applicability to real problem. In particular, a system of coupled buildings with external disturbances can be described in this way. To model high-rise buildings, we will use a chain of elastically connected material points. To simulate seismic excitation, we will say that one material point (base) makes translational movements. For the sake of simplicity, we will consider a model of two coupled buildings. An example of such a design is shown in Fig.3

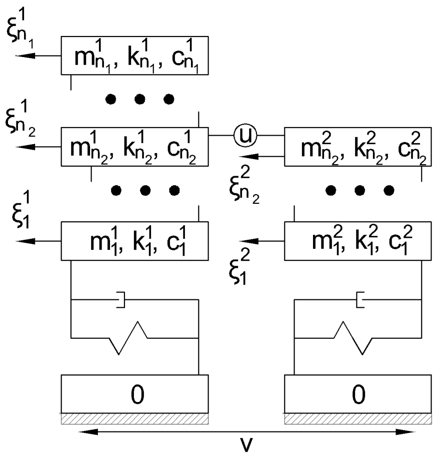

Figure 3. The model of coupled buildings

Here is a mathematical model describing the dynamics of two coupled buildings

$$
\begin{aligned}
\ddot{\xi}_1^j &= -\beta^j \dot{\xi}_1^j - \eta^j \xi_1^j + \beta^j(\dot{\xi}_2^j - \dot{\xi}_1^j) + \eta^j(\xi_2^j - \xi_1^j) \\
&\quad + v + u_1^j \\
\ddot{\xi}_i^j &= -\beta^j(\dot{\xi}_i^j - \dot{\xi}_{i-1}^j) - \eta^j(\xi_i^j - \xi_{i-1}^j) + \beta^j(\dot{\xi}_{i+1}^j - \dot{\xi}_i^j) \\
&\quad + \eta^j(\xi_{i+1}^j - \xi_i^j) + v + u_i^j \\
\ddot{\xi}_n^j &= -\beta^j(\dot{\xi}_n^j - \dot{\xi}_{n-1}^j) - \eta^j(\xi_n^j - \xi_{n-1}^j) + v + u_n^j \\
\xi_i^j(0) &= \dot{\xi}_i^j(0) = 0, \quad i \in [0; n], \quad j \in \{1, 2\} \\
u_i^j &= \begin{cases} u; & j = 1,\ i = n_2 \\ -u, & j = 2,\ i = n_2 \\ 0, & else \end{cases}
\end{aligned}
\tag{13}
$$

where ξ are the coordinates of the points, β, η are the parameters describing buildings, v is the disturbance, u is the control, n is total dimension of the system i.e. $n = n_1 + n_2$

For convenience, we will write down the system in general form. The system will appear as follows:

$$
\begin{aligned}
&\ddot{\Xi}(t) = -BQ\dot{\Xi} - NQ\Xi + Pv + Wu \\
&B = \begin{pmatrix} \beta_1 I_{n_1} & 0 \\ 0 & \beta_2 I_{n_2} \end{pmatrix}, \quad N = \begin{pmatrix} \nu_1 I_{n_1} & 0 \\ 0 & \nu_2 I_{n_2} \end{pmatrix} \\
&W = col(0, ..., 1, ..., -1, ...), \quad P = col(1...1) \\
&Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}, Q_i = \begin{pmatrix} 2 & -1 & 0 & 0 & ...0 & \\ -1 & 2 & -1 & 0 & ... & 0 \\ 0 & -1 & 2 & -1 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & 0 & -1 & 2 & -1 \\ ... & ... & ... & .... & -1 & 1 \end{pmatrix}
\end{aligned}
\tag{14}
$$

where Ξ is the state of the system, $\beta_1, \beta_2, \nu_1, \nu_2, N$ are the parameters of the buildings, v is the disturbance, u is the control.

We will select the maximum deformations of each building as criteria. We will define the following objective functions

$$
J_k(u) = \sup_{v \in L_2} \frac{\max\{\sup_{t \geq 0} |\xi_1^k(t)|_2, \sup_{t \geq 0} |\xi_j^k(t) - \xi_{j-1}^k(t)|_2\}}{||v||_2}
$$
$$
j \in [2, n_k], \quad k = 1, 2 \tag{15}
$$

where ξ are coordinates of the points, v is disturbance.

3.2 Solution and result

To illustrate the results, we will examine three systems consisting of buildings with varying stiffness:

System 1 is buildings of low and medium stiffness
System 2 is buildings of medium and hard stiffness
System 3 is buildings of low and hard stiffness

We will begin with a simple example: two single-point bodies (see Fig. 4)

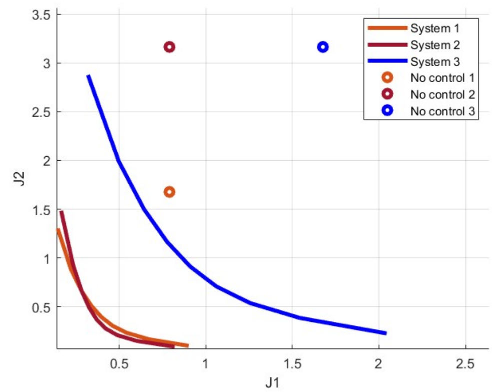

Figure 4. The single-point bodies

The graph depicts the Pareto-optimal front for this task. The unique aspect of this graph is its presentation of a set of all feasible solutions, each of which stands as the best possible. Any solutions above the curve are inferior by at least one metric. Anything below the graph is unattainable.

The graphs also show the value of the criteria in the absence of any control.

As one can see from the above graph, the application of the developed control method leads to a significant improvement in the values of the selected criteria. Now, we will test this hypothesis for buildings of a larger scale.

Consider two dual-points buildings (see Fig. 5).

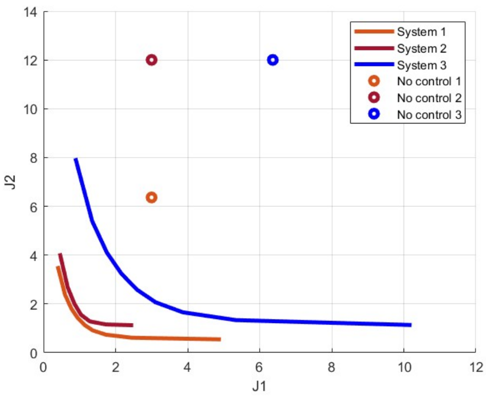

Figure 5. The dual-points buildings

One can notice that the magnitude of the criteria values obtained has changed, but the general trend has remained. We will check on the model of multi-points building (see Fig. 6)

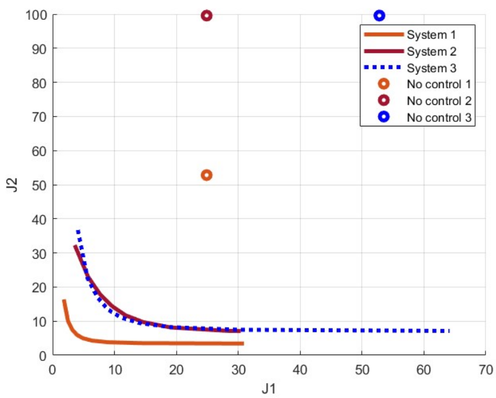

Figure 6. The multi-points buildings

The obtained results allow us to conclude that the proposed method is highly effective for systems of various scales.

4 Modeling based on real seismic excitation

4.1 Data preparation

The earthquake data was sourced from [Institut de physique du globe de Paris (IPGP) and Ecole et Observatoire des Sciences de la Terre de Strasbourg (EOST), 1982]. This source offers seismogram data collected at various stations. For our demonstration, we will utilize data on the earthquake that occurred in Turkey on 2023/06/02. We will select the station closest to the epicenter. Fig. 7 illustrates the velocity data as a function of time.

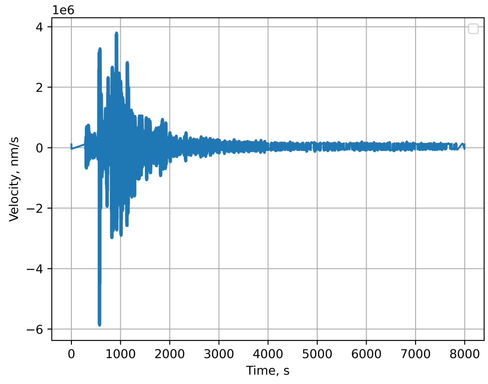

Figure 7. Data on the velocity of soil movement during an earthquake

For analysis, we will take an interval from zero to 8000 seconds. We will plot the acceleration graph of ground movement (see Fig. 8)

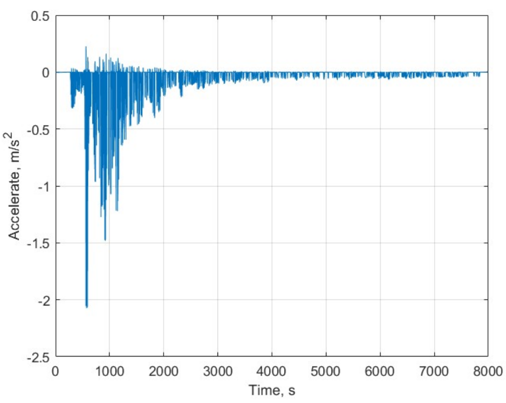

Figure 8. Data on the accelerate of soil movement during an earthquake

4.2 Transients

During the research, we applied the proposed algorithm to real data on external disturbances. For this purpose, we used the model of two multi-points buildings described earlier.

Let's take a closer look at transients using the example of multi-points buildings with different α parameters.

The graph for the first building is shown in Fig. 9, and for the second one in Fig. 10

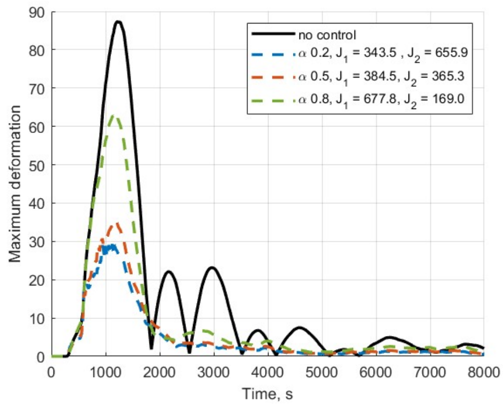

Figure 9. Transients of the first building

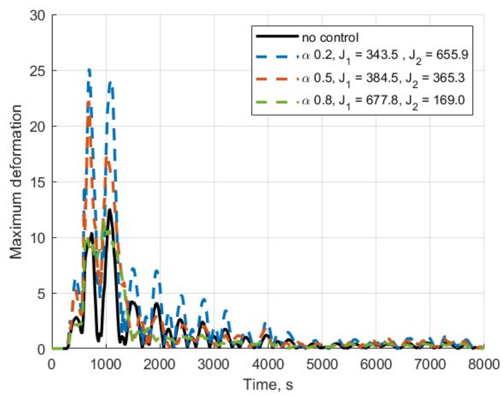

Figure 10. Transients of the second building

Recall that the first building featured a greater number of points, leading to a more pronounced sway during an earthquake. Consequently, when connecting buildings, a portion of the energy transferred from the first structure to the second, causing it to move more vigorously. Simultaneously, this process significantly dampened the vibrations of the original building.

5 Conclusion

This article proposed an approach to the development of smart control for vibration protection in multiple buildings. The approach is based on solving a multi-criteria control problem, with the maximum deformation of each building serving as the quality criterion. These deformations are determined using the generalized H_2-norm. An algorithm for solving optimization problems using LMIs was also presented. Numerical simulations were conducted for systems involving two buildings of different sizes and using real earthquake data. The results demonstrated the effectiveness of smart control in significantly reducing vibrations in connected buildings, highlighting the potential of this approach for enhancing the structural integrity and safety of multi-building systems.

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