BIAS-COMPENSATED ADAPTIVE OBSERVER FOR A CONTINUOUS-TIME MODEL ESTIMATION

Kenji Ikeda [*,1] **Yoshio Mogami** [*] **Takao Shimomura** [*]

[*] *The University of Tokushima*

Abstract: In this paper, a bias-compensating method for a continuous-time model estimation by using adaptive observer is proposed. It is assumed that the observation noise is a white Gaussinan signal while there are no process noises. The proposed method is applicable for the identification in the closed loop environment.

Keywords: Continuous-time systems, Parameter estimation, Closed loop systems, Bias compensation, Adaptive filters

1. INTRODUCTION

There are many results on the direct identification of a continuous-time model from sampled I/O data for more than three decades (Young, 1981; Unbehauen and Rao, 1990; Sinha and Rao, 1991; Pintelon *et al.*, 1994). The importance of the continuous-time model identification increases as the computer becomes faster and as the sampling period becomes smaller because the conventional discrete-time model identification method becomes numerically unstable when the sampling period becomes small.

On the other hand, there are increasing demands on the development of the identification method in the closed loop environment(Forssell and Ljung, 1999). In general, the sampling period for the feedback control is faster than the sampling period which is considered to be optimal for the parameter estimation of the plant. Thus, the closed loop identification requires the techniques developed for the continuous-time model identification.

Among the many approaches to the direct identification of continuous-time models, a use of state variable filters is one of the basic approaches and has a long history(Wang and Gawthrop, 2001; Young, 1981). In order to recover the information loss incurred by the sampling, the authors have proposed a continuous-time model identification method by using an adaptive observer, which estimates the inter-sample output of the plant as well as the plant parameters with the ZOH input assumption (Ikeda *et al.*, 2006*a*; Ikeda *et al.*, 2006*b*). The output of the state variable filter becomes a pseudo regression vector and a bootstrap method is adopted. The parameter convergence and the parameter estimation error of the proposed method are analyzed in (Ikeda *et al.*, 2006*a*; Ikeda *et al.*, 2006*b*).

In this paper, bias compensating method is applied for the previously proposed method(Ikeda *et al.*, 2006*a*; Ikeda *et al.*, 2006*b*) in order to achieve the consistency of the estimate. Bias compensating least squares methods were origninally developed for the discrete-time model estimation(Sagara and Wada, 1977) and applied for the continuous-time model estimation(Zhao *et al.*, 1991; Garnier *et al.*, 2000).

Section 2 states the problem. An adaptive observer for the estimation of the continuous-time model is introduced in section 3. Section 4 an-

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alyzes the asymptotic bias of the least squares estimator in the closed loop envorionment. The magnitude of the variance of the noise is also estimated from the output estimation error. Section 5 introduces an iteration algorithm to estimate the continuous-time model with bias compensation. A numerical example is presented in section 6. Finally, section 7 concludes the paper.

Notation: Let τ be a sampling period throughout the paper. A function of continuous time t will be denoted by $x(t)$ while its sampled value $x(k\tau)$ will be denoted by $x[k]$.

Let $\|x\|$ denote an Euclidean norm of $x \in R^n$. For a discrete-time signal $x[k] \in \mathrm{R}^n$, define its norm as

$$\|x\|_{[1,N]} = \sqrt{\sum_{k=1}^{N} \|x[k]\|^2}. \tag{1}$$

Let $\mathcal{O}(A, c^{\mathrm{T}})$ denote an observability matrix:

$$\mathcal{O}(A, c^{\mathrm{T}}) = [\, c \quad A^{\mathrm{T}}c \quad \ldots \quad (A^{\mathrm{T}})^{n-1}c \,]^{\mathrm{T}}. \tag{2}$$

Let $E\{X\}$ denote an expectation of X.

2. PROBLEM STATEMENT

Consider an SISO continuous-time system:

$$\dot{x}_p(t) = Ax_p(t) + bu(t), \tag{3}$$
$$y(t) = c^{\mathrm{T}}x_p(t) + \nu(t), \tag{4}$$

where $u(t) \in \mathrm{R}$, $y(t) \in \mathrm{R}$, $x_p(t) \in \mathrm{R}^n$, and $\nu(t) \in \mathrm{R}$ are the input, the output, the state of the system, and the observation noise, respectively. System matrices A, b, and c have appropriate dimensions. Assume the followings for this system.

(A1) (A, c^{T}) is observable,
(A2) an upper bound of the dimension n is known. (It is denoted n as such upper bound.)

Without loss of generality, (A, b, c^{T}) is assumed to be the observer canonical form:

$$A = \begin{bmatrix} -a_1 & 1 & & \\ \vdots & & \ddots & \\ \vdots & & & 1 \\ -a_n & & & \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix},$$
$$c = [\, 1 \quad 0 \quad \ldots \quad 0 \,]^{\mathrm{T}}.$$

Define a coefficient vector of the characteristic polynomial of A as $a = (a_1, \ldots, a_n)^{\mathrm{T}}$. Furthermore, sampled I/O data available for the identification is assumed as follows:

(A3) the output $y(t)$ can be measured at the discrete-time instants $t = k\tau$ $(k = 0, 1, 2, \ldots, N)$.
(A4) the input $u(t)$ is a ZOH of a discrete-time signal, *i.e.*,

$$u(t) = u(\lfloor t/\tau \rfloor \tau), \quad \forall t \in [0, N\tau]. \tag{5}$$

(A5) The observation noise $\nu[k]$ is a zero mean white gaussian noise with covariance:

$$E\{\nu[k]\nu[l]\} = \sigma_\nu^2 \delta_{kl} \tag{6}$$

where δ_{kl} is a Kronecker delta.
(A6) The I/O data is collected in the close loop environment, where the controller is a (discrete-time) linear time invariant system and the closed loop system is asymptotically stable.

From the assumption (A3) and (A4), the plant has a discrete-time representation:

$$x_p[k+1] = A_p x_p[k] + b_p u[k] \tag{7}$$
$$y[k] = c_p^{\mathrm{T}} x_p[k] + \nu[k] \tag{8}$$

and from (A6), there exists a feedback controller:

$$x_c[k+1] = A_c x_c[k] - b_c y[k] + b_{c2} r[k] \tag{9}$$
$$u[k] = c_c^{\mathrm{T}} x_c[k] - d_c y[k] + d_{c2} r[k]. \tag{10}$$

such that

$$A_{cl} := \begin{bmatrix} A_p - b_p d_c c_p^{\mathrm{T}} & b_p c_c^{\mathrm{T}} \\ -b_c c_p^{\mathrm{T}} & A_c \end{bmatrix} \tag{11}$$

has its all eigenvalues on the open unit disc. A reference signal $r[k]$ is deterministic.

(A7) It is assumed that $\{r[k]\}$ is independent of $\{v[k]\}$.

Problem formulation: System identification requires the determination of the unknown plant parameters $a = (a_1, \ldots, a_n)^{\mathrm{T}}$ and $b = (b_1, \ldots, b_n)^{\mathrm{T}}$ from sampled I/O data $\{u[k], y[k]\}$ for $k = 0, \ldots, N$ and to estimate the state variable $x_p(t)$ for $t \in [0, N\tau]$.

3. ADAPTIVE OBSERVER FOR THE ESTIMATION OF A CONTINUOUS-TIME MODEL

The proposing adaptive observer is based on the structure depicted in Fig. 1, in which there is an estimation mechanism of the intersample output(Ikeda *et al.*, 2006*a*). When the parameter estimate $\hat{\theta}_i$ is a constant vector, the continuous-time signals $z_i(t)$ can be easily discretized without any approximations because of the zeroth order hold input assumption.

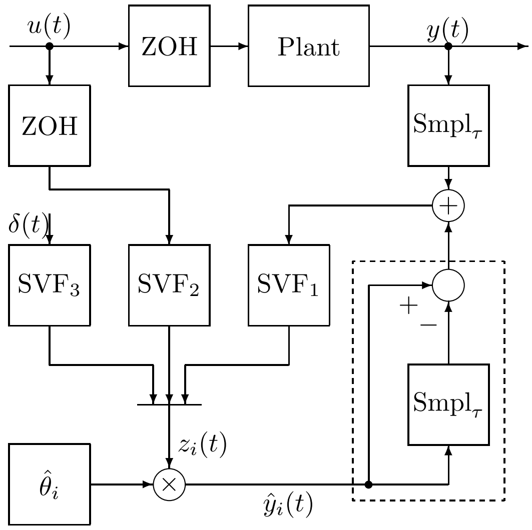

Fig. 1. Observer using SVF

Define a discrete-time state variable filter

$$\mathrm{SVF}_D : (F, g, \tau, \hat{\theta}_i, \{u[k]\}, \{y[k]\}) \mapsto \{z_i[k]\} \quad (12)$$

as

$$z_i[k+1] = \bar{F}_d(\hat{\theta}_i) z_i[k] + \bar{G}_d(\hat{\theta}_i) \begin{bmatrix} y[k] \\ u[k] \end{bmatrix} \quad (13)$$

$$z_i[0] = [\, 0^{\mathrm{T}} \quad 0^{\mathrm{T}} \quad g^{\mathrm{T}} \,]^{\mathrm{T}} \quad (14)$$

where $\bar{F}_d(\theta)$ and $\bar{G}_d(\theta)$ are

$$\bar{F}_d(\theta) = e^{\bar{F}_\theta \tau} - \int_0^\tau e^{\bar{F}_\theta t} dt\, \bar{g} \theta^{\mathrm{T}}, \quad (15)$$

$$\bar{G}_d(\theta) = \int_0^\tau e^{\bar{F}_\theta t} dt\, \bar{G}, \quad (16)$$

$$\bar{F}_\theta = \bar{F} + \bar{g}\theta^{\mathrm{T}}, \quad \bar{g} = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix}, \quad (17)$$

$$\bar{F} = \begin{bmatrix} F & O & O \\ O & F & O \\ O & O & F \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} g & 0 \\ 0 & g \\ 0 & 0 \end{bmatrix}. \quad (18)$$

The true value of the estimated parameter is given by

$$\theta_* = \begin{bmatrix} \theta_{*1} \\ \theta_{*2} \\ \theta_{*3} \end{bmatrix} = \begin{bmatrix} \mathcal{O}(F^{\mathrm{T}}, g^{\mathrm{T}})^{-1} \mathcal{O}(F_o, c^{\mathrm{T}})(f - a) \\ \mathcal{O}(F^{\mathrm{T}}, g^{\mathrm{T}})^{-1} \mathcal{O}(F_o, c^{\mathrm{T}}) b \\ \mathcal{O}(F^{\mathrm{T}}, g^{\mathrm{T}})^{-1} \mathcal{O}(F_o, c^{\mathrm{T}}) x_p(0) \end{bmatrix} \quad (19)$$

where f is a coefficient vector of the characteristic polynomial of F and (F_o, c^{T}) is an observer canonical form of $(F^{\mathrm{T}}, g^{\mathrm{T}})$. For the sake of simplicity, (F, g) is assumed to be a controller canonical form.

In order to analyze the parameter estimation error in the following section, decompose the error term $\tilde{z}_i[k] = z_i[k] - z_*[k]$ as

$$\tilde{z}_i[k] = \tilde{z}_{i\nu}[k] + \tilde{z}_{i\Delta}[k]. \quad (20)$$

where $\{\tilde{z}_{i\nu}[k]\}$, $\{\tilde{z}_{i\Delta}[k]\}$ and $\{z_*[k]\}$ are defined by

$$\tilde{z}_{i\nu}[k+1] = \bar{F}_d(\hat{\theta}_i)\tilde{z}_{i\nu}[k] + \bar{G}_d(\hat{\theta}_i) \begin{bmatrix} \nu[k] \\ 0 \end{bmatrix} \quad (21)$$

$$\tilde{z}_{i\Delta}[k+1] = \bar{F}_d(\hat{\theta}_i)\tilde{z}_{i\Delta}[k] + \Delta(\hat{\theta}_i, \theta_*)(\dot{z}_*)[k] \quad (22)$$

$$(\dot{z}_*)[k] = [\bar{F}, \bar{G}][\, z_*^{\mathrm{T}}[k] \quad c^{\mathrm{T}} x_p[k] \quad u[k] \,]^{\mathrm{T}}. \quad (23)$$

$$\Delta(\hat{\theta}_1, \hat{\theta}_2) = \sum_{m=1}^{\infty} \frac{\tau^{m+1}}{(m+1)!} \sum_{i=0}^{m-1} \bar{F}_{\hat{\theta}_2}^{m-1-i} \bar{g} \tilde{\theta}_{12}^{\mathrm{T}} \bar{F}_{\hat{\theta}_1}^{i}, \quad (24)$$

$$\{z_*[k]\} = \mathrm{SVF}_D(F, g, \tau, \theta_*, \{u[k]\}, \{c^{\mathrm{T}} x_p[k]\}). \quad (25)$$

Consider the following least-squares criterion:

$$J(\theta, k) = \sum_{i=1}^{k} \left[y[k] - z_\theta^{\mathrm{T}}[k]\theta \right]^2. \quad (26)$$

Because $z_\theta[k]$ is a pseudo-linear regression vector, a bootstrap method is adopted here. First, define a discrete-time Recursive Least Square (RLS) algorithm:

$$\mathrm{RLS} : (\{z_i[k]\}, \{y[k]\}, \hat{\theta}_i, \gamma) \mapsto (\bar{\theta}_{i+1}, \Gamma_{i+1}) \quad (27)$$

as

$$\bar{\theta}_{i+1} = \hat{\theta}[N], \qquad \Gamma_{i+1} = \Gamma[N], \quad (28)$$

$$\hat{\theta}[k] = \hat{\theta}[k-1] - \frac{\Gamma[k-1] z_i[k]}{1 + z_i^{\mathrm{T}}[k]\Gamma[k-1] z_i[k]} e_{y-}[k], \quad (29)$$

$$\Gamma[k] = \left\{ I - \frac{\Gamma[k-1] z_i[k] z_i^{\mathrm{T}}[k]}{1 + z_i^{\mathrm{T}}[k]\Gamma[k-1] z_i[k]} \right\} \Gamma[k-1], \quad (30)$$

$$e_{y-}[k] = z_i^{\mathrm{T}}[k]\hat{\theta}[k-1] - y[k], \quad (31)$$

$$\hat{\theta}[0] = \hat{\theta}_i, \qquad \Gamma[0] = \gamma I. \quad (32)$$

The design parameter $\gamma > 0$ will be chosen to be a very large number, say $10^2 \sim 10^5$ in general. The output of the RLS algorithm $\hat{\theta}_{i+1}$ and Γ_{i+1} are the estimated parameter vector and the recursively estimated inverse of the covariance matrix of the regression vector $z_i[k]$, $k = 0, \ldots, N$.

By using this RLS algorithm, parameter vector is estimated as follows:

$$\bar{\theta}_{i+1} - \theta_* = \frac{1}{\gamma}\Gamma_{i+1}(\hat{\theta}_i - \theta_*) - \Gamma_{i+1} \sum_{k=1}^{N} z_i[k]\varepsilon_i[k], \quad (33)$$

$$\varepsilon_i[k] = \theta_*^{\mathrm{T}} \tilde{z}_i[k] - \nu[k]. \quad (34)$$

Decompose $\varepsilon_i[k]$ into the determinitic part and the stochastic part as

$$\varepsilon_i[k] = \varepsilon_{i\Delta}[k] + \varepsilon_{i\nu}[k] \quad (35)$$
$$= \tilde{z}_{i\Delta}^{\mathrm{T}}[k]\theta_* + \tilde{z}_{i\nu}^{\mathrm{T}}[k]\theta_* - \nu[k] \quad (36)$$

by using $\tilde{z}_{i\Delta}[k]$ and $\tilde{z}_{i\nu}[k]$ in (20). It can be shown that $\|\varepsilon_{i\Delta}\| \to 0$ when $\hat{\theta}_i \to \theta_*$ (Ikeda *et al.*, 2006*a*). The asymptotic bias is caused by $\varepsilon_{i\nu}[k]$ and is analyzed in the next section.

4. ASYMPTOTIC BIAS OF THE LEAST SQUARES ESTIMATE IN CLOSED LOOP ENVIRONMENT

Consider an innovations process $\nu^\dagger[k]$ defined by

$$\hat{\xi}[k+1] = (A_p - K_d c_p^{\mathrm{T}})\hat{\xi}[k] + K_d\nu[k], \quad (37)$$
$$\nu^\dagger[k] = -c_p^{\mathrm{T}}\hat{\xi}[k] + \nu[k], \quad (38)$$

where

$$K_d = A_p\Sigma c_p[c_p^{\mathrm{T}}\Sigma c_p + \sigma_\nu^2]^{-1}, \quad (39)$$
$$\Sigma = A_p(\Sigma - \Sigma c_p[c_p^{\mathrm{T}}\Sigma c_p + \sigma_\nu^2]^{-1}c_p^{\mathrm{T}}\Sigma)A_p^{\mathrm{T}}. \quad (40)$$

It is well known that the covariance of the innovations process is given by

$$E\{\nu^\dagger[k]\nu^\dagger[l]\} = (c_p^{\mathrm{T}}\Sigma c_p + \sigma_\nu^2)\delta_{kl}. \quad (41)$$

Rewrite the closed loop system whose input is $\nu^\dagger[k]$ instead of $\nu[k]$. In this section, we consider the case when $r[k] = 0$ because $\{r[k]\}$ is independent of $\{\nu[k]\}$. Define $x_p'[k] = x_p[k] + \hat{\xi}[k]$, then the plant model (7) and (8) together with the innovations representation (37) and (38) can be rewritten as

$$x_p'[k+1] = A_p x_p'[k] + b_p u[k] + K_d\nu^\dagger[k], \quad (42)$$
$$y[k] = c_p^{\mathrm{T}}x_p'[k] + \nu^\dagger[k]. \quad (43)$$

From this and the controller (9) and (10), we obtain

$$\begin{bmatrix} x_p'[k+1] \\ x_c[k+1] \end{bmatrix} = A_{cl}\begin{bmatrix} x_p'[k] \\ x_c[k] \end{bmatrix} + B_{cl}^\dagger\nu^\dagger[k] \quad (44)$$
$$\begin{bmatrix} y[k] \\ u[k] \end{bmatrix} = C_{cl}\begin{bmatrix} x_p'[k] \\ x_c[k] \end{bmatrix} + D_{cl}\nu^\dagger[k] \quad (45)$$

where

$$B_{cl}^\dagger = [\,(K_d - b_p d_c)^{\mathrm{T}} \quad -b_c^{\mathrm{T}}\,]^{\mathrm{T}}, \quad (46)$$
$$C_{cl} = \begin{bmatrix} c_{cl,y}^{\mathrm{T}} \\ c_{cl,u}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} c_p^{\mathrm{T}} & 0^{\mathrm{T}} \\ -d_c c_p^{\mathrm{T}} & c_c^{\mathrm{T}} \end{bmatrix}, \quad (47)$$
$$D_{cl} = \begin{bmatrix} 1 \\ -d_c \end{bmatrix}. \quad (48)$$

Because A_{cl} is stable from the assumption (A4), the following lemma holds.

Lemma 1. In the feedback system (44) and (45), $E\{y[k]\nu^\dagger[l]\} = 0$ $(l > k)$ and $E\{u[k]\nu^\dagger[l]\} = 0$ $(l > k)$.

Proof. It is obvious from the discussions above.

Next, consider the equation error $\varepsilon_{i\nu}[k]$ in eq. (35) when the true value of the estimated parameter is known, which will be denoted by $\varepsilon[k]$ instead of $\varepsilon_{i\nu}[k]$. Equation error $\varepsilon[k]$ has the following representation:

$$\varepsilon[k] = (\zeta_1[k])^{\mathrm{T}}\theta_{*1} - \nu[k] \quad (49)$$

where $\zeta_1[k] \in R^n$ is the first subvector of

$$\{\zeta[k]\} = \mathrm{SVF}_D(F, g, \tau, \theta_*, \{0\}, \{\nu[k]\}). \quad (50)$$

Replacing the input $\nu[k]$ by the innovation $\nu^\dagger[k]$, define

$$\varepsilon^\dagger[k] = (\zeta_1^\dagger[k])^{\mathrm{T}}(\theta_{*1} - K_c) - \nu^\dagger[k] \quad (51)$$

where $\zeta_1^\dagger[k] \in R^n$ is the first subvector of

$$\{\zeta^\dagger[k]\} = \mathrm{SVF}_D(F, g, \tau, \theta_*, \{0\}, \{\nu^\dagger[k]\}) \quad (52)$$

and $K_c \in R^n$ is

$$K_c = \left(\int_0^\tau e^{At}dt\right)^{-1} K_d. \quad (53)$$

For this signal, the following lemma holds.

Lemma 2. Let $\varepsilon[k]$ be defined by eqs. (49) and (50) and $\tilde{\varepsilon}[k]$ be defined by eqs. (51) and (52) together with the innovations model (37) and (38). Then, $\tilde{\varepsilon}[k] = \varepsilon[k]$.

Proof. See (Ikeda, 2007).

Now, calculate $E\{z[k]\varepsilon[k]\}$ where regression vector $z[k]$ is defined by

$$\{z[k]\} = \mathrm{SVF}_D(F, g, \tau, \theta_*, \{u[k]\}, \{y[k]\}). \quad (54)$$

From Lemmas 1 and 2, we obtain

$$E\{z[k]\varepsilon[k]\} = -E\{z[k]\zeta_1^\dagger[k]^{\mathrm{T}}\}(\theta_{*1} - K_c). \quad (55)$$

In order to calculate the expectation of $z[k]\zeta_1^\dagger[k]^{\mathrm{T}}$, define

$$\bar{X}[k] = [z[k]^{\mathrm{T}}, (\zeta_1^\dagger[k])^{\mathrm{T}}, [x_p'[k]^{\mathrm{T}}, x_c[k]^{\mathrm{T}}]]^{\mathrm{T}}. \quad (56)$$

Then, we obtain the following equation:

$$\bar{X}[k+1] = \bar{A}\bar{X}[k] + \bar{B}\tilde{\nu}[k] \quad (57)$$

where

$$\bar{A} = \begin{bmatrix} \bar{F}_d(\theta_*) & O & \bar{G}_d(\theta_*)C_{cl}^{\mathrm{T}} \\ O & \bar{F}_{d11}(\theta_*) & O \\ O & O & A_{cl} \end{bmatrix}, \quad (58)$$

$$\bar{B} = \begin{bmatrix} \bar{G}_d(\theta_*)D_{cl} \\ \bar{G}_{d11}(\theta_*) \\ B_{cl}^{\dagger} \end{bmatrix}. \quad (59)$$

From the definition of $\bar{X}[k]$, $E\{z[k]\zeta_1^{\dagger}[k]^{\mathrm{T}}\}$ is a 1-2 block of $E\{\bar{X}[k]\bar{X}[k]^{\mathrm{T}}\}$. Therefore,

$$E\{z[k]\varepsilon[k]\} = -P_{12}(\theta_{*1} - K_c)\sigma_{\nu^{\dagger}}^2 \quad (60)$$

where P_{12} is a 1-2 block of $P = P^{\mathrm{T}} > 0$ which is a positive definite solution of the Lyapunov equation:

$$P = \bar{A}P\bar{A}^{\mathrm{T}} + \bar{B}\bar{B}^{\mathrm{T}}, \quad (61)$$

and $\sigma_{\nu^{\dagger}}^2 = c_p^{\mathrm{T}}\Sigma c_p + \sigma_{\nu}^2$.

The variance of the innovation $\sigma_{\nu^{\dagger}}^2$ can be estimated from the output estimation error

$$\begin{aligned} \tilde{y}_i[k] &:= \hat{y}_i[k] - y[k] & (62)\\ &= (\bar{\theta}_{i+1} - \theta_*)^{\mathrm{T}} z_i[k] + \varepsilon_i[k]. & (63) \end{aligned}$$

The summation of the squared error becomes

$$\begin{aligned} &\frac{1}{N}\sum_{k=1}^{N}\tilde{y}_i[k]^2 \\ &= \frac{1}{N}\sum_{k=1}^{N}\varepsilon_i[k]^2 + 2(\bar{\theta}_{i+1} - \theta_*)\frac{1}{N}\sum_{k=1}^{N} z_i[k]\varepsilon_i[k] \\ &\quad + \frac{1}{N}(\bar{\theta}_{i+1} - \theta_*)^{\mathrm{T}}\sum_{k=1}^{N} z_i[k]z_i[k]^{\mathrm{T}}(\bar{\theta}_{i+1} - \theta_*). \end{aligned} \quad (64)$$

Assuming $\|\tilde{z}_{i\Delta}\|_{[0,N]}$ is small enough compared with $\|\tilde{z}_{i\nu}\|_{[0,N]}$, the expectation of the r.h.s. of the equation above becomes

$$\begin{aligned} &E\left\{\frac{1}{N}\sum_{k=1}^{N}\tilde{y}_i[k]^2\right\} \\ &= \{1 + (\theta_{*1} - K_c)^{\mathrm{T}}P_{22}(\theta_{*1} - K_c)\}\sigma_{\nu^{\dagger}}^2 \\ &\quad - N(\theta_{*1} - K_c)^{\mathrm{T}}P_{12}^{\mathrm{T}}\Gamma_{i+1}P_{12}(\theta_{*1} - K_c)\sigma_{\nu^{\dagger}}^4 \end{aligned} \quad (65)$$

Γ_{i+1} is very small in general, we may define

$$\hat{\sigma}_{\nu^{\dagger}}^2 = \frac{\sum_{k=1}^{N}\tilde{y}_i[k]^2}{N\{1 + (\theta_{*1} - K_c)^{\mathrm{T}}P_{22}(\theta_{*1} - K_c)\}}. \quad (66)$$

Remark 3. The proposed method is no more a direct approach of the closed loop identification because the bias compensating term requires the information on the feedback controller even though the least squares estimate does not require controller parameters. However, when the prefilter is designed as an optimal prefilter by using the Kalman filter theory, the parameter vector θ_{*1} becomes K_c. This means there is no bias to be compensated and the controller information is not required.

5. ITERATION OF ADAPTIVE ESTIMATION

As seen in section 4, $\bar{\theta}_{i+1}$ is asymptotically biased due to the term $\sum_{k=1}^{N}\tilde{z}_{i\nu}[k]\varepsilon_{i\nu}[k]$. However, the asymptotic bias of the parameter estimate $\bar{\theta}_{i+1}$ can be compensated and define $\hat{\theta}_{i+1}$ by

$$\hat{\theta}_{i+1} = \bar{\theta}_{i+1} + \Gamma_{i+1}\hat{\beta}_i, \quad (67)$$

where

$$\hat{\beta}_i = N\hat{P}_{12i}(\hat{\theta}_{i1} - \hat{K}_{ci})\hat{\sigma}_{\nu^{\dagger}}^2,$$

and $\hat{P}_{12i}$ and $\hat{K}_{ci}$ are calculated by using eqs. (39), (40), and (61), where plant parameters are replaced by their estimates.

A bootstrap algorithm is defined as follows.

Iteration Algorithm:

(1) Let $\hat{\theta}_0$ be an R^{3n} vector, and (F, g) be an n-dimensional single input controllable pair where F is a Hurwitz matrix. Let $i = 0$.
(2) $\{z_i[k]\} = \mathrm{SVF}_D(F, g, \tau, \hat{\theta}_i, \{u[k]\}, \{y[k]\})$
(3) $(\bar{\theta}_{i+1}, \Gamma_{i+1}) = \mathrm{RLS}(\{z_i[k]\}, \{y[k]\}, \hat{\theta}_i, \gamma)$
(4) Compensate the bias, and define $\hat{\theta}_i$ as in (67).
(5) Increase i by 1 and goto step (2)

The regression vectors are assumed to be persistently exciting (Ljung, 1999) as follows:

(A8) Assume each $\{z_i[k]\}$ satisfies the following condition:

$$k_{\min}I \leq \sum_{k=1}^{N} z_i[k]z_i^{\mathrm{T}}[k] \leq k_{\max}I. \quad (68)$$

Under the assumptions (A1) to (A8), the proposed estimate $\hat{\theta}_{\infty}$ becomes unbiased and consistent(Ikeda, 2007), *i.e.*,

$$E\{\hat{\theta}_{\infty}\} = \theta_*. \quad (69)$$

6. NUMERICAL EXAMPLE

In order to illustrate the proposed adaptive observer, a numerical example of an identification of a 3rd order system is presented. The plant to be estimated is

$$y(t) = \frac{2}{(p+2)(p-1)p}u(t) + \nu(t).$$

The sampling period is $\tau = 1/64 = 0.015625$[s] and the number of samples N varies from 1024 to

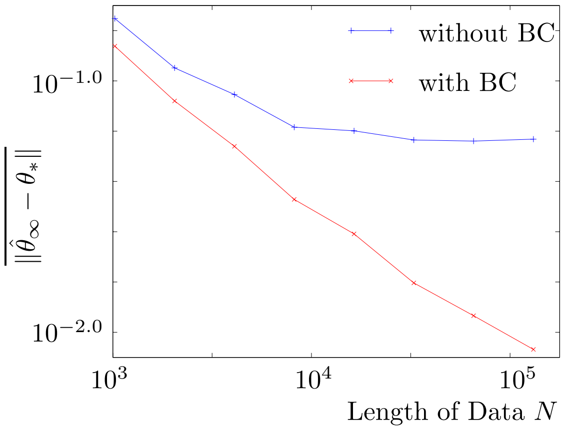

Fig. 2. Consistency of the proposed method

131072. The observation noise $\nu[k]$ is generated as zero mean white Gaussian noise with its variance $\sigma_\nu^2 = 1.0$. The I/O data is collected in the closed loop environment. The controller is designed by using LQG/LTR method and is given by

$$u[k] = \frac{q^3 - 2.906q^2 + 2.816q - 0.9098}{q^3 - 2.868q^2 + 2.744q - 0.8757} r[k] - \frac{0.865q^2 - 1.702q + 0.8372}{q^3 - 2.868q^2 + 2.744q - 0.8757} y[k].$$

The reference input sequence $r[k]$ is a pseudo-random binary signal(PBRS) taking values ± 10 with its bandwidth 0.25[Hz]. The initial state of the plant is assumed to be known as 0 for the sake of simplicity. For this problem, the state variable filters are designed by pole placement at $s = -1.5$ and is given by

$$F = \begin{pmatrix} -4.5 & -6.75 & -3.375 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad g = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The estimation results are presented in Fig.2. RSE (Root Squared Error) of the parameter estimates are plotted versus the number of data both in logarithmic scale. RSE of the parameter estimates with bias compensation becomes small as N becomes large with coefficient about -0.5, while RSE of the parameter estimates without bias compensation does not become small even if N becomes large. From this simulation, it can be concluded that the proposed method with BC works as a consistent estimate.

7. CONCLUSION

In this paper, a bias compensating method for the identification of the closed loop environment by using adaptive observer(Ikeda *et al.*, 2006*a*; Ikeda *et al.*, 2006*b*), which estimates the continuous-time model from the samled I/O data, is proposed. When the plant is unstable, the bias compensating term can be calculated based on the innovations instead of the noise itself.

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