Oscillatory media properties' influence on speed of excitation propogation.

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We study behavior of locally diffusively coupled Bonhoeffer - Van der Pol (BvdP) oscillators. We investigate how the properties of cells, more concretely, individual frequencies influence on the excitation propogation through the 1D and 2D media. Speed of excitation propogation depends on the frequency mismatch between individual cells and the rhythm, which synchronises the media. Qualitative and quantitative results, describing this dependence are observed in numerical simulations and explained.

I. INTRODUCTION

The question of excitation propogation from the pacemaker in geterogeneous media is interesting not only from the viewpoint of presense or absense cells' synchronization. Spatio-temporal parameters of excitation (speed of propogation, wave length) can cause strongly different patterns, and a few of such cases are shown in this paper. First we analyze well known patterns (spiral and target waves) in lattices with equal parameters. Then pulses propogation in 1D ensebmles are considered. Effects in 2D ensembles are considered as well.

II. MODEL

We investigate a locally diffusively coupled Bonhoeffer - Van der Pol (BvdP) oscillators [1]:

$$\begin{cases} \dot{x}_{i,j} = F(x_{i,j}, y_{i,j}) + d(x_{i,j+1} + x_{i,j-1} + \\ +x_{i+1,j} + x_{i-1,j} - 4x_{i,j}), \\ \dot{y}_{ij} = G_{ij}(x_{i,j}), \\ i = 1, ..., M, \\ j = 1, ..., N, \end{cases} \tag{1}$$

where $F(x_{i,j}, y_{i,j}) = x_{i,j} - x_{i,j}^3/3 - y_{i,j}$, $G_{i,j}(x_{i,j}) = \varepsilon(x_{i,j} + a_{i,j})$, M and N are the numbers of elements across each dimension in the lattice, and d is the coupling between the elements, $\varepsilon \ll 1$, $0 < a_{i,j} < 1$. We used free boundary conditions.

III. EXCITATION PROPOGATION AT DIFFERENT CONDITIONS

2D media in known to demonstrate target ans spiral waves (Fig. 1). Excitation time of the element is approximately constant in both these cases. But the length of excitation is much higher at the target pattern. We suggested that there is dependence of excitation speed on the mismatch between frequenies if individual cells and the synchronization frequency.

Let us consider, why it is happening. Typical phase portrait of a BvdP element is shown in Fig. 2. Let us consider two coupled oscillatory elements (the first is a bit faster then the second). Inidially they both are located in dot (a). Their frequency mismatch is conditioned by the unstable state position (it is near h_- part of the curve of slow motions. That is why both elements will go approximately together to h_+, then through h_+, and then to h_- via fast motions approximately together. During their moving through h_- their position mismatch is forming, it is conditioned by their individual frequencies. Let us consider two cases: weakly nonidentical, and strongly nonidentcal elements. In the first case when the first element is in (a), the second is in (b) point. In the second case when the first element is in (a), the second is in (c) point. So when in the next moment the first element appears at h_+, element in (b) needs much less time to be synchronized, than element in (c). So the less elements' nonidentity the less time of excitation propogation between them.

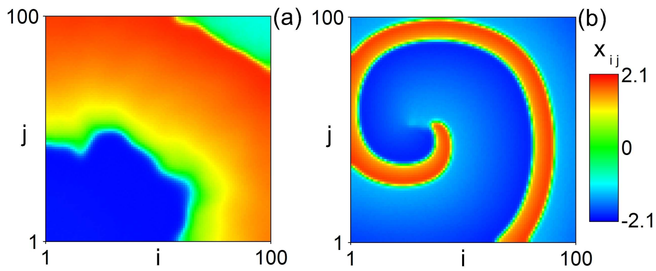

FIG. 1: Typical snapshot of target (a) and spiral (b) wave in two equal lattices.

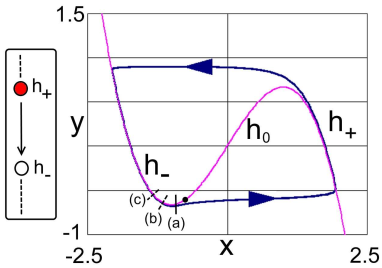

FIG. 2: The phase portrait of a BvdP element. The purple curve is the slow motions, the blue curve is the limit cycle. (a) - initial conditions.

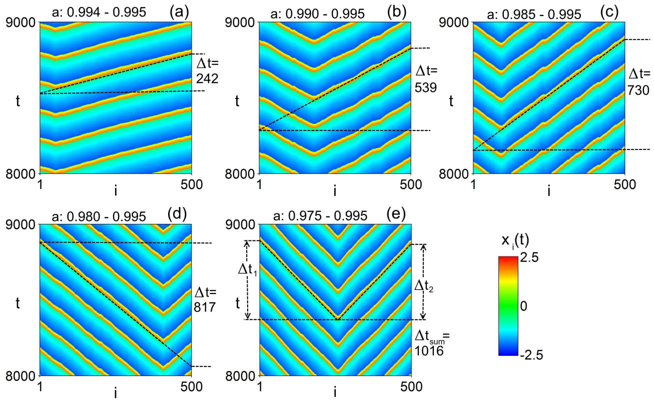

FIG. 3: Excitation propogation in the chain and time which pulse is needed to propogate through the chain. The frequency of the slowest element is constant, and individual frequency of the fastest element was increased from (a) to (e).

IV. CHAIN

We performed series of experiments with the chain of nonidentical elements. Pacemaker is the fastest element here. We measure the dependence of excitation speed on the difference between the average individual period of oscillations and the minimal one (period of synchronization). We used random even distribution of the individual frequencies in some interval between maximal and minimal ones.

Previous suggestions allow us to expect the following:

(i) The more nonidentity, the more $< T > -T_{min}$, and the less speed of excitation front V.

(ii) If $< T > -T_{min} \to 0$, $V \to +\inf$.

(iii) If $< T > -T_{min}$ becomes more than some critical value (synchronization disappears) there is no point to measure V.

The first series of experiments was performed with the following conditions: frequency of the slowest element is constant, individual frequency of the fastest element is varied. The results are shown in Fig. 3.

At the second series we kept frequency of the fastest element to be constant, and individual frequency of the slowest element was varied. The third series corresponds to the average period is approx. constant. Individual frequencies of the slowest and the fastest element were varied. The depencence of excitation speed and the time of propogation on period mismatch is shown at Fig. 4.

Excitation speed doesn't strongly depends on average frequency or the pacemaker frequency themselves. It depends on nonidentity.

V. LATTICE

Let us consider 2D lattice consists from two oscillatory parts. The left one consists of weakly nonidentical slow oscillators, and the right one consists of weakly nonidentical fast oscillators. Such things take place in nature, e.g. mammalian small intestine sections [2]. In our case coupling strength is enough for global synchronization. Typical snapshot is shown at Fig. 5

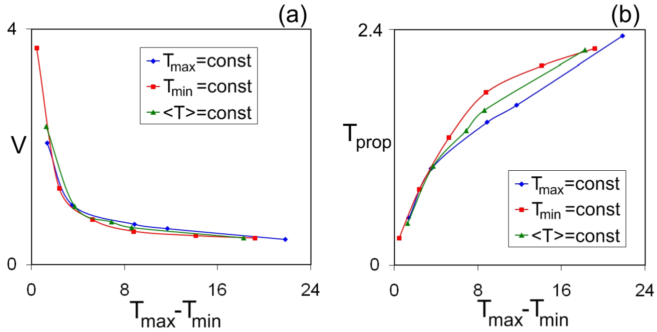

FIG. 4: The depencence of excitation speed (a) and the time of propogation (b) on period mismatch between the fastest and the slowest element in the lattice.

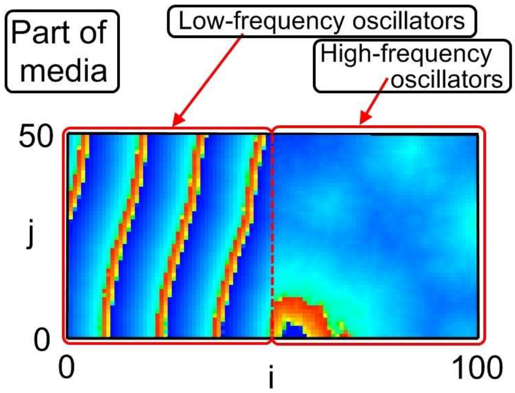

FIG. 5: Snapshot in 2D media which consists from two oscillatory parts. The left one consists of weakly nonidentical slow oscillators, and the right one consists of weakly nonidentical fast oscillators.

Result here is that only one spatial period is in the fast part of the lattice and three spatial periods are in the slow part of the lattice.

Another example is the lattice of nonidentical oscillatory elements surrounded by excitable elements (Fig. 6). Such structure represents sin-atrial node of heart.

Oscillatory elements in (a) are weakly non-identical. Spatial excitation period in the oscillatory lattice is much higher than its size. As the result target patterns realize in (a). Oscillatory elements in (b) are more non-identical than in (a). Spatial excitation period in the oscillatory lattice is comparable with its size. As the result free-end waves realize in (b), which cause forming spirals.

VI. CONCLUSION

The properties of cells, more concretely, individual frequencies have influence on the excitation propogation

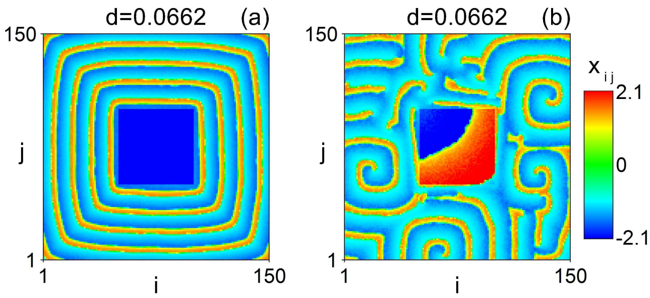

FIG. 6: Target pattern formation at excitable media under influence of oscillatory lattice in the center (a). Spiral patterns formation at excitable media under influence of strongly heterogeneous oscillatory lattice in the center (b).

through the media. Speed of excitation propogation depends on the frequency mismatch between individual cells and the rhythm, which synchronises the media. The more the nonidentity, the less speed of propogation. It was shown using 1D and 2D media and explained on the expample of two coupled elements. One of possible scenarios of transition from target to spiral patters was shown as one of the consequencies of this effect.

VII. ACKNOWLEDGMENTS

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