BUMPLESS TRANSFER FOR A DISCRETE-TIME SWITCHED SYSTEM WITH ROBUST CONTROL

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Abstract
A bumpless transfer for a discrete-time switched system with robust controller is presented. The proposed method is based on pre-setting an off-line controller with reference model respective to an inactive mode, so that oscillations in output transient response are damped, when this mode is activated at switching time. The proposed approach is illustrated by simulation experiments, where the results obtained with and without bumpless transfer are shown and compared for a model of switched DC motor laboratory plant.

Key words
Bumpless transfer, Switched systems, Robust control

1 Introduction

It is common practice to design more than one linear controller for a controlled plant and to switch between them (different operating point, slow/fast controllers, changing a dynamics of system). Such system then belong to a class of switched systems, where several operation modes and switching between them are considered. The problem is, that a switching between individual modes introduces a nonlinearity into the loop [Campo, Morari and Nett, 1989], which may cause undesirable transient effects. The suppression of these effects is called bumpless transfer. Important contribution on bumpless transfer in early stages was provided by Hanus, e.g. [Hanus, 1988].

Various approaches have been proposed for bumpless transfer. In [Turner and Walker (2000); Cheong and Safonov (2008); Zaccarian and Teel, 2002], bumpless transfer is proposed for the case of switching controllers, while the plant dynamics does not change. The authors employ the scheme, where the off-line controller is pre-set, so that on switching instant, the control variable does not change. The approach proposed in [Malloci et al, 2009] considers a discrete-time switched system, where the bumpless transfer aim was to avoid step changes of control variable. To reach this aim, the authors proposed additional controller, modifying the control signal for a determined period of time.

In this paper we study the bumpless transfer (BT) for a robust control of a discrete-time switched system, assuming that the control signal can be switched to the value respective to other mode within the sampling period and the aim of the bumpless transfer is to reduce oscillations of controlled output caused by switching.

The paper is organized as follows. In Section 2 the basic results from literature on LMI robust stability conditions guaranteeing stability of the discrete-time switching system are summarized. Section 3 provides robust PI controller design guaranteeing robust stability. The bumpless transfer scheme for switching system is proposed in Section 4. The respective simulation results for switched laboratory DC motor system with and without proposed bumpless transfer method are shown and compared in Section 5.

2 Problem Formulation and Preliminaries

Let us consider an uncertain switched system described by linear discrete-time state space model:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{N} \xi_i(k)\big(A_i x(k) + B_i u(k)\big) \\ y(k) &= \sum_{i=1}^{N} \xi_i(k) C_i x(k) \end{aligned} \qquad (1)$$

where

$$[A_i, B_i] \in \Omega_i$$

$$\Omega_i = \left\{ \begin{matrix} (A_i, B_i) = \sum_{l(i)=1}^{M(i)} \alpha_{l(i)} \left[A_{l(i),i}, B_{l(i),i}\right], \\ \alpha_{l(i)} \geq 0, \sum_{l(i)=1}^{M(i)} \alpha_{l(i)}(k) = 1 \end{matrix} \right\} \qquad (2)$$

$$C_i = C \text{ for } i = 1, \dots, N$$

$\xi_i(k)$ is the logic parameter and it has value 1, when the state matrix belongs into *i-th* domain; $x(k)$ is the state vector; i is the operation mode; N is number of system modes; in each mode, a polytopic uncertainty domain Ω_i with $M(i)$ vertices is considered for system matrices.

Stability and stabilization of switched discrete-time systems have been extensively studied in literature, e.g. [Geromel and Colaneri (2006); Maherzi, Bernussou and Mhiri (2007)], the latter is used below.

Stabilizing robust static output feedback control $u(k) = K_i C x(k)$ is to be designed in the first step. (In the second step, bumpless transfer will be proposed.) The respective closed loop system matrix in mode i is $A_i + B_i K_i$. Output feedback gain matrices for individual modes can be calculated via Linear matrix inequalities and equalities (LMIs and LMEs), using recent result given by Theorem 1.

Theorem 1 [Maherzi, Bernussou and Mhiri, 2007]
Discrete-time switching system (1) can be stabilized by a static output feedback if there exist M symmetric positive definite matrices $S_1, S_2, \dots S_M$, M matrices $G_1, G_2, \dots, G_M$ and M matrices $U_1, U_2, \dots, U_M$ of appropriate dimensions such that the following LMIs and LMEs hold:

$$\begin{bmatrix} G_i + G_i^T - S_i & \left(A_{l(i),i} G_i + B_{l(i),i} U_i C_i\right)^T \\ * & S_j \end{bmatrix} > 0 \quad (3)$$

$$V_i C_i = C_i G_i \quad (4)$$

Note that equality (4) is added to change the order of unknown matrices and thus convexify the static output feedback problem. Therefore the resulting LMIs are only sufficient for stabilization of the given switched system.
The respective static output feedback gain is defined by:

$$K_i = U_i V_i^{-1} \quad (5)$$

3 Design of Robust PI Controller

This section describes application of Theorem 1 to design robust stabilizing PI controller.

A discrete-time PI controller is described by control algorithm

$$u(k) = K_p e(k) + K_I \sum_{i=0}^{k} e(i)$$

The respective description of the discrete-time PI controller in state space is

$$\begin{aligned} q(k+1) &= [\,I\,]\, q(k) + [\,I\,]\, e(k) \\ u(k) &= K_I\, q(k) + (K_p + K_I)\, e(k) \end{aligned} \quad (6)$$

or

$$\begin{aligned} q(k+1) &= A_r q(k) + B_r e(k) \\ u(k) &= C_r q(k) + D_r e(k) \end{aligned}$$

The dynamics of PI controller is included into controlled system dynamics, thus system matrices are augmented by integral part of PI controller, which enables tracking reference trajectory $r(k)$.

$$e(k) = y(k) - r(k) \quad (7)$$

Integral action for set-point tracking is described by

$$q(k+1) = q(k) + y(k) - r(k). \quad (8)$$

Augmented system for i-th mode corresponding to (1) and (8) is:

$$\begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ C_i & I \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ -I \end{bmatrix} r(k)$$

$$\begin{bmatrix} y(k) \\ q(k) \end{bmatrix} = \begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} \quad (9)$$

PI controller design for augmented system (9) is transformed into a static output controller design for augmented system:

$$u(k) = K_i \begin{bmatrix} e(k) \\ q(k) \end{bmatrix}, \; i=1,\dots,N \quad (10)$$

By solving (3), (4) and (5), parameters of robust PI controllers for all modes are obtained. The respective LMIs can be solved by SeDuMi solver via YALMIP.

Comparison of (6) and (10) provides the following terms for PI controller parameters:

$$K_i = [K_i(1) \quad K_i(2)] = [K_{P,i} \quad K_{I,i}]$$

4 Bumpless transfer for a discrete-time switched system

In this section, the bumpless transfer scheme for a switched system is proposed, following the result from literature of [Turner and Walker (2000)].

The basic idea for bumpless transfer is to minimize transients produced by switching controller from off-line to on-line control [Turner and Walker (2000)]. This can be done by synthesizing a static feedback gain F, which drives the off-line controller as shown in scheme 1 (Fig. 1). We assume that the off-line controller is described by the difference equations:

$$\begin{aligned} q(k+1) &= A_r q(k) + B_r \alpha(k) \\ \tilde{u}(k) &= C_r q(k) + D_r \alpha(k) \end{aligned}$$

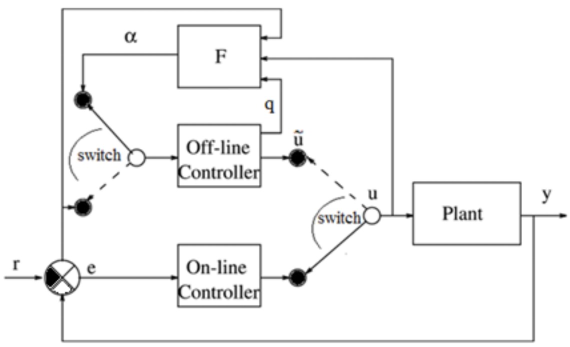

Fig. 1 Bumpless transfer scheme 1

In [Turner and Walker (2000)], the equations determining static feedback gain F for the discrete-time system were developed using quadratic cost function minimization.

$$\alpha(k) = F \begin{bmatrix} q(k) \\ u(k) \\ e(k) \end{bmatrix} \quad (11)$$

$$F = (I - \Delta B_r{}'\Pi B_r)^{-1}\Delta \begin{bmatrix} (D_r{}'W_u C_r + B_r{}'\Pi A_r)' \\ -\left(D_r{}'W_u + B_r{}'(I - M)^{-1}\hat{U}\right)' \\ -\left(W_e + B_r{}'(I - M)^{-1}\hat{E}\right)' \end{bmatrix}'$$

where

$$\Delta = -(D_r{}'W_u D_r + W_e)^{-1}$$
$$\tilde{A} = A_r + B_r\Delta D_r{}'W_u C_r$$
$$\tilde{B} = B_r\Delta B_r{}'$$
$$\tilde{C} = C_r{}'W_u C_r + C_r{}'W_u D_r\Delta D_r{}'W_u C_r$$
$$M = \tilde{A}'(I - \Pi\tilde{B})^{-1}$$
$$\hat{U} = M\Pi B_r\Delta D_r{}'W_u + C_r{}'W_u + C_r{}'W_u D_r\Delta D_r{}'W_u$$
$$\hat{E} = M\Pi B_r\Delta D_r{}'W_e + C_r{}'W_u D_r\Delta W_e$$

W_u, W_e are weighting matrices and Π is a stabilizing solution to the discrete-time algebraic Riccati equation:

$$\tilde{A}'\left(I - \Pi\tilde{B}\right)^{-1}\Pi\tilde{A} - \Pi + \tilde{C} = 0$$

Note that if $D = 0$, the expression for F reduces.

The described approach is appropriate in the case, when the controllers are switched, but the dynamics of the controlled system does not change. For a switched system (1), the problem is different, since the dynamics of the system is also changed by switching between modes. In this case, we propose the bumpless transfer scheme for switching from *i*-th to *j*-th mode, inspired by the above described approach. The main idea is to drive the off-line (*j*-th) controller, so that the value of controlled variable is preset to the value, respective to the *j*-th mode. The *j*-th mode dynamics is provided by the respective reference model (off-line plant). The proposed scheme 2 for switching from the first to the second mode is shown in Fig. 2.

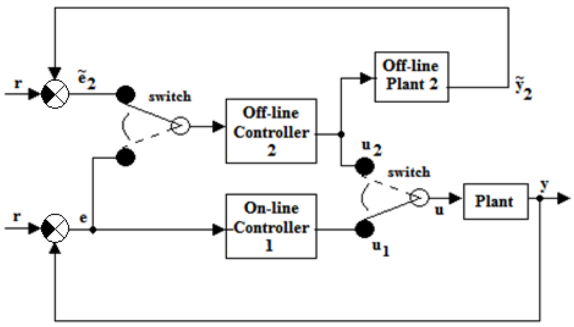

Fig. 2 Bumpless transfer scheme 2 for switching system dynamics

Applying these two approaches described by schemes in Figs 1 and 2, the bumpless control can be designed for switched systems, when the system has two or more modes that represent different system dynamics. In the first case, the transient effects are eliminated when the PI controllers are changed, while the system dynamics does not change. In the second case (the system dynamics and also PI controllers are switched), it is assumed that the control variable can be changed within the sampling period, but the output signal transient effect (due to switching) should be with minimal error. The output oscillations due to switching are caused by old state values (before changing dynamics) of plant.

Note that these two approaches are appropriate when switching comes after sufficient dwell time, when the steady state can be considered. Since robust stabilizing controllers have been designed for switched system (Section 2), stability is not influenced by implementing scheme 2 for bumpless transfer.

5 Switched System - Simulation Results

The proposed approach was tested on a laboratory system shown in Fig. 3. It consists of two DC motors and on/off switching rule. One of the motors is active in each mode and the other one is active only when the switch is on. The input to the system is voltage [V] and output of this switched system is electrical power.

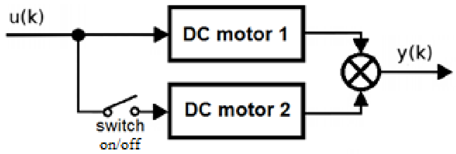

Fig. 3 Scheme of switched DC motors

The real plant was identified in two discrete modes which represent the switching. The measured input and output data were used to obtain ARMAX model. Uncertainty of this system corresponds to changing the value of load (2-6). That means, for each operation mode two vertices of polytopic system were considered, corresponding to upper and lower value of load.

In the first mode, the system was identified for two different load values respective to two vertices of the respective uncertainty domain. The following discrete-time transfer functions were obtained for sample time $Ts = 0.1s$:

Value of load = 2

$$G_{1,1}(z) = \frac{-0.002738z^2 + 0.02474z + 0.05236}{z^3 - 1.654 + 0.756z - 0.05814}$$

Value of load = 6

$$G_{2,1}(z) = \frac{-0.001168z^2 + 0.009795z + 0.02475}{z^3 - 1.731z^2 + 0.873z - 0.1205}$$

The system dynamics is changed, when the switch position is changed. In the second mode, the following discrete-time transfer functions were obtained:

Value of load = 2

$$G_{1,2}(z) = \frac{-0.001049z^2 + 0.04709z + 0.1091}{z^3 - 1.704z^2 + 0.8573z - 0.1069}$$

Value of load = 6

$$G_{2,2}(z) = \frac{0.0002408z^2 + 0.03457z + 0.06102}{z^3 - 1.947z^2 + 1.289z - 0.3128}$$

By using LMI approach from Section 3, the robust PI controller was designed for uncertain switched system. The values of PI controller parameters respective to a state-space form are:

$$A_{r1} = 1\,; B_{r1} = 1\,; C_{r1} = 0.1642\,; D_{r1} = 1.0949$$
$$A_{r2} = 1\,; B_{r2} = 1\,; C_{r2} = 0.0859\,; D_{r2} = 0.6681$$

To verify the designed robust controllers, simulations in each vertex of the polytopic system were performed. The obtained results are shown in Fig. 4.

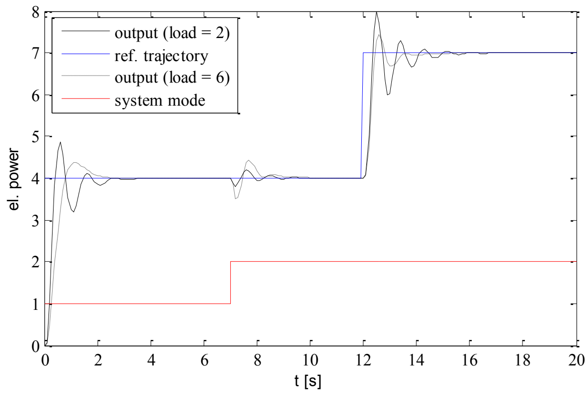

Fig. 4 Time responses of the system output in different vertices of the polytopic system

Time responses of the system output when both system mode and a load value are changed at the same time are depicted in Fig. 5.

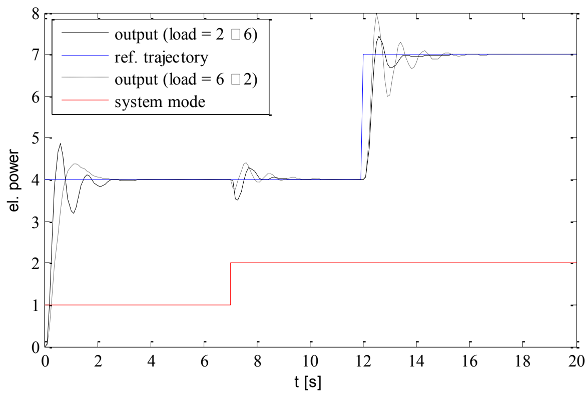

Fig. 5 Time responses of the system output during switching, load change from 2 to 6 (solid line), load change from 6 to 2 (dotted line)

Presented simulation results illustrate the fact that the designed controllers guarantee robust stability of the uncertain switched system during switching and dynamical load change.

5.1 Bumpless transfer application

In the following, we consider constant value of load, which represents one vertex of polytopic model. For illustrating the bumpless transfer in the case, when the controllers are switched in the same mode, another robust PI controller was designed for the first mode with the following parameters:

$$A_{r11} = 1; B_{r11} = 1\,; C_{r11} = 0.12\,; D_{r11} = 0.52$$

Figure 6 shows a signal respective to changing the system modes, switching controllers (in mode 1) and reference trajectory behaviour.

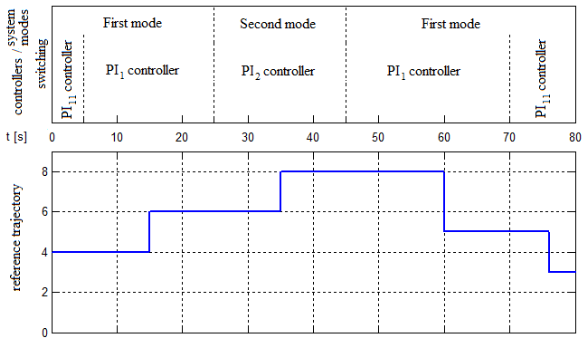

Fig. 6 Graphical illustration of system modes, controllers switching and the reference trajectory

Simulation results without bumpless transfer on switched system are depicted in Figs. 7 and 8 (Fig. 7 – reference trajectory tracking; Fig. 8 – controlled variable response).

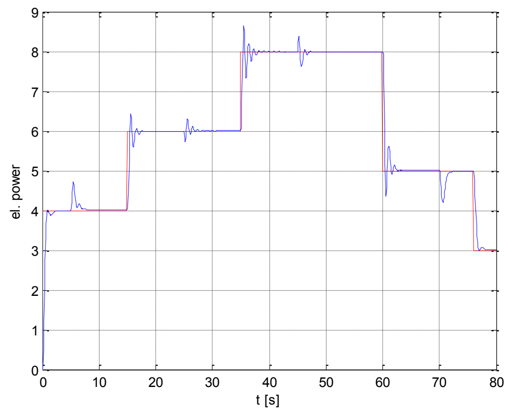

Fig. 7 Time responses of the electric power and reference trajectory

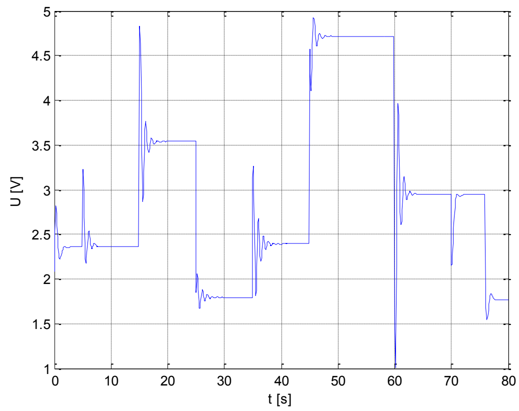

Fig. 8 Time responses of of the controlled variable

As can be seen, changing parameters of controllers and system modes (based on Fig. 6) bring some transient effects, based on control discontinuities.

Figures 9 and 10 show the results with bumpless transfer according to scheme 1 in Fig. 1. (Fig. 9 –

reference trajectory tracking; Fig. 10 – controlled variable response)

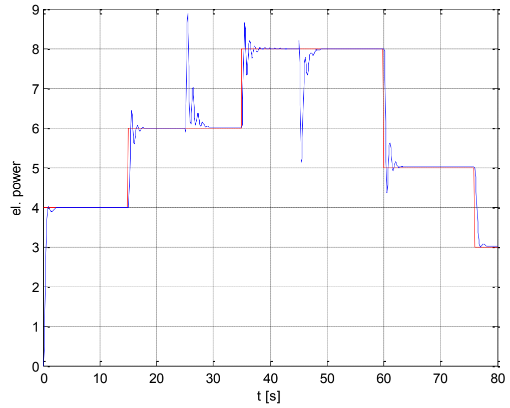

Fig. 9 Time responses of the electric power and reference trajectory (with BT), respective to scheme 1

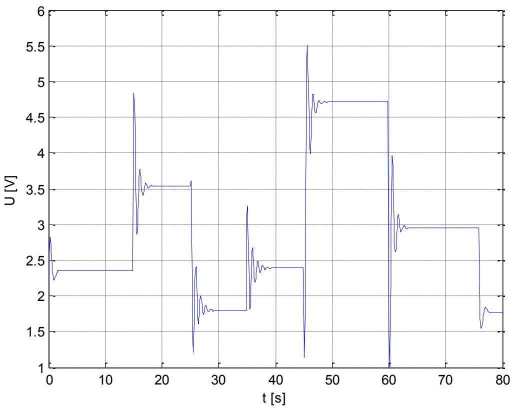

Fig. 10 Time responses of the controlled variable (with BT), respective to scheme 1

As is shown in figures 9 and 10, the control with bumpless transfer suppress transient effects only when the PI controllers are switched, but dynamics of the system remains. In the case when the dynamics of system as well as the PI controllers were switched in the same time (25s and 45s), the results are unacceptable. From these results it can be concluded that scheme 1 does not concern switched systems appropriately.

Next simulation results are based on the proposed method of bumpless transfer for switching systems according to scheme 2 in Fig. 2. It means that this method was combined with the previous one. (Fig. 11 – reference trajectory tracking; Fig. 12 – controlled variable response)

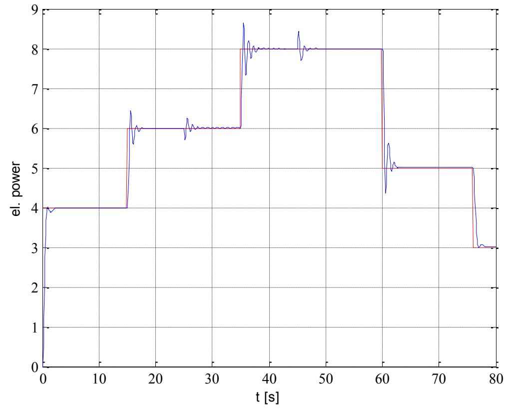

Fig. 11 Time responses of the electric power and reference trajectory (using both approaches: scheme 1 and 2)

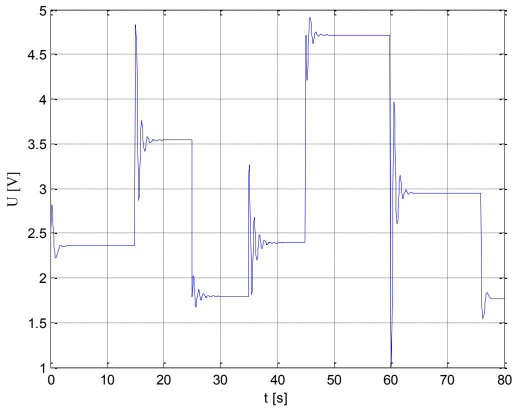

Fig. 12 Time responses of the controlled variable (using both approaches: scheme 1 and 2)

As illustrated by the above simulation results, the proposed combined method (scheme 1 and 2) outperforms for switched system results without BT scheme, or scheme 1 alone. In comparison with simulations in the first case – without BT, the time responses seems to be very close to each other, when the system is switching, but as shown below, this is specific for the designed controller parameters.

Let us now consider redesigned robust controller parameters:

$$A_{r1} = 1 \; ; B_{r1} = 1 \; ; C_{r1} = 0.045 \; ; D_{r1} = 0.245$$
$$A_{r2} = 1 \; ; B_{r2} = 1 \; ; C_{r2} = 0.08 \; ; D_{r2} = 0.78$$
$$A_{r11} = 1 \; ; B_{r11} = 1 \; ; C_{r11} = 0.16 \; ; D_{r11} = 0.5$$

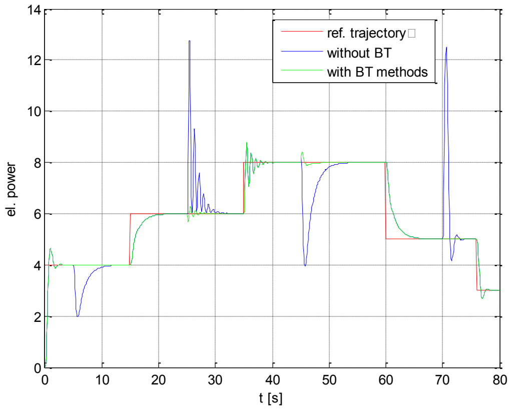

Fig. 13 Time responses of the electric power and reference trajectory (with and without BT methods)

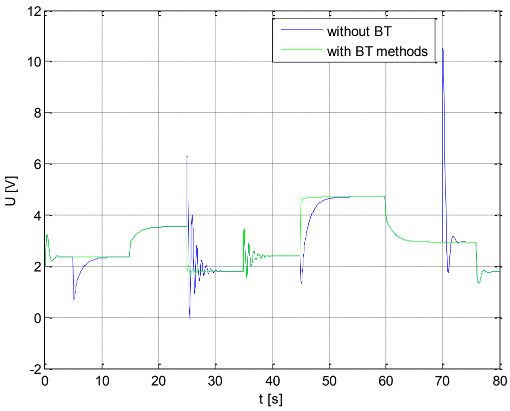

Fig. 14 Time responses of the controlled variable (with and without BT methods)

For redesigned controllers, simulation results show significant differences between the cases with and without the proposed BT scheme. Though the problem of output transient effects when the system is switched (at time 25s and 45s) cannot be totally removed, the oscillations due to switching are significantly reduced.

6 Conclusion

The main aim of this paper was to present bumpless transfer in application for switched system. For the control purpose, robust PI controllers were designed using LMI approach for a discrete-time switched system. The previously proposed bumpless transfer approach (scheme 1) provides positive results when the system dynamics does not change (switching modes) but in the case when dynamics are changed, the transient effects persist. Therefore another method has been proposed which yields to better simulation results.

Acknowledgements

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