PIEZOELECTRIC ACTUATOR BASED ADAPTIVE VIBRATION CONTROL OF FLEXIBLE ARM

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Abstract: This paper presents an adaptive control experiment by piezoelectric actuator to cancel the vibration from a flexible arm. In real systems with external disturbances, the vibration effect due to the inherent elastic deformation of flexible arm makes the satisfactory control results can not be obtained. To reduce the vibration effect, based on an external disturbance model with consideration of dynamics of the piezoelectric actuator, adaptive vibration control experimental system designed, where the controller is based on Youla-Kucera parametrization and right coprime factorization. As a practical appeal, experimental results are shown to support the proposal on the control system design.

Keywords: Coprime Factorization, Adaptive Vibration Control, Disturbances, Flexible Arm, Piezoelectric Actuator

1. INTRODUCTION

Since piezoelectric material actuator is light weight and high operational speed, the use of this kind of actuator has been paid attention. Also, since the piezoelectric actuator can be bonded or embedded along a robot arm easily, in robotic research approach, many researches have been undertaken. As a result, better control performance can be obtained. Concerning with the selection of a piezoelectric actuator, it is useful to know how the physical parameters of the actuator can affect control performance (Dadfarnia, *et al.*, 2003). That is, the dynamics of the actuator needs to be considered. In this paper, piezoelectric actuator based adaptive vibration control of flexible arm is considered, where the arm is modeled from the flexible arm used for wafer conveyance in semiconductor manufacture process. The importance of control of the arm is summarized as follows. Since the semiconductor wafer under conveyance is damaged by vibration of the flexible arm, the vibration control is a very interesting subject (Tomoda *et al.*, 2001). The vibration of the arm tip can be considered as periodic disturbance, in order to remove the influence of the periodic disturbance, a disturbance model needs to be included in a control system by the internal model principle. However, the disturbance model contains unknown parameter, such as frequency, so it is difficult to apply conventional control techniques, e.g. PID control, to the disturbance. In this paper, an adaptive vibration control experimental system is designed, where the disturbance removal compensator in which a disturbance model is included as an internal model is designed using Youla-Kucera parametrization and right coprime factorization (Inoue *et al.*, 2004). Further, the piezoelectric element as an actuator is employed. As a practical appeal, experimental result is shown in order to verify the validity of the designed control system.

The organization of the paper is as follows. In Section 2, experimental system is introduced, and

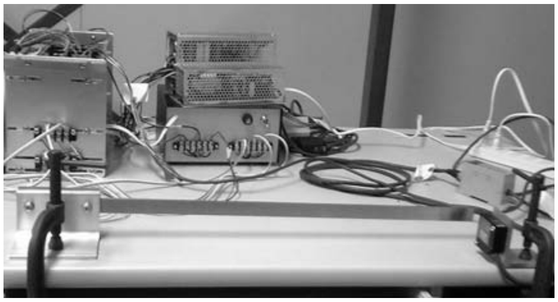

Fig. 1. The experimental system actuated by a piezoelectric actuator

preliminary of this paper is considered. The design of adaptive vibration control system is given in Section 3. In Section 4, the results of experiment are given.

2. EXPERIMENTAL SYSTEM AND PROBLEM SETUP

The experimental system (see Fig.1) has roughly three parts: 1) Flexible arm; 2) Interface; 3) Computer. The flexible arm part consists of a arm $(500(mm) \times 20(mm) \times 3(mm))$, a piezoelectric actuator $(50(mm) \times 20(mm) \times 0.2(mm))$ bonded at the end part of the arm, an amplifier linking with the actuator, and a laser sensor for measuring the vibrating displacement of the arm. The interface part consists of A/D, D/A and Buffer boards. Computer (Pentium 4, 2.8GHz, 512MB, Windows XP) demands to process an adaptive control by using the controller will be given in Section 3, where the software is Visual C++. The schema of the experimental system is shown in Fig.2. In Fig. 3, a control input is a moment M_p generated with the input voltage to the piezoelectric element. An observation output is the displacement $y(l,t)$ at the tip of an arm. The control purpose is presuming a disturbance causes vibration on the arm, estimating the disturbance with an adaptive compensator, and removing the influence, described by (1).

$$\lim_{t\to\infty} |y(t) - y_M(t)| = 0 \tag{1}$$

where $y_M(t)$ is an ideal output to an input r where disturbance is not added.

$$y_M(t) = T(s)r(t) \tag{2}$$

3. ADAPTIVE VIBRATION CONTROL SYSTEM DESIGN

In this section, model of piezoelectric actuator and flexible arm is derived, and adaptive vibration controller based on Youla-Kucera parametrization and right coprime factorization is given.

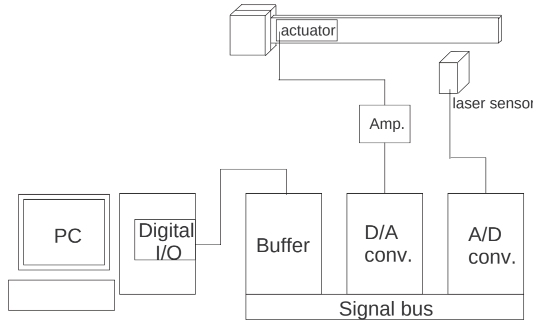

Fig. 2. Schema of the experimental system

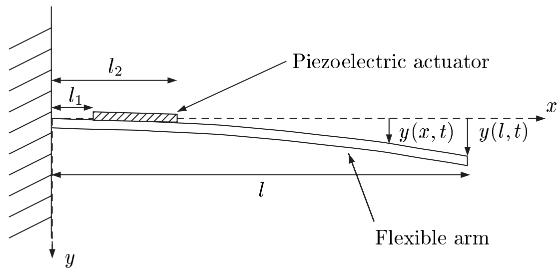

Fig. 3. Schema of the flexible arm with the piezoelectric actuator

3.1 Model of piezoelectric actuator and flexible arm

The input moment to an arm is generated by expansion and contraction of a piezoelectric element which is supplied by input voltage. The moment acts on the attachment part of a piezoelectric element uniformly (Dadfarnia, *et al.*, 2003; Inoue *et al.*, 2004). The moment $M_p(t)$ generated with input voltage $V(t)$ is described by (3).

$$M_p(t) = M_{p0} \cdot V(t) \tag{3}$$

where M_{p0} is a constant decided by the characteristic of the arm and the piezoelectric element.

$$M_{p0} = -\frac{1}{2} b E^p d_{31}(t_b + t_p) \tag{4}$$

b : Width of piezoelectric element
d_{31} : Piezoelectric charge constant
t_p : Thickness of piezoelectric element
t_b : Thickness of arm
E^p : Youngs modulus of piezoelectric element

The dynamics of the arm is described by partial differential equations.

$$\rho S \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI\left(1 + C\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial x^2}\right] = \frac{\partial^2}{\partial x^2}\left[M_p\left\{H(x-l_1) - H(x-l_2)\right\}\right] \tag{5}$$

where $H(\cdot)$ is Heavyside function and others are defined below.

l : Length of arm

l_1, l_2 : Attachment position of piezoelectric element

ρ : Density of arm

S : Cross-section area of arm

E : Youngs modulus of arm

I : Moment of inertia of area

C : Damping modulus

The detailed calculation is shown in Appendix (Inoue *et al.*, 2004).

3.2 Adaptive vibration controller design

The arm dynamics considered in this paper is

$$y(l,t) = T(s)\{M_p(t) + M_d(t)\} \tag{6}$$

where $M_p(t)$ is the control input,

$$T(s) = \sum_{m=1}^{\infty} \left(\frac{\frac{\omega_m(l)}{\rho S \psi_m}\{\omega_m{}'(l_2) - \omega_m{}'(l_1)\}}{s^2 + k_m{}^2 Cs + k_m{}^2} \right) \tag{7}$$

where $M_d(t)$ is a virtual disturbance which causes the vibration and unknown and unmeasurable. Construct the disturbance removal compensator to make the influence of disturbance $M_d(t)$ not appear in output $y(l,t)$.

In this section, assuming that the characteristic polynomial of disturbance is known, a disturbance removal compensator is made using Youla Parametrization based on coprime factrization. In order to remove disturbance completely, it is necessary to include a disturbance model in a control system by the internal model principle.

First, disturbance which satisfies (8) is considered.

$$\Delta(s)M_d(t) = 0 \tag{8}$$

where Δ is the model of disturbance of M_d.

Considering $T(s)$ factored over the ring of proper stable rational functions given as

$$T(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s) \tag{9}$$

$$Y(s)D(s) + X(s)N(s) = I \tag{10}$$
$$\tilde{D}(s)\tilde{Y}(s) + \tilde{N}(s)\tilde{X}(s) = I \tag{11}$$

Since $T(s)$ is a proper stable rational function, $\tilde{N}(s)$, $\tilde{D}(s)$ and the solution of a Bezout equation $\tilde{X}(s)$, $\tilde{Y}(s)$ can be decided as follows, respectively.

$$\tilde{N}(s) = T(s) \tag{12}$$
$$\tilde{D}(s) = I \tag{13}$$
$$\tilde{X}(s) = 0 \tag{14}$$
$$\tilde{Y}(s) = I \tag{15}$$

As mentioned above, the Youla-Kucera expression of a stabilization compensator is expressed as (16).

$$\begin{aligned} C(s) &= [C_1(s), -C_2(s)] \\ C_1(s) &= (D(s) + C_2(s)N(s)K(s)) \\ C_2(s) &= (\tilde{X}(s) + D(s)Q(s))(\tilde{Y}(s) - N(s)Q(s))^{-1} \end{aligned} \tag{16}$$

where $K(s)$ and $Q(s)$ are the free parameters which a designer can set up arbitrarily. Here, the flexibility of $Q(s)$ is used. From (14)∼(16), a control input becomes (17)

$$M_p(t) = D(s)K(s)r(t) - D(s)Q(s)T(s)M_d(t) \tag{17}$$

Using this control input, the output of the plant is as follows.

$$y(l,t) = N(s)K(s)r(t) + [I - N(s)Q(s)]T(s)M_d(t) \tag{18}$$

If defined as

$$K(s) = D^{-1}(s), \tag{19}$$

the output is as follows.

$$\begin{aligned} y(t) &= y_M(t) + y_d(t) \\ y_d(t) &= [I - N(s)Q(s)]T(s)M_d(t) \end{aligned} \tag{20}$$

$y_d(t)$ is the influence of the disturbance to an output. In order to remove this influence, $Q(s)$ is determined from (8) so that $\Delta(s)$ is included in $y_d(t)$.

If $N(s)$ and $\Delta(s)$ are right coprime, there exist $Q(s) \in RH_\infty$ such that

$$N(s)Q(s) + \Delta(s)Z(s) = I \tag{21}$$

and

$$y_d(t) = [I - N(s)Q(s)]T(s)M_d(t) \tag{22}$$
$$= Z(s)T(s)\Delta(s)M_d(t) \tag{23}$$

The influence of disturbance is removed from (8).

$Q(s)$ which satisfies (21) should be used so that the disturbance compensator contains a disturbance model in a closed loop as an internal model. However, when disturbance model $\Delta(s)$ is unknown, $Q(s)$ which satisfies (21) is also unknown. Then, expressing the unknown coefficients of $Q(s)$ as unknown parameter matrix θ^*, $Q(s)$ is identified by identifying θ^* using an adaptive adjustment rule.

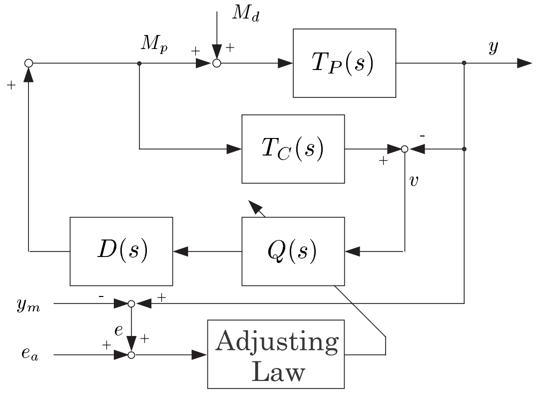

Fig. 4. Block diagram of the control system

If disturbance $M_d(t)$ is defined below,

$$M_d(t) = a \sin(\omega t + \nu), \tag{24}$$

disturbance model $\Delta(s)$ becomes (25).

$$\Delta(s) = \frac{(s^2 + \omega^2)}{(s - \lambda)^2} \tag{25}$$

Using this model, adaptive compensator is designed.

$$Q(s) = \theta^{*T}[1, s]^T \frac{1}{(s + \lambda)^{2l}} \tag{26}$$

$$\theta^* = [\theta_1, \theta_2]^T \tag{27}$$

θ^* is presumed with the following adaptive compensator (Kroumov *et al.*, 1993).

$$\begin{array}{rl}
\text{Internal signal} & v(t) = T(s)u(t) - y(t) \\
\text{1st filter} & \zeta(t) = \dfrac{1}{(s+\lambda)^2}[v(t), sv(t)] \\
\text{Identification value} & \theta(t) \\
\text{Control input} & u(t) = D(s)\theta^T(t)\zeta(t) \\
\text{Output error} & e(t) = y(t) - y_M(t) \\
\text{Extended error} & e_a(t) = \theta^T(t)N(s)\zeta(t) \\
 & \qquad -N(s)\theta^T(t)\zeta(t) \\
\text{2nd filter} & \xi(t) = N(s)\zeta(s) \\
\text{Identification error} & \epsilon(t) = e(t) + e_a(t) \\
\text{Identification rules} & \\
 & \dot{\theta}(t) = -\dfrac{\Gamma(t)\xi(t)\epsilon(t)}{c + \xi^T(t)\Gamma(t)\xi(t)} \\
 & \dot{\Gamma}(t) = -\dfrac{\Gamma(t)\xi(t)\xi^T(t)\Gamma(t)}{c + \xi^T(t)\Gamma(t)\xi(t)}
\end{array}$$

The above compensator is shown in the block diagram Fig.4. This adaptive compensator removes the influence of the disturbance to a plant output.

4. EXPERIMENTAL RESULTS

In this section, two vibration control experiments are performed. Namely, natural vibration and forced vibration of the arm are considered.
In the experiment, we set $T(s) = T_C(s)$ as follows.

$$T(s) = \frac{1}{\alpha s^2 + \beta s + \gamma} \tag{28}$$

where

$$\alpha = 4166.6667; \beta = 623.8553$$
$$\gamma = 12604030.0054$$

The adaptive compensator is designed for the modeled portion $T_C(s)$ and the dynamics of the 2nd, 3rd and high order mode in real plant is considered as unmodeled portion. The experiment is performed using the following parameters.

$$\begin{aligned}
E &= 7 \times 10^{10}[\mathrm{N/m^2}] \\
\rho &= 2700[\mathrm{kg/m^3}] \\
l &= 0.5[\mathrm{m}] \\
S &= 50 \times 10^{-6}[\mathrm{m^2}] \\
l_1 &= 0[\mathrm{m}] \\
l_2 &= 0.1[\mathrm{m}] \\
C &= 0.0007
\end{aligned}$$

The experimental results for natural vibration are shown in Fig. 5. Fig.5(top) shows the arm top point output (dashed line) for without control and the output (solid line) for the same conditions using the proposed method, Fig.5(bottom) is the control input. Further, the experimental results for forced vibration are shown in Fig. 6. Fig.6(top) shows the arm top point output (dashed line) for without control and the output (solid line) for the same conditions using the proposed method, Fig.6(bottom) is the control input. Comparing the results, the proposed compensation algorithm shows a better vibration control performance. We note that the controller output to the piezoelectric actuator is limited as $-200[V] \sim +200[V]$ in the above experiments.

5. CONCLUSION

Adaptive vibration control experiment by piezoelectric actuator to cancel the vibration from a flexible arm has been considered in this paper. The vibration controller is designed by using Youla-Kucera parametrization and right coprime factorization approach. Two experimental results on natural vibration and forced vibration show the validity of the adaptive vibration controller with piezoelectric actuator.

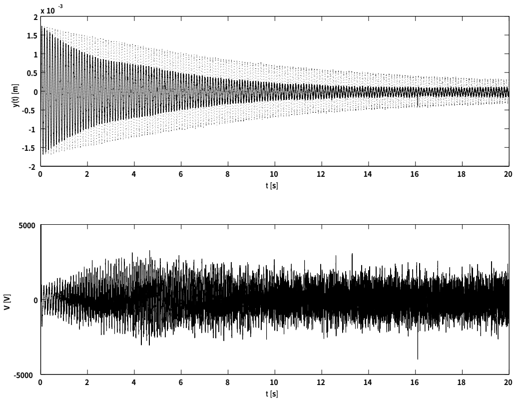

Fig. 5. Experimental result (Natural Vibration)

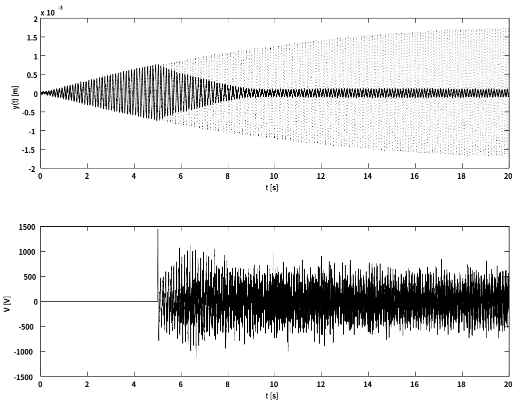

Fig. 6. Experimental result (Forced Vibration)

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6. APPENDIX

First, consider the following natural vibration equation (Inoue *et al.*, 2004).

$$\rho S \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI\left(1 + C\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial x^2}\right] = 0 \quad (29)$$

Using functions $\omega(x)$ and $f(t)$, separate $y(t) = \omega(x)f(t)$ by the variables x and t.

$$\frac{d^2 f(t)}{dt^2} + k^2 C\frac{df(t)}{dt} + k^2 f(t) = 0 \quad (30)$$

$$\frac{d^4\omega(x)}{dx^4} - \lambda^4\omega(x) = 0, \quad \lambda^4 = \frac{k^2\rho S}{EI} \quad (31)$$

Considering the boundary conditions of an arm, (32) is derived.

$$\begin{gathered} \omega(0) = 0, \frac{d\omega(0)}{dx} = 0 \\ \frac{d^2\omega(l)}{dx^2} = 0, \frac{d^3\omega(l)}{dx^3} = 0 \\ 1 + \cos\lambda l \cosh\lambda l = 0 \end{gathered} \quad (32)$$

The solution of (32) becomes as follows.

$$\lambda_1 l = 1.875,\ \lambda_2 l = 4.697,\ \lambda_3 l = 7.855,\ \cdots$$

With $\lambda_m (m = 1, 2, 3, \cdots)$, $\omega_m(x)$ can be expressed like the following formula using an arbitrary constant B.

$$\begin{aligned} \omega_m(x) = B[&(\sinh\lambda_m l + \sin\lambda_m l)(\cosh\lambda_m x \\ &- \cos\lambda_m x) - (\cosh\lambda_m l + \cos\lambda_m l) \\ &(\sinh\lambda_m x - \sin\lambda_m x)] \end{aligned} \quad (33)$$

$\omega_m(x)$ is called the mth order mode function. Real vibration becomes the added vibration from the 1st mode to the infinity mode. Therefore, the displacement of an arm $y(x,t)$ becomes (34).

$$y(x,t) = \sum_{m=1}^{\infty}\omega_m(x) f_m(t) \quad (34)$$

Substituting (34) in to (5), (35) is obtained.

$$\sum_{m=1}^{\infty}\left(\frac{d^2 f_m}{dt^2} + {k_m}^2 C\frac{df_m}{dt} + {k_m}^2 f_m(t)\right)\omega_m(x)$$

$$= \frac{1}{\rho S} \frac{\partial^2}{\partial x^2} \left[M_p(t) \left\{ H(x - l_1) - H(x - l_2) \right\} \right] \tag{35}$$

Taking into account that

$$\int_0^l \omega_m(x) \omega_n(x) dx = \begin{cases} 0 & (m \neq n) \\ \psi_m & (m = n) \end{cases}$$

it is apparent that

$$\left\{ \frac{d^2 f_m}{dt^2} + {k_m}^2 C \frac{df_m}{dt} + {k_m}^2 f_m(t) \right\} \psi_m = \frac{M_p(t)}{\rho S} \left(\frac{d\omega_m(l_2)}{dx} - \frac{d\omega_m(l_1)}{dx} \right) \tag{36}$$

Then, we have

$$y(x,t) = \sum_{m=1}^{\infty} \left(\frac{\frac{\omega_m(x)}{\rho S \psi_m} \{ {\omega_m}'(l_2) - {\omega_m}'(l_1) \}}{s^2 + {k_m}^2 C s + {k_m}^2} \right) M_p(t) \tag{37}$$

where ${\omega_m}'(l_2) = \frac{d\omega_m(l_2)}{dx}$,${\omega_m}'(l_1) = \frac{d\omega_m(l_1)}{dx}$.