OPTIMAL PERIODIC MOTIONS OF TWO-MASS SYSTEMS IN RESISTIVE MEDIA

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Abstract: Progressive motions of two-mass systems in resistive media are analyzed. The motion control is implemented by means of periodic relative displacements of the masses. Different kinds of resistance forces acting upon the system are considered, including linear and nonlinear resistance depending on the velocity, as well as Coulomb's dry friction forces. Constraints are imposed on the relative displacements and velocities of the masses. Optimal periodic motions are determined that correspond to the maximal average speed of the system as a whole. Experimental data confirm the theoretical results obtained. Models of mobile mini-robots are described which are based on the principle presented in the paper.

Keywords: Multibody System, Periodic Motions, Periodic Control, Optimal Control, Mobile Robots

1. INTRODUCTION

A system of two or more bodies can move progressively in a resistive medium, if the bodies perform periodic motions relative to each other. One of these bodies (an inner one) can be contained within a certain closed cavity inside the other (outer) body, so that the system has no outward moving parts such as screws, wheels, legs, wings, etc. This well-known principle of motion is utilized in various projects of mobile robots and underwater vehicles (see, e.g., Breguet and Clavel, 1998; Schmoeckel and Worn, 2001; Vartholomeos and Papadopoulos, 2006).

In this paper, simple models of this phenomenon are analyzed. The mechanical system under consideration consists of two rigid bodies of masses M and m. For brevity, these bodies will be called body M and body m, respectively. Body m moves periodically relative to the main body M which interacts with the outward medium and is subject to resistance forces.

Different kinds of resistance forces acting upon body M are considered, including linear and nonlinear resistance depending on the velocity of the body, and also Coulomb's dry friction. The forces can be anisotropic, i.e., dependent on the direction of the velocity of body M.

The progressive motion of the system as a whole is controlled by the periodic motion of body m relative to body M. Simple relative periodic motions are analyzed, and constraints are imposed on the relative displacements and velocities. Under the constraints imposed, optimal parameters of the periodic motions are determined that correspond to the maximal average speed of the system as a whole. The results obtained (see also Chernousko

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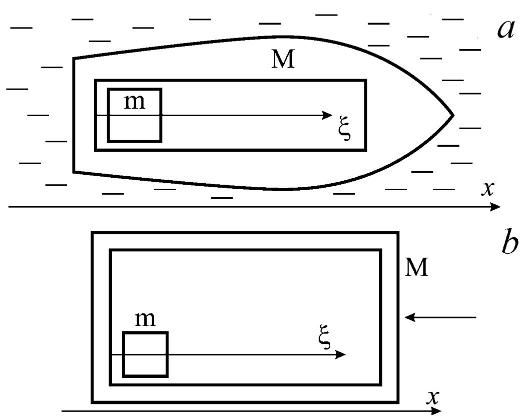

Fig. 1. Mechanical models

2005, 2006a,b) enable one to evaluate the maximal possible speed of mobile mechanical systems that utilize the principle of motion based on relative oscillations of parts of the system moving in a resistive medium.

Experimental results confirm the practical implementability of this principle of motion.

2. EQUATIONS OF MOTION

The system consists of two rigid bodies that can move along a straight line in a resistive medium (Fig. 1). Denote by x and v the absolute coordinate and velocity of the main body M, respectively, and by ξ, u, and w the displacement of the inner body m relative to body M, its relative velocity and acceleration, respectively.

The kinematic equations of motion of body m relative to body M are

$$\dot{\xi} = u, \qquad \dot{u} = w. \tag{1}$$

The dynamic equations for body M can be written as follows:

$$\begin{aligned} &\dot{x} = v, \qquad \dot{v} = -\mu w - r(v) \\ &\mu = m/(M+m), \end{aligned} \tag{2}$$

where $r(v)$ is the resistance force acting upon body M divided by the total mass of the system, $M+m$.

For the anisotropic linear resistance (Fig. 1a), the function $r(v)$ is given by

$$\begin{aligned} r(v) &= k_+ v, \quad \text{if} \quad v \geq 0; \\ r(v) &= k_- v, \quad \text{if} \quad v < 0. \end{aligned} \tag{3}$$

Similarly, for the anisotropic quadratic resistance, this function has the form

$$\begin{aligned} r(v) &= æ_+ |v| v, \quad \text{if} \quad v \geq 0; \\ r(v) &= æ_- |v| v, \quad \text{if} \quad v < 0. \end{aligned} \tag{4}$$

In Eqs. (3) and (4), k_+, k_-, $æ_+$, and $æ_-$ are positive coefficients. For the isotropic case, $k_+ = k_-$ and $æ_+ = æ_-$.

For the case of anisotropic Coulomb's friction (Fig. 1b), the function $r(v)$ is given by

$$\begin{aligned} r(v) &= f_+ g, \quad \text{if} \quad v > 0; \\ r(v) &= -f_- g, \quad \text{if} \quad v < 0, \end{aligned} \tag{5}$$

where g is the acceleration due to gravity, f_+ and f_- are coefficients of friction that can be different for onward and backward motions. If the inequalities

$$-f_- g \leq \mu w \leq f_+ g \tag{6}$$

hold and body M is at rest ($v = 0$), then it will stay at rest.

In what follows, the motion of body m relative to body M is supposed to be periodic with a period T and bounded within a fixed internal:

$$0 \leq \xi(t) \leq L, \tag{7}$$

where $L > 0$ is given. Without loss of generality, it is assumed that at the beginning and at the end of the period body m is at the left end of the internal, so that

$$\xi(0) = \xi(T) = 0, \qquad u(0) = u(T) = 0. \tag{8}$$

The maximal admissible displacement $\xi(0) = L$ is reached at some instant $\theta \in (0, T)$.

The motion of the system is controlled by the relative motion of body m, i.e., by functions $\xi(t)$, $u(t)$, and $w(t)$ subject to Eqs. (1) and conditions (7) and (8).

We will find the relative motions such that the velocity $v(t)$ of body M is T-periodic, i.e., $v(0) = v(T) = v_0$, and the average velocity of the system $V = \Delta x/T$, where $\Delta x = x(T) - x(0)$, is maximal.

3. LINEAR RESISTANCE

Note that the anisotropic resistance (3) is, in fact, nonlinear, if $k_+ \neq k_-$. In the case of the linear resistance, $k_+ = k_- = k$. Substitute $r(v)$ from Eq. (3) and w from Eq. (1) into (2) and integrate the resulting equation to obtain

$$v(T) - v(0) = -\mu[u(T) - u(0)] - k[x(T) - x(0)].$$

Since $u(t)$ and $v(t)$ should be T-periodic, the relation $x(T) = x(0)$ holds and, therefore, $V = 0$.

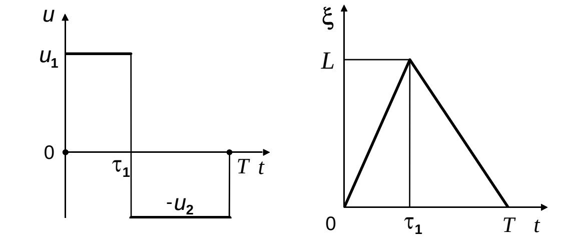

Fig. 2. Relative motion

Hence, in the case of the isotropic linear resistance for an arbitrary periodic relative motion of body m, the system cannot move progressively and will only oscillate about some mean position.

4. RELATIVE MOTION

Only the simplest class of relative periodic motions of mass m, which will be called two-phase motions (Fig. 2), will be considered. Suppose that the period $[0, T]$ consists of two intervals, where $u(t)$ is constant, and denote by τ_1 and τ_2 the durations of these intervals and by u_1 and $(-u_2)$ the values of the relative velocity for these intervals. Accordingly,

$$\begin{aligned} &u(t) = u_1 \quad \text{for} \quad t \in (0, \tau_1); \quad u_1 > 0, u_2 > 0; \\ &u(t) = -u_2 \quad \text{for} \quad t \in (\tau_1, T); \quad T = \tau_1 + \tau_2. \end{aligned} \tag{9}$$

Note that the function $u(t)$ has jumps at $t = 0$ and $t = T$. For convenience, without loss of generality, we define $u(t) = u(T) = 0$ at these instants.

The relative acceleration $w = \dot{u}$ of body m for the two-phase motion (9) is given by

$$\begin{aligned} &w(t) = u_1\delta(t) - (u_1 + u_2)\delta(t - \tau_1) \\ &+u_2\delta(t - T), \end{aligned} \tag{10}$$

where $\delta(t)$ is Dirac's delta function.

The two-phase motion is determined by two independent parameters: u_1 and u_2, or τ_1 and τ_2, which are related by

$$\begin{aligned} &\tau_1 = \theta = L/u_1, \quad \tau_2 = L/u_2, \\ &T = L(u_1^{-1} + u_2^{-1}). \end{aligned} \tag{11}$$

If the relative velocity is bounded, the parameters u_1 and u_2 should be subjected to the constraints

$$0 < u_i \leq U, \qquad i = 1, 2, \tag{12}$$

where U is the maximal velocity allowed for the relative motion.

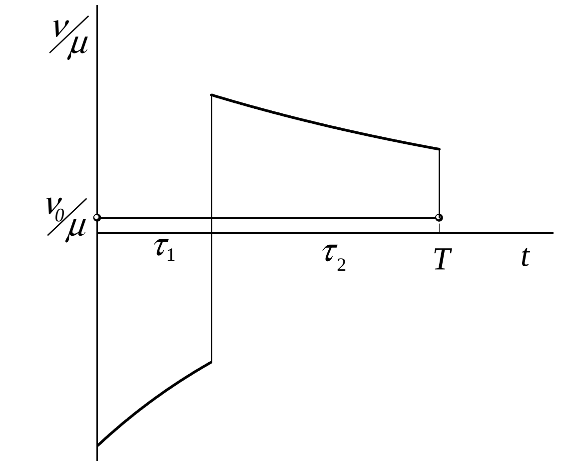

Fig. 3. Velocity of body M

5. NONLINEAR RESISTANCE

To find the periodic motions of body M for the case (3), substitute Eqs. (3) and (10) into Eq. (2) and integrate the resulting equations under the initial condition $v(0) = v_0$. Choose the parameter v_0 such that $v(t)$ is T-periodic to obtain the desired periodic solution (Chernousko, 2006).

$$\begin{aligned} &v(t) = -\frac{\mu(u_1 + u_2)(1 - e_2)}{1 - e_1e_2} \exp(-k_- t) \\ &\text{for} \qquad t \in (0, \tau_1); \\ &v(t) = \frac{\mu(u_1 + u_2)e_2(1 - e_1)}{1 - e_1e_2} \exp[-k_+(T - t)] \\ &\text{for} \qquad t \in (\tau_1, T); \\ &v_0 = \frac{\mu[u_1e_2(1 - e_1) - u_2(1 - e_2)]}{1 - e_1e_2}, \\ &e_1 = \exp(-k_-\tau_1), \qquad e_2 = \exp(-k_+\tau_2), \end{aligned} \tag{13}$$

where the parameters u_1, u_2, τ_1, τ_2, and T satisfy equations (11). The function $v(t)$ from Eq. (13) is shown in Fig. 3. To calculate the total displacement Δx, integrate $v(t)$ from Eq. (13) over the period $[0, T]$. Divide the resulting expression by T and use equations (11) to obtain

$$V = \frac{\Delta x}{T} = \frac{\mu L(1 - e_1)(1 - e_2)(k_+^{-1} - k_-^{-1})}{(1 - e_1e_2)\tau_1\tau_2}. \tag{14}$$

Hence, $V > 0$ only if $k_+ < k_-$, which is physically quite natural. For given μ, L, k_+, and k_-, the average speed V from Eq. (14) depends on two parameters τ_1 and τ_2, or u_1 and n_2. The maximization of V with respect to these parameters subject to the constraint (12) provides

$$\begin{aligned} &V_{\max} = \frac{\mu U^2 L^{-1}(1 - e_1)(1 - e_2)(k_+^{-1} - k_-^{-1})}{1 - e_1e_2}, \\ &e_1 = \exp(-k_-L/U), \quad e_2 - \exp(-k_+L/U). \end{aligned}$$

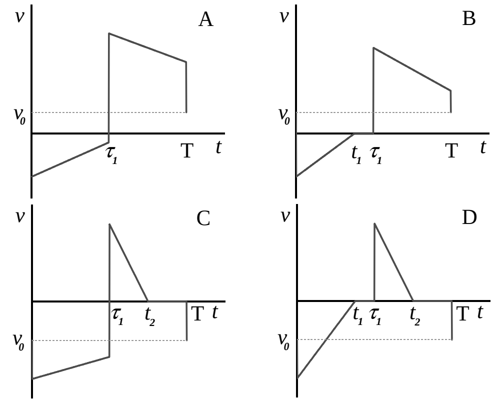

Fig. 4. Modes of motion in the presence of dry friction

In contrast to the case (3), for the quadratic resistance (4) the average speed is positive even for the isotropic case. If $\varkappa_+ = \varkappa_- = \varkappa$ and $\mu L\varkappa < 1$, the maximum average velocity in this case is given by

$$V_{\max} = -U(2L\varkappa)^{-1}(1 - \mu L\varkappa)\log(1 - \mu^2 L^2\varkappa^2) > 0$$

Note that $V_{\max} \to \infty$ as $U \to \infty$ for both cases (3) and (4).

6. COULOMB'S FRICTION

Consider now the case of Coulomb's dry friction (5) for the relative motion defined by Eqs. (9) and (10).

According to Eqs. (2), (5), and (10), the velocity $v(t)$ of body M has two jumps at the ends of the period $[0, T]$ and one jump at the instant $t = \tau_1 \in (0, T)$ inside the period. Between these jumps, body M is subjected only to constant friction force. The absolute value of its velocity here either decreases linearly in time or is equal to zero, if the condition (6) is satisfied. Hence, the period $[0, T]$ can include not more than two intervals of rest where $v = 0$, one of these intervals can be placed before the instant $t = \tau_1$ and the other before $t = T$. Thus, the four modes shown in Fig. 4 are possible:

A — no intervals of rest,

B — one interval of rest (t_1, τ_1),

C — one interval of rest (t_2, T),

D — two intervals of rest (t_1, τ_1) and (t_2, T).

Here, $0 \le t_1 \le \tau_1$ and $\tau_1 \le t_2 \le T$.

Introduce the notation

$$a_+ = f_+ g, \qquad a_- = f_- g, \qquad c = a_+/a_- \tag{15}$$

and consider first mode A.

Using Eqs. (2), (5) and (10), and also the initial condition $v(0) = v_0$ and notation (15), we calculate successively

$$\begin{aligned}
&v(\tau_1 - 0) = v_0 - \mu u_1 + a_-\tau_1,\\
&v(t_1 + 0) = v(\tau_1 - 0) + \mu(u_1 + u_2)\\
&= v_0 + \mu u_2 + a_-\tau_1,\\
&v(T - 0) = v(\tau_1 + 0) - a_+\tau_2\\
&= v_0 + \mu u_2 + a_-\tau_1 - a_+\tau_2,\\
&v(T) = v(T - 0) - \mu u_2 = v_0 + a_-\tau_1 - a_+\tau_2.
\end{aligned} \tag{16}$$

It follows from Eqs. (16) and the periodicity condition $v(T) = v_0$ that $a_-\tau_1 = a_+\tau_2$. Taking into account relations (11) and (12), we obtain

$$cu_1 = u_2. \tag{17}$$

For mode A, the inequalities $v(\tau_1 - 0) \le 0$ and $v(T - 0) \ge 0$ must hold. These inequalities, together with Eqs. (15) – (17), imply

$$-\mu c u_1 \le v_0 \le \mu u_1 - a_-\tau_1. \tag{18}$$

Introduce the non-dimensional variables

$$\begin{aligned}
&u_i = u_0 x_i, \quad u_0 = (a_- L/\mu)^{1/2}, \quad i = 1, 2,\\
&v_0 = \mu u_0 x_0, \qquad V = \mu u_0 F,
\end{aligned} \tag{19}$$

and express τ_1 by means of Eq. (11) to rewrite inequality (18) as follows:

$$-cx_1 \le x_0 \le x_1 - x_1^{-1}. \tag{20}$$

The left-hand side of inequality (20) should not exceed its right-hand side, which implies

$$x_1 \ge (1 + c)^{-1/2}, \qquad cx_1 = x_2. \tag{21}$$

The second equality (21) follows from Eqs. (17) and (19).

Thus, the non-dimensional parameters x_0, x_1, and x_2 of mode A must satisfy conditions (20) and (21). Integrate the function $v(t)$ from Fig. 4 over the interval $[0, T]$ for mode A to evaluate the distance $\Delta x = x(T) - x(0)$ and the average speed $V = \Delta x/T$ of body M. After certain calculations using notation (19), we find

$$F = x_0 + (2x_1)^{-1}. \tag{22}$$

Modes B–D are analyzed analogously to mode A. For each of the modes, three conditions are obtained — one equality and two inequalities imposed on three parameters x_0, x_1, and x_2, and also the expression for the non-dimensional average

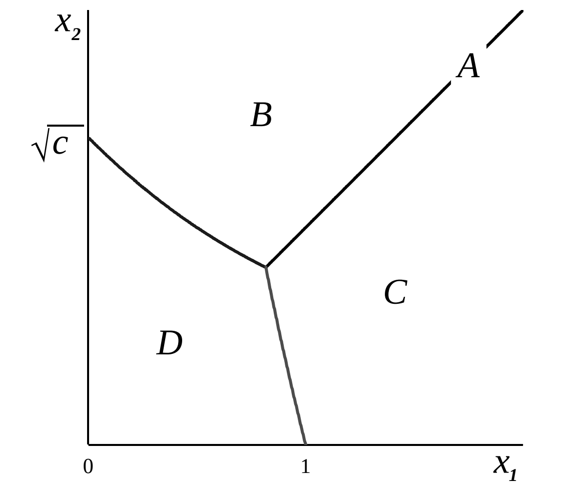

Fig. 5. Domains A–D in x_1, x_2-plane

speed F. The respective relations, similar to those of (20) – (22), have the form

$$\begin{aligned}
&B: x_0 = x_1 - c x_2^{-1},\ c x_1 \le x_2,\\
&(x_1 + x_2) x_2 \ge c,\\
&F = x_1 - \frac{c(c+1)x_1}{2x_2(x_1+x_2)};\\
&C: x_0 = -x_2,\ c x_1 \ge x_2,\ (x_1+x_2)x_1 \ge 1,\\
&F = \frac{(1+c)x_2}{2c(x_1+x_2)x_1} - x_2;\\
&D: x_0 = -x_2,\ (x_1+x_2)x_1 \le 1,\\
&(x_1+x_2)x_2 \le c,\ F = \frac{(1-c)(x_1+x_2)x_1x_2}{2c}.
\end{aligned} \tag{23}$$

Modes A–D take place in the respective domains (23) in the plane of parameters $x_1 > 0$ and $x_2 > 0$. These domains are shown in Fig. 5 for $c > 1$. According to equations (21), mode A occurs on the ray which is the boundary between domains B and C. The boundaries between domain D and domains B and C are the arcs of hyperbolas $(x_1 + x_2)x_2 = c$, $x_1 \le (1+c)^{-1/2}$, and $(x_1+x_2)x_1 = 1$, $x_1 \ge (1+c)^{-1/2}$. These arcs and the ray $x_2 = cx_1$ meet at the point with the coordinates (Fig. 5)

$$x_1 = (1+c)^{-1/2}, \qquad x_2 = c(1+c)^{-1/2}.$$

Thus, for each pair of the non-dimensional parameters x_1, x_2, or the dimensional ones u_1, u_2 or τ_1, τ_2, see Eqs. (19) and (11), one can determine the respective mode of motion using Eqs. (21) and (23) or Fig. 5. Moreover, for modes B–D, one can also find the non-dimensional average speed F by means of Eqs. (23). As for mode A, one should first choose x_0 according to inequalities (20) and then evaluate F by means of Eq. (22). The values of the dimensional velocities u_1, u_2, v_0 and V are determined by equations (19).

7. OPTIMIZATION

Determine the optimal values of the parameters u_1, u_2, and v_0 that correspond to the maximal possible average speed of body M under the constraints (12). In terms of the non-dimensional parameters (19), the optimization problem is stated as follows: find the parameters x_0, x_1, and x_2 that correspond to the maximal value of F under the constraints

$$0 < x_1 \le X, \quad 0 < x_2 \le X, \quad X = U/u_0. \tag{24}$$

The function F is defined be equations (20)–(23) in the respective domains A–D.

In (Chernousko, 2005, 2006a), additional assumption $v_0 = 0$ was made. It follows from Eqs. (20) and (23) that this assumption is compatible with inequalities (24) only for modes A and B. In this paper we will not impose this additional assumption.

Consider the behavior of the function F in domains A–D. Note that this function grows monotonically with x_0 in domain A, see Eq. (22). Hence, the optimal value of x_0 is given by the upper bound in (20) and, therefore,

$$x_0 = x_1 - x_1^{-1}, \qquad F = x_1 - (2x_1)^{-1} \tag{25}$$

for mode A. The function F from Eq. (25) increases with x_1. Therefore, the required maximum of F can be reached in domain A only at the maximal possible x_1 satisfying inequalities (24).

In domain B, the function F increases with x_2. Hence, its maximum can be attained in B only at the maximal possible x_2 compatible with conditions (24). Furthermore, we have $\partial^2 F/\partial x_1^2 > 0$ for all $x_1 > 0$, $x_2 > 0$. Thus, F is a convex function of x_1, and its maximum can be reached only at the ends of the permissible interval of the parameter x_1.

For mode C, the function F, according to Eq. (23), decreases monotonically with x_1. Hence, its maximum is never reached within domain C and can be attained only on its boundaries with domains A and D, see Fig. 5.

In domain D, the function F increases with x_1 and x_2, if $c < 1$. If $c > 1$, this function is negative and decreases with x_1 and x_2 in D.

Summarize now our observations and find the required maximum of F over x_1 and x_2 subject to constraint (24).

If $c \le 1$ and the point $x_1 = X$, $x_2 = X$ lies within domain D (it happens, if $X \le (c/2)^{1/2}$), the

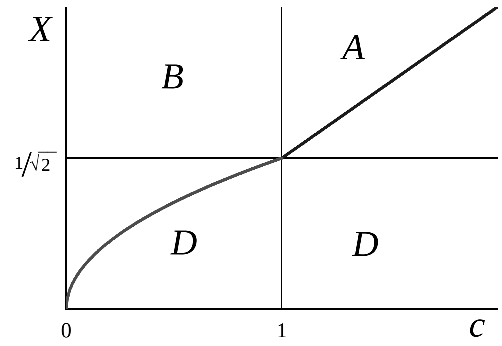

Fig. 6. Domains in c, X-plane

required maximum of the function F is reached at this point. If this point is outside D, then it lies in domain B, and the maximum can be reached either at the same point or at the intersection of the line $x_2 = X$ with the boundary between domains B and D. A comparison of the respective values of F defined by Eq. (23) leads to the conclusion that this maximum is always attained at $x_1 = x_2 = X$.

If $c \geq 1$ and $X < 2^{-1/2}c$, then the function F is always negative. Its zero upper bound is approached, if $x_1 \to 0$ or $x_2 \to 0$. If $c \geq 1$ and $X \geq 2^{-1/2}c$, then the required maximum is attained in domain A at $x_1 = X/c$, $x_2 = X$.

The results obtained are presented in Fig. 6 and by the formulas

$$
\begin{aligned}
&c \leq 1,\ X \leq (c/2)^{1/2} :\ (x_1, x_2) \in D,\\
&x_1 = x_2 = X,\ x_0 = -X,\\
&F = (1-c)X^3/c;\\
&c \leq 1,\ X \geq (c/2)^{1/2} :\ (x_1, x_2) \in B,\\
&x_1 = x_2 = X,\ x_0 = -X,\\
&F = X - c(c+1)(4X)^{-1};\\
&c \geq 1,\ X < c/2^{1/2} :\ (x_1, x_2) \in D,\\
&x_1 \to 0 \text{ or } x_2 \to 0,\ x_0 \to 0,\ F \to 0;\\
&c \geq 1,\ X \geq c :\ (x_1, x_2) \in A,\\
&x_1 = X/c,\ x_2 = X,\\
&x_0 = (X^2 - c^2)/(cX),\\
&F = (2X^2 - c^2)/(2cX).
\end{aligned}
\qquad (26)
$$

We can return to the original dimensional variables in (26) using the notation (19) and (24).

8. GENERALIZATIONS AND EXPERIMENTS

A number of other problems of optimal periodic motions of two-mass systems in resistive media were considered in (Chernousko, 2002, 2005, 2006a; Figurina, 2007). Optimal relative motions with piecewise constant relative accelerations were analyzed (Chernousko, 2002, 2005a, 2006a). A problem of optimal control for a two-mass system was solved in (Figurina, 2007).

The cases of one or more internal masses moving in horizontal and vertical directions inside body M were considered in (Chernousko *et al.*, 2005; Bolotnik *et al.*, 2006). Due to the vertical motion of the internal mass, the pressure of body M

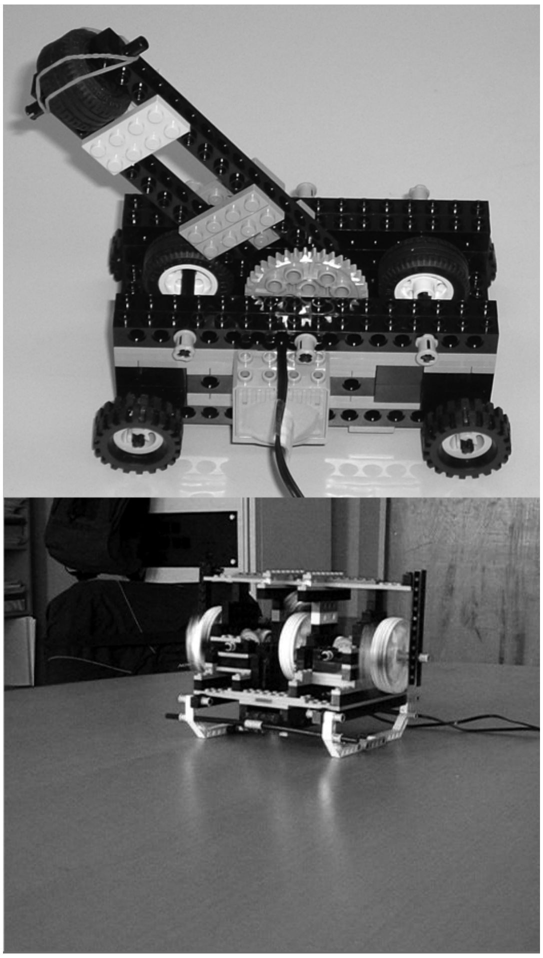

Fig. 7. Experimental models

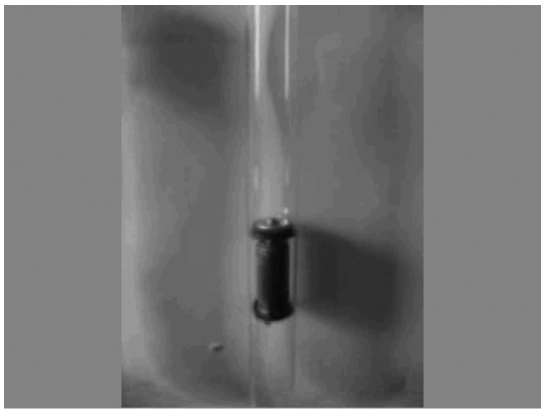

Fig. 8. Mini-robot in a tube

exerted upon the horizontal plane changes and, hence, the friction force changes too. Thus, an additional increment of the average speed is attained.

The principle of motion described above is implemented in experimental models shown in Fig. 7. The internal motions are performed either by an inverted pendulum (Li *et al.*, 2005) or by eccentric rotating wheels. The experiments have shown the implementability of motions induced by moving internal masses.

Mini-robots that utilize the same principle and can move inside tubes (Fig. 8) have been designed and tested by Gradetsky *et al.* (2003).

9. CONCLUSIONS

Progressive motions of a rigid body controlled by periodic oscillations of internal masses are analyzed. For a simple class of periodic relative motions, optimal controls are found that correspond to the maximal average speed of the system in various resistive media.

Experimental data confirm the theoretical results. The principle of motion considered above is of practical use for mobile robots, especially, for mini-robots moving inside tubes and in corrosive media.

REFERENCES

Bolotnik, N.N., I. Zeidis, K. Zimmermann, and S.F. Yatsun (2006). Dynamics of controlled motions of vibration-driven systems. *Journal of Computer and Systems Sciences International*, **45**, 831–840.

Breguet, J.-M., and R. Clavel (1998). Stick and slip actuators: design, control, performances and applications. In: *Proc. International Symposium on Micromechatronics and Human Science (MHS)*, IEEE, New York, pp. 89–95.

Chernousko, F.L. (2002). The optimum rectilinear motion of a two-mass system. *Journal of Applied Mathematics and Mechanics*, **66**, 1–7.

Chernousko, F.L. (2005). On the motion of a body containing a movable internal mass. *Doklady Physics*, **50**, 593–597.

Chernousko, F.L. (2006a). Analysis and optimization of the motion of a body controlled by means of a movable internal mass. *Journal of Applied Mathematics and Mechanics*, **70**, 915–941.

Chernousko, F.L. (2006b). Optimization of the motion in a resistive medium of a body with a movable internal mass. *Proc. of the Institute of Mathematics and Mechanics, Russian Academy of Sciences, Urals Branch*, **12**, 242–248.

Chernousko, F.L., K. Zimmermann, N.N. Bolotnik, S.F. Yatsun, and I. Zeidis (2005). Vibration-driven robots. In: *Proc. of the Workshop on Adaptive and Intelligent Robots: Present and Future, Moscow, Russia, Nov. 24–26, 2005*, pp. 26–31.

Figurina, T.Yu. (2007). Optimal control of the motion of a two-body system along a straight line. *Journal of Computer and Systems Sciences International*, **46**, 227–233.

Gradetsky, V., V. Solovtsov, M. Kniazkov, G.G. Rizzotto, and P. Amato (2003). Modular design of electro-magnetic mechatronic microrobots. In: *Proc. of the 6th International Conference on Climbing and Walking Robots CLAWAR, Catania, Italy, Sept. 17–19, 2003*, pp. 651–658.

Li, H., K. Furuta, and F.L. Chernousko (2005). A pendulum-driven cart via internal force and static friction. In: *Proc. of the International Conference "Physics and Control", St.-Petersburg, Russia, Aug. 24–26, 2005*, pp. 15–17.

Schmoeckel, F., and H. Worn (2001). Remotely controllable mobile microrobots acting as nano positioners and intelligent tweezers in scanning electron microscopes (SEMs). In: *Proc. International Conference on Robotics and Automation*, New York, pp. 3903–3913.

Vartholomeos, P., and E. Papadopoulos (2006). Dynamics, design and simulation of a novel microrobotic platform employing vibration microactuators. *Trans. ASME. Journal of Dynamic Systems, Measurement, and Control*, **128**, 122–133.