Algorithm of multiple synchronization for two-rotor vibration unit with time-varying payload

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1. Introduction

A conventional way to increase quality of vibration units for screening with several vibroactuators is improvement of the quality of hardware. One of the main arising problems is keeping stable synchronous working mode in order to achieve maximum working amplitude of the platform vibrations. Recently an approach based on development of special control algorithms has become rather popular [1]. Additional opportunities for development of vibration equipment, especially for vibrational transportation of materials can be provided by using multiple synchronous modes. It keeps constant the ratio of average velocities and/or phases of vibroactuators. Unlike the simple synchronization modes which can arise spontaneously, stable multiple synchronous mode can only be achieved by means of advanced control systems. A time-varying payload attached to a platform allows to analyze dynamics of processing material.

During recent years the speed-gradient algorithms developed in control engineering area [2] have been applied intensively to control of oscillatory motion and particularly to control of vibration units. Among problems solved by speed-gradient approach are control of vibration units in start-up modes (swing-up and passage through resonance) [3-5], synchronization [6-8], etc.

In this paper an algorithm of multiple synchronization of two-rotor vibration unit with time-varying payload is proposed. The performance of the proposed system is analyzed by computer simulation for model of the 2-rotor vibration set-up.

2. Model of two-rotor vibration set-up dynamics

The scheme of the two-rotor vibration set-up is presented in Fig.1. Here φ_1, φ_2 are rotation angles of the rotors measured from the lowest vertical position, y is the vertical displacement of the supporting body from the equilibrium position, m, M are the masses of the rotors and the supporting body, respectively, J_1, J_2 are the inertia moments of the rotors, ρ is the eccentricity of rotors, c_0, c_1 are the spring stiffness, g is the gravitational acceleration, m_r is the mass of the payload, y_1 is the vertical displacement of the payload, $m_0 = M + 2m$. Let us consider only vertical motion of the system.

Kinetic and potential energies T and Π are as follows:

$$T = \frac{1}{2} m_0 \dot{y}^2 + \frac{1}{2} J_1 \dot{\varphi}_1^2 + \frac{1}{2} J_2 \dot{\varphi}_2^2 + m\rho\, \dot{y}[\sin\varphi_1\, \dot{\varphi}_1 + \sin\varphi_2\, \dot{\varphi}_2] + \frac{1}{2} m_r(t)\, \dot{y}_1^2 .$$
$$\Pi = m_0 g y - m g \rho(\cos\varphi_1 + \cos\varphi_2) + \frac{1}{2} c_1 (y - y_1)^2 + c_0 y^2 + m_r(t) g y_1 \tag{1}$$

Denoting friction coefficient in the bearings of unbalanced rotors by k_c and dissipation of the lower springs by b, we obtain dynamics equations of two-rotor vibration set-up with payload:

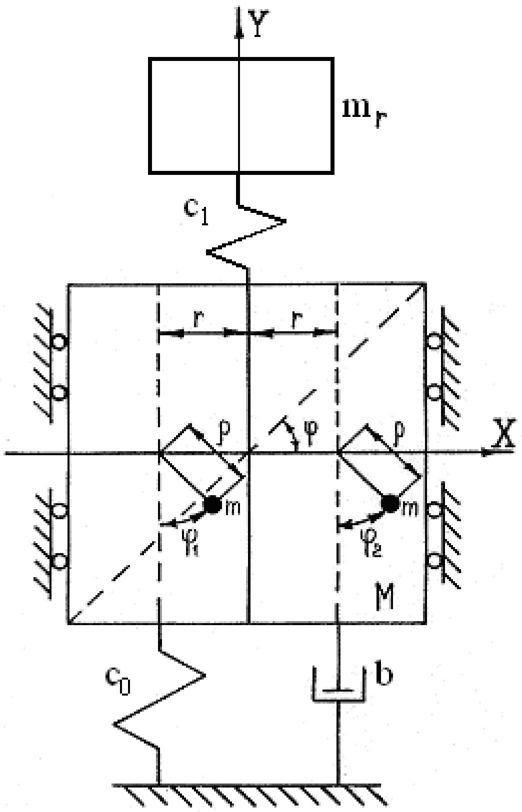

Fig.1. Scheme of the two-rotor vibration set-up.

$$
\begin{cases}
m_0\ddot{y} + m\rho\sin\varphi_1\ddot{\varphi}_1 + m\rho\sin\varphi_2\ddot{\varphi}_2 + m\rho\cos\varphi_1\dot{\varphi}_1^{\,2} + m\rho\cos\varphi_2\dot{\varphi}_2^{\,2} + \\
\qquad + 2c_0 y + c_1(y - y_1) + m_0\cdot g + b\cdot\dot{y} = 0 \\
m\rho\sin\varphi_1\ddot{y} + J_1\ddot{\varphi}_1 + mg\rho\sin\varphi_1 + k_c\dot{\varphi}_1 = M_1; \\
m\rho\sin\varphi_2\ddot{y} + J_2\ddot{\varphi}_2 + mg\rho\sin\varphi_2 + k_c\dot{\varphi}_2 = M_2; \\
m_r(t)\ddot{y}_1 + c_1(y_1 - y) + m_r(t)g + \dot{m}_r(t)\dot{y}_1 = 0,
\end{cases}
\tag{2}
$$

where M_1, M_2 are the motor torques (controlling variables).

3. Control algorithm for multiple synchronization

The first step of design of control algorithm by the speed-gradient method [1, 2] is the choice of the goal functional according to the desired control goal. In our case the control goal is achievement of the desired ratio of angular velocities of the rotors. For n-ple synchronization it means achievement of minimum (zero) value of the term $(\dot{\varphi}_1 \pm n\dot{\varphi}_2)^2$.

Another goal is to achieve the desired level of average angular velocities of the rotors. It corresponds to achievement of the desired average kinetic energy or total energy of the system. Therefore the goal functional can be chosen as follows:

$$Q(z) = 0.5\{(1-\alpha)(H - H^*)^2 + \alpha(\dot{\varphi}_1 \pm n\dot{\varphi}_2)^2\}, \tag{3}$$

where $z = (\varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2, y, \dot{y})^T$ is the state vector of the system; $0 < \alpha < 1$ is weighting coefficient; H^* is the desired level of total mechanical energy.

Obviously the goal is achieved if Q (z) = 0, otherwise Q (z)> 0. At this stage of design we neglected friction ($k_c = 0, b = 0$). Applying the speed-gradient methodology we evaluate the speed of changing (3) along trajectories of controlled system, assuming that payload mass is frozen. Then evaluate the gradient of the speed with respect to controlling variables (torques). The designed control algorithm is as follows

$$M_1 = \gamma_1\{(1-\alpha_1)(H-H^*)\dot{\varphi}_1 + \frac{\alpha_1}{J_1}(\dot{\varphi}_1 \pm n\dot{\varphi}_2)\};$$

$$M_2 = \gamma_2\{(1-\alpha_2)(H-H^*)\dot{\varphi}_2 \pm \frac{\alpha_2 n}{J_2}(\dot{\varphi}_1 \pm n\dot{\varphi}_2)\}, \qquad (4)$$

where $\gamma_1>0$ and $\gamma_2>0$ are control gains.

4. Analysis of double synchronization algorithm by means of computer simulation

Let us analyze efficiency of proposed algorithm of double synchronization *(n = 2)*, designed based on total mechanical energy of the controlled system.

The table 4.1 contains values of varying parameters: rate of payload mass change *V* and final loading time t_2, as well as experimental results taken from simulation plots: synchronization time t_{sync} which is the time until the multiplied phase shift enters 5% zone near its steady state mode, usually a multiple of π), t_p – which is the transient time for rotor velocities, $\Delta\varphi_{st}$ which is the steady state value of the multiplied phase shift.

The final payload mass in our experiments varies up to the value of *25%* from the mass of the supporting platform which is equal to *9 kg*. The maximum value of the rate of payload mass change is *V*=0.33kg/s*.

It is seen from the pictures that multiple synchronization is achieved after *1-3 s*. The payload oscillates with the steady state amplitude from *0* to *5 cm*. Controlling torques vary from *0,8* to *13 N·m*. The payload started changing at $t_1 = 5$ *s* after start. Final loading time t_2 was chosen to achieve the given final value of payload mass change $\Delta m_r = 1$ *kg.* Initial rotor positions were $\varphi_1 = 1 rad, \varphi_2 = 0.75 rad$.

It is seen from the table that the steady state value of multiple phase shift $(\varphi_1 - 2\varphi_2)$ is about 0 rad, i.e. it is close to 3π. Such a difference corresponds to a stable phase shift of π under condition that initial phase shift is 2π.

Table 4.1 – Characteristics of the system with time varying payload and unconstraint controlling torques.

Algorithm parameters	Constraints for controlling torque	Rate of payload mass change	t_2, s	t_{sync}, s	t_p, s	$\Delta\varphi_{st}$, rad
$\gamma_1 = \gamma_2 = 0.01$	M_1 M_2 – no constraints	V = 1/5	$t_2 = 10$	1.9	1.1	10.02
$\alpha_1 = 0.05$ $\alpha_2 = 0.003$		V = 1/4	$t_2 = 9$	1.7	1	9.95
$\varphi_1 = 1$ $\varphi_2 = 0.75$		V = 1/3	$t_2 = 8$	1.3	1.2	9.81

The following plots are shown in the pictures:

a) Platform position *y(t), m* ; b) Platform velocity $\dot{y}$, *m/s*; c) Current value of rotor phases φ_1, φ_2, *rad*; d) Current value of multiple phase shift $\Delta\varphi = (\varphi_1 - 2\varphi_2)$, *rad*; e) Total mechanical energy *Energy, J*; f) Controlling torques *M1, M2, N·m*; g) Rotor velocities $\dot{\varphi}_1, \dot{\varphi}_2$, s^{-1}; h) Multiple velocity shift $\Delta\dot{\varphi} = (\dot{\varphi}_1 - 2\dot{\varphi}_2)$, s^{-1}; i) Payload position $y_1(t)$, *m*; k) Payload mass change m_r , *kg*.

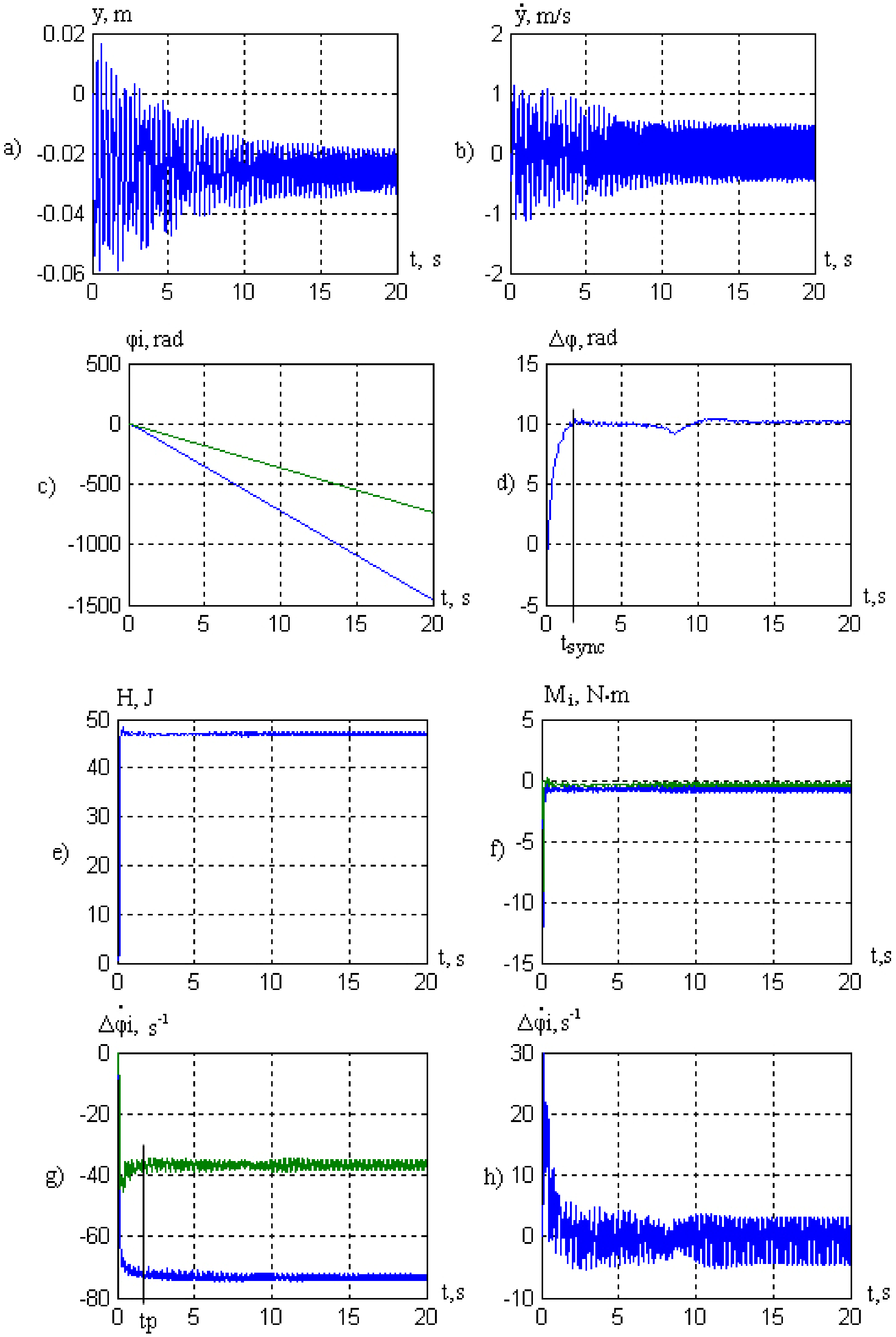

Fig.2. Simulation results with unconstraint controlling torques $V = 1/3\ kg/m$; $t_2 = 8s$.

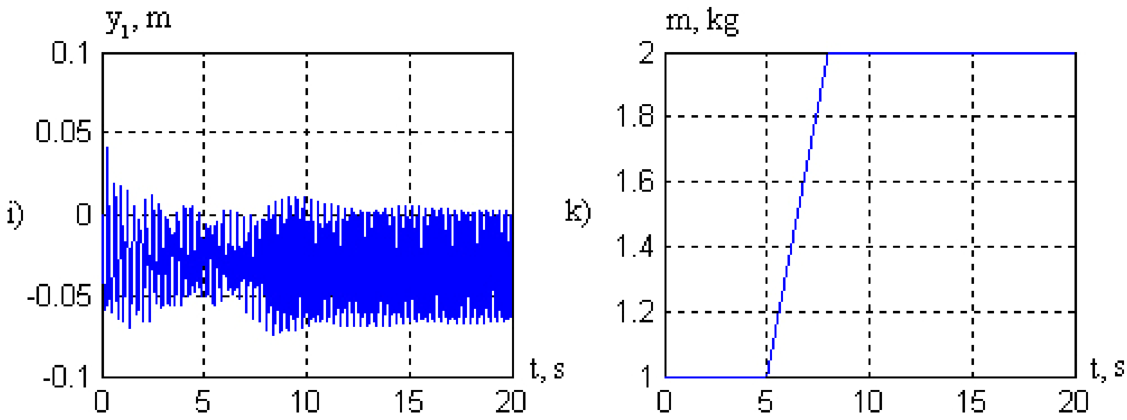

Fig.2 (cont) – Simulation results for system with unconstraint controlling torques $V = 1/3$ *kg/m*; $t_2 = 8s$.

Speed-gradient algorithms possess adaptive properties with respect to time-varying parameters. Their drawback is that they require rather large values of control action at the early stage of the transient process to achieve the specified control goal.

However, other methods of control design, e.g. dominating control have the same drawback. Therefore it is important for practice to perform a comparative analysis of control algorithms under condition of bounded level of control.

The results of analysis are as follows. The table 4.2 contains the values of varying parameters: the rate of change of the payload mass V, final loading time t_2 , upper level of the controlling torques M. It also contains simulation results: synchronization time t_{sync}, transient time in rotor velocities t_p , steady-state value of multiple phase shift $\Delta\varphi_{st}$. Simulation results for the case $M_1<2N\cdot m$; $M_2<2N\cdot m$; $V = 1/3$ *kg/m*; $t_2 = 8s$ are shown in Fig.3.

Table 4.2. Characteristics of a system with time-varying payload and bounded level of controlling torques.

Algorithm parameters	Bound on control level	Rate of payload change	t_2, s	t_{sync}, s	t_p, s	$\Delta\varphi_{st}$
$\gamma_1 = \gamma_2 = 0.01$ $\alpha_1 = 0.05$ $\alpha_2 = 0.003$ $\varphi_1 = 1$ $\varphi_2 = 0.75$	$M_1 < 3$ $M_2 < 3$	V = 1/5	$t_2 = 10$	3.4	1.1	22.70
		V = 1/4	$t_2 = 9$	3.1	1	22.70
		V = 1/3	$t_2 = 8$	3	1.1	22.70
	$M_1 < 2$ $M_2 < 2$	V = 1/5	$t_2 = 10$	4	2.1	22.64
		V = 1/4	$t_2 = 9$	3.8	2	22.59
		V = 1/3	$t_2 = 8$	3.7	2.2	22.57
	$M_1 < 1$ $M_2 < 1$	V = 1/5	$t_2 = 10$	3.2	3.2	116.88
		V = 1/4	$t_2 = 9$	2.9	3.3	116.96
		V = 1/3	$t_2 = 8$	3	3.2	116.85

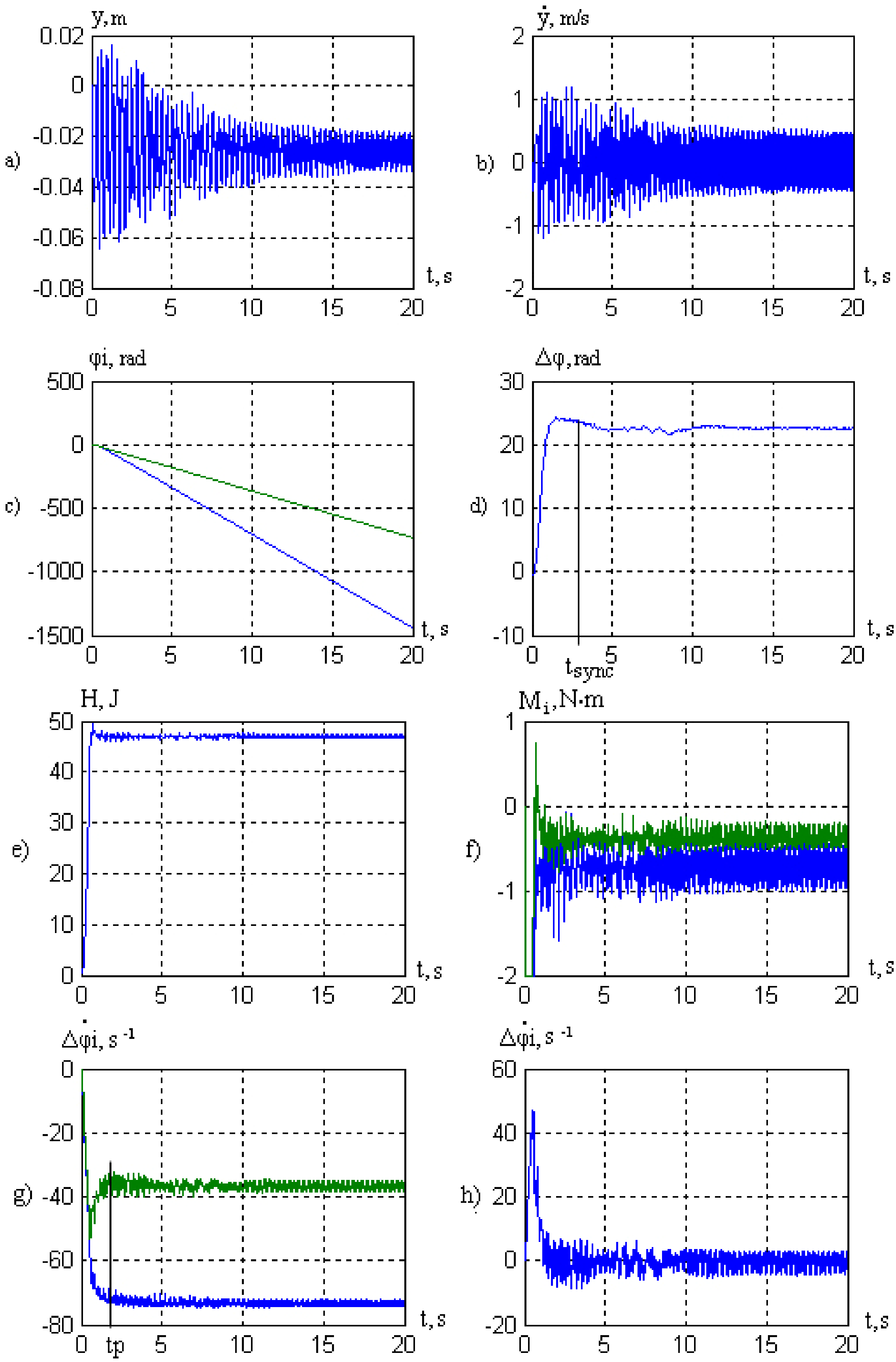

Fig.3. Simulation results for a system with bounded controlling torques, $M_1<2$ $N{\cdot}m$; $M_2<2$ $N{\cdot}m$, $V = 1/3$ kg/m ; $t_2 = 8s$.

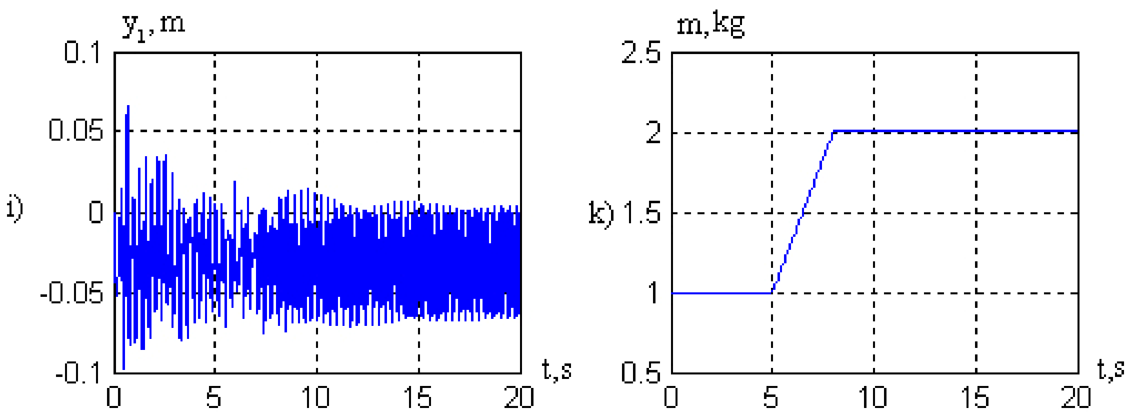

Fig.3 (cont). Simulation results for a system with bounded controlling torques $M_1<2N{\cdot}m$; $M_2<2N{\cdot}m$, $V = 1/3\ kg/m$; $t_2 = 8s$.

5. Conclusions

The main result of our study is demonstration of a stable multiple synchronous mode in vibration units with changing payload mass. It holds if the unit is loaded in a synchronous mode and controlling torques are bounded. The main dynamical properties of the multiple synchronous mode do not depend on the rate of payload mass changing and t_{sync}, is less than $3 – 4\ s$ if the unit is loaded in synchronous mode. Comparison of systems with bounded and unbounded controls shows that the system dynamics in both cases are equivalent. They differ only by number of rotor turns made before the multiple synchronization is achieved.

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