AN EXPERIMENTAL STUDY ON THE OUT-OF-PLANE MOTION UNDER THE EXTERNAL PRIMARY RESONANCE IN STRINGS

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Abstract

Strings were resonated in the neighborhood of the natural frequencies by external excitation. The phenomena of the out-of-plane motion due to the coupling effect of the non-linear stiffness in strings has been theoretically clarified by using the equations of motion which are the non-linear partial differential equations. In this study, we consider the string which is fixed at one upper end and external excited harmonic periodically at the other lower end. We show the equations of motion considering the characteristic of geometrical cubic non-linear restoring force under the external excitation in the string, and analyze theoretically the vibrational mechanism of the out-of-plane motion with method of the multiple scales to find out the equilibrium states and their stabilities from the amplitude equations. Finally, we compare the experimental results and the theoretical ones.

Key words

Strings, Non-linear stiffness, Out-of-plane motion, External excitation, Primary resonance, Restoring force, Coupling effect, Continuous systems

1 Introduction

Recently, they are really easy to occur particular oscillations such as non-linear vibrations and most mechanical systems are possible to regard as continuous systems. Strings are highly interested in the engineering fields and the most fundamental isotropic elements of the continuous systems [Nayfeh, 1979]. It is well-known that the out-of-plane motion in strings is produced in a previous study [O'Reilly and Holmes, 1992]. Making the non-linear dynamical characteristics like strings and beams with respect to continuous systems clear in detail is very important problems to design the mechanical systems and control the infinite degree of freedom. In this study, we consider the out-of-plane motion caused by non-linearity of strings and compare the theoretical results and experimental ones.

2 Theoretical analysis

2.1 Analytical model and equation of motion

Figure 1 is the analytical model of the string for this study. ρ is density of the string, A is cross-sectional area of the string, l is total length of the string, and N_o is initial tension of the string. We set the X-Y-Z coordinate system with respect to time(t). O(upper end) is the origin of the coordinate system. And the lower end is excited periodically. The stiffness of the string is followed the Hook's law. We consider the characteristic of geometrical cubic non-linear restoring force under the external excitation of the string. The gravity can be ignored, because it is too small rather than the tension of the string. In order to non-dimensionalize, we use the representative time and representative length. The dimensionless equations of motion are as follows: [Anand, 1969]

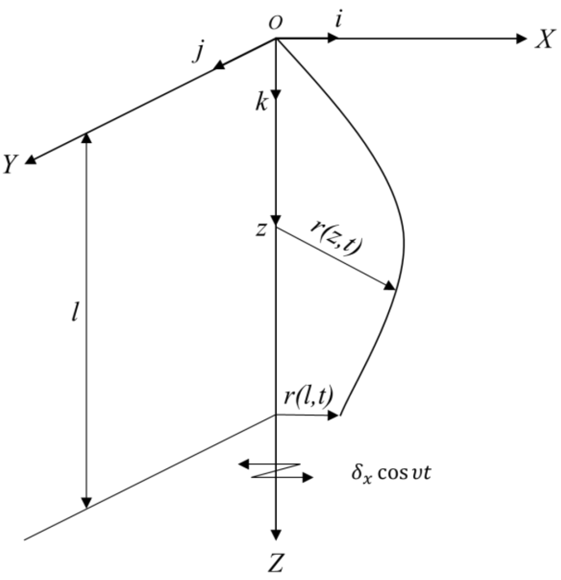


Figure 1. Analytical model of out-of-plane motion

$$\frac{\partial^2 \xi}{\partial t^2} + 2\mu \frac{\partial \xi}{\partial t} - c^2 \frac{\partial^2 \xi}{\partial z^2} = 0$$
$$\frac{\partial^2 \eta}{\partial t^2} + 2\mu \frac{\partial \eta}{\partial t} - c^2 \frac{\partial^2 \eta}{\partial z^2} = 0 \quad (1)$$

where μ is damping coefficient of the system, ξ is displacement of X direction in the string, η is displacement of Y direction in the string. c^2 and β are expressed as follows:

$$c^2 = 1 + \frac{\beta}{2}\int_0^1 \left[\left(\frac{\partial \xi}{\partial z}\right)^2 + \left(\frac{\partial \eta}{\partial z}\right)^2\right] dz, \beta = \frac{EA}{N_0} \quad (2)$$

where E is Young's modulus of the material, and boundary conditions of the system are as follows:

$$\xi(0,t) = 0, \;\; \xi(1,t) = \delta_x \cos \upsilon t$$
$$\eta(0,t) = 0, \;\; \eta(1,t) = 0 \quad (3)$$

where δ_x is the external excitation amplitude, and ν is frequency of the external excitation. The out-of-plane motion occur around the natural frequency of the string due to the non-linear term of the equation including the coupling effect term.

2.2 Theoretical analysis based on method of the multiple scales

The amplitude equations are derived through the method of the multiple scales as follows:

$$\frac{d}{dt}a = -\mu a + \frac{1}{4}C_1 a b^2 \sin 2(\gamma - \theta) - \frac{\delta_x}{2C_{1111}} \sin \gamma \quad (4)$$

$$a\frac{d}{dt}\gamma = \frac{1}{4}C_1 a\{3a^2 + b^2 \cos 2(\gamma - \theta) + 2b^2\} - \sigma a - \frac{\delta_x}{2C_{1111}} \cos \gamma \quad (5)$$

$$\frac{d}{dt}b = -\mu b - \frac{1}{4}C_1 a^2 b \sin 2(\gamma - \theta) \quad (6)$$

$$b\frac{d}{dt}\theta = \frac{1}{4}C_1 b\{3b^2 + 2a^2 + a^2 \cos 2(\gamma - \theta)\} - \sigma b \quad (7)$$

which govern the time variation about amplitude and phase of the both directions as follows:

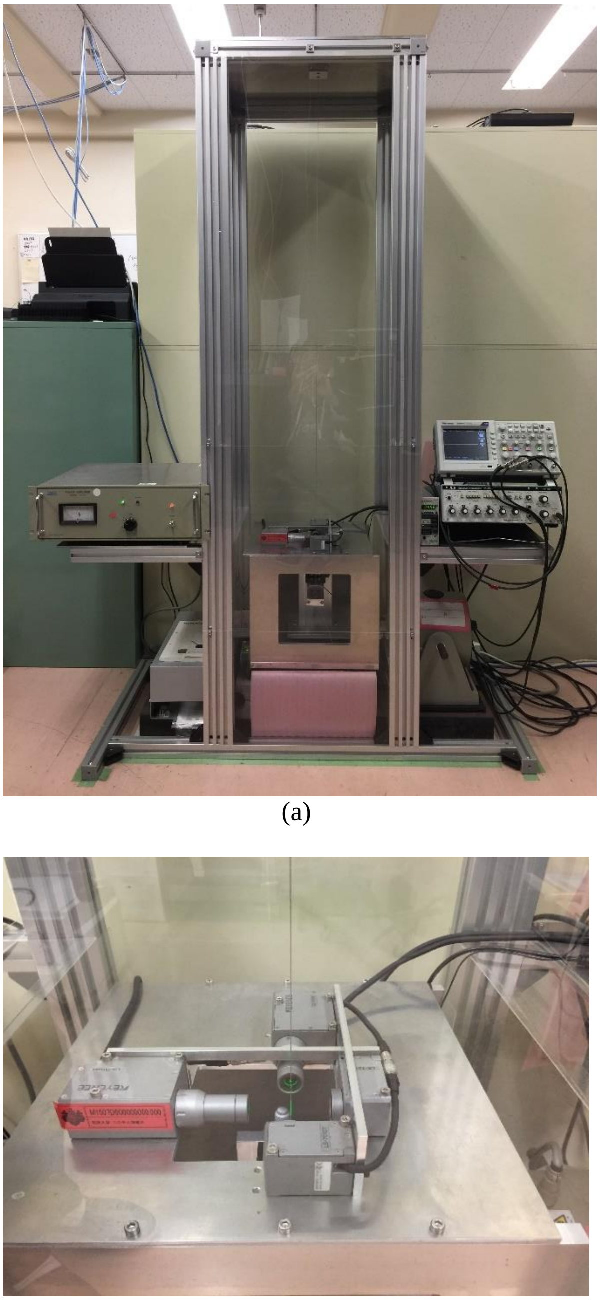
$$\xi = a(t)\cos\{\upsilon t + \gamma(t)\} \sin \pi z$$
$$\eta = b(t)\cos\{\upsilon t + \theta(t)\} \sin \pi z \quad (8)$$

If the equations are solved, we can show the frequency response curve and bifurcation phenomena.

3 Experiment

3.1 Experimental method

Figure 2 is the experimental equipment of the string for this study. The diameter of the string is 5.4 × 10^3 m, and the total length of the string is 1.47 × 10^{-3} m. The material of the string is stainless wire. We use two laser displacement sensors for X and Y directions to measure the displacement in each direction of the string, and set the load cell for the tension variation of the string. It can be changed the tension of the string freely.



(a)

(b)

Figure 2. String externally excited at the lower end

3.2 Experimental result

We obtain experimental results by using FFT. The experimental conditions are set as followings. The initial tension of the string is 10N, and the natural frequency of the string is 37.6Hz. We apply external exciting in the string between 35Hz and 38Hz. Figure 3 to 5 are results of the experiment. First, we can see the spatial motion of the string as Figure 3. The string vibrates both directions in this time. In this case, the string spins around. And we plot the frequency response curves by solving the non-linear equations as Figure 4 and 5.

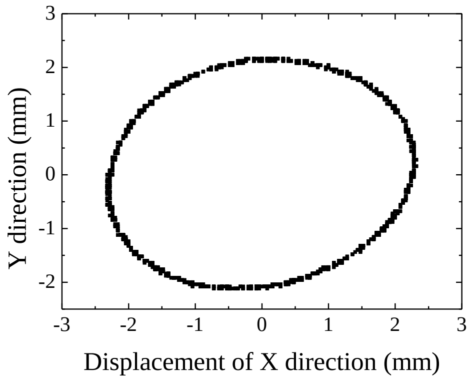


Figure 3. Out-of-plane motion in a string

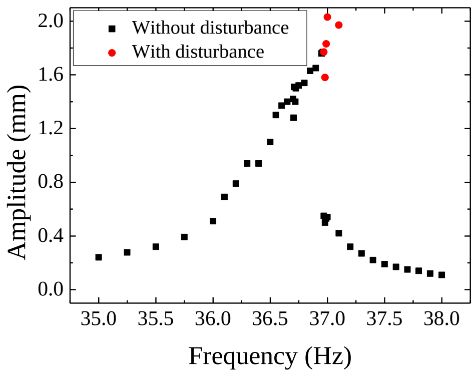


Figure 4. Frequency response curve of X direction by experimental results

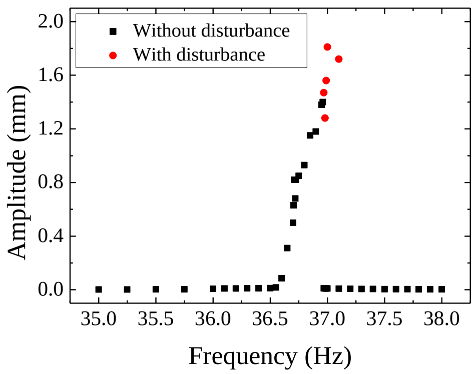


Figure 5. Frequency response curve of Y direction by experimental results

4 Conclusion

In this study, we showed the equations of motion for a string subjected to externally periodic excitation. By using the method of multiple scales, we expressed the amplitude equations and theoretically predict the out-of-plane motion through non-linear coupling. And we also confirmed the occurrence of the out-of-plane motion by using an experimental apparatus.

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