ON DYNAMICS OF DOUBLE PENDULUM IN AIRFLOW

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Abstract

Dynamics of two-section aerodynamic pendulum is studied. It is assumed that the flow acts upon only one section of the pendulum. Aerodynamic load is supposed to be quasi-steady. All equilibrium positions are found, and their stability is analyzed. Numerical simulation is performed. Parameter ranges are found where periodical motions exist, as well as ranges where non-regular motions exist. Obtained results are in qualitative agreement with experimental data.

Key words

Solid body, airflow, aerodynamic pendulum, stability, quasi-steady approach.

1. Introduction

Investigation of interaction of solid bodies with the flow of resisting medium belongs to the range of classical problems of mechanics. Double aerodynamic pendulum is one of the simplest mechanical objects for which appearance of non-linear and chaotic oscillations is possible. On the other hand, steadily growing interest to usage of renewable energy sources, in particular, wind, encourages searching of new directions in construction of wind power generators. Double aerodynamic pendulum is one of perspective systems allowing convert wind energy into mechanic or electric power.

2. Problem statement

Consider the motion of a double aerodynamic pendulum (fig. 1) in horizontal plane OXY. The pendulum is placed in flow of resisting medium [Gertsenstein, Dosaev, Nekrasov, 2004]. The flow speed is constant: $\mathbf{V} = V\mathbf{e}_x$. The first section ("thin" rod OO_1 of length l_1) can rotate about fixed vertical axis OZ. The moment of inertia of the rod about this axis is I_1. The second section is connected to the rod with the joint O_1 and consists of "thin" frame inside of which a plate is fixed. The plate width (chord) is b, its area is S (fig. 1). Let m and I_G be mass of the second section and its moment of inertia about its center of mass G ($|O_1G| = l_2$). Assume that the flow only acts upon the plate, and this aerodynamic load of the flow can be reduced to a force that passes through the center of pressure C situated on the line O_1G.

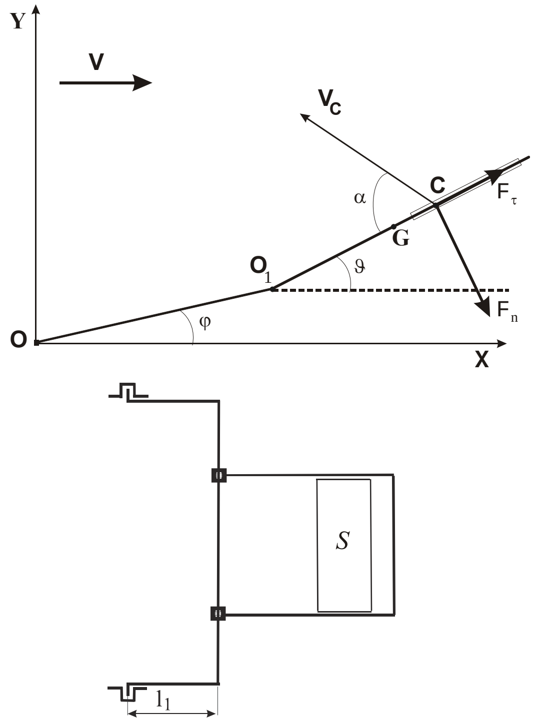


Figure 1

The pendulum position is described by two angles: φ is the angle between OX and the first section, ϑ is the angle between OX and the second section. Introduce also effective angle of attack α, that is the angle between O_1G and the speed $\mathbf{V_C}$ of the center of pressure C with respect to the flow. Thus, state of motion of the pendulum is defined by angles (φ,ϑ), velocities ($\dot\varphi$ and $\dot\vartheta$) and pseudo-velocities α, V_C. These variables are related by kinematical equations:

$$\begin{cases} V_C\cos\alpha = V\cos\vartheta - l_1\dot\varphi\sin(\vartheta-\varphi) \\ V_C\sin\alpha = V\sin\vartheta + l_1\dot\varphi\cos(\vartheta-\varphi) + r(\alpha)\dot\vartheta \end{cases} \tag{1}$$

Here $|O_1C| = r(\alpha)$.

We will take into account only aerodynamic forces and neglect friction. Represent the aerodynamic force applied to the plate in the center of pressure C as a sum of normal force $\mathbf{F_n}$ and tangential force $\mathbf{F_\tau}$:

$$F_n = \tfrac{1}{2}\rho S V_C^2 C_n(\alpha),\ F_\tau = \tfrac{1}{2}\rho S V_C^2 C_\tau(\alpha),$$

where ρ is the medium density, $C_n(\alpha)$ and $C_\tau(\alpha)$ are non-dimensional coefficients of normal and tangential forces. For description of the aerodynamic load we, like in [Samsonov, Sapounkov, 1996], use the quasi-steady model. That is, dependences of aerodynamic coefficients and of the center of pressure position r on α are assumed to be the same as in the static case.

Then motion equations of the system look as follows:

$$\begin{cases} \ddot\varphi\left(I_1+ml_1^2\right)+\ddot\vartheta ml_1l_2\cos(\vartheta-\varphi)-\dot\vartheta^2ml_1l_2\sin(\vartheta-\varphi)= \\ =\dfrac{\rho S l_1}{2}\Big[-C_n(\alpha)\Big(V^2\cos(\vartheta-\varphi)+2Vl_1\dot\varphi\sin\varphi\cos(\vartheta-\varphi)+ \\ +2Vr(\alpha)\dot\vartheta\sin\varphi+(l_1^2\dot\varphi^2+r(\alpha)^2\dot\vartheta^2)\cos(\vartheta-\varphi)+2l_1r(\alpha)\dot\varphi\dot\vartheta\Big)+ \\ +C_\tau(\alpha)\Big(V^2\sin(\vartheta-\varphi)+2Vl_1\dot\varphi\sin\varphi\sin(\vartheta-\varphi)+ \\ (l_1^2\dot\varphi^2-r(\alpha)^2\dot\vartheta^2)\sin(\vartheta-\varphi)\Big)\Big] \\ \ddot\vartheta\left(I_G+ml_2^2\right)+\ddot\varphi ml_1l_2\cos(\vartheta-\varphi)+\dot\varphi^2ml_1l_2\sin(\vartheta-\varphi)= \\ =-\dfrac{\rho S r(\alpha)}{2}C_n(\alpha)\big[V^2+2Vl_1\dot\varphi\sin\varphi+2Vr(\alpha)\dot\vartheta\sin\vartheta+ \\ +2l_1r(\alpha)\dot\varphi\dot\vartheta\cos(\vartheta-\varphi)+l_1^2\dot\varphi^2+r(\alpha)^2\dot\vartheta^2\big] \end{cases} \tag{2}$$

Equations (2) along with relations (1) form closed system. Equations for determining of equilibrium equations of this system are as follows:

$$\begin{cases} C_\tau(\alpha)\sin(\vartheta-\varphi)=0 \\ C_n(\alpha)=0 \\ V_C\cos\alpha = V\cos\vartheta \\ V_C\sin\alpha = V\sin\vartheta \end{cases} \tag{3}$$

Hence, the system has four equilibrium positions where both sections are directed parallel to the speed of the flow:

$$\alpha(t)\equiv\vartheta(t)\equiv 0;\ \varphi(t)\equiv 0;\dot\varphi(t)\equiv\dot\vartheta(t)\equiv 0; V_C(t)\equiv V \tag{4}$$

$$\alpha(t)\equiv\vartheta(t)\equiv\pi;\ \varphi(t)\equiv 0;\dot\varphi(t)\equiv\dot\vartheta(t)\equiv 0; V_C(t)\equiv V \tag{5}$$

$$\alpha(t)\equiv\vartheta(t)\equiv 0;\ \varphi(t)\equiv\pi;\dot\varphi(t)\equiv\dot\vartheta(t)\equiv 0; V_C(t)\equiv V \tag{6}$$

$$\alpha(t)\equiv\vartheta(t)\equiv\pi;\ \varphi(t)\equiv\pi;\dot\varphi(t)\equiv\dot\vartheta(t)\equiv 0; V_C(t)\equiv V \tag{7}$$

Let us call the first equilibrium (4) the position «along the flow».

3. Stability of equilibrium positions

Introduce units of measurement so that the following equalities would be satisfied: $b=1,\quad V=1,\quad \tfrac{1}{2}\rho S=1$.

Study the character of stability of (4) in the first approximation with respect to disturbances of variables $\varphi(t),\vartheta(t),\dot\varphi(t),\dot\vartheta(t)$. Therefore, linearized the ODE system (1-2) and determine stability domains of the solution with the help of Hurwitz criterion. The linearized system looks as follows (the kinematical relations are taken into account):

$$\begin{aligned} &\ddot\varphi\left[I_1+ml_1^2\right]+\ddot\vartheta ml_1l_2=-l_1\left[C_n^\alpha(\theta+l_1\dot\varphi+r_0\dot\vartheta)+C_{\tau0}(\varphi-\theta)\right] \\ &\ddot\vartheta\left[I_G+ml_2^2\right]+\ddot\varphi ml_1l_2=-r_0\left[C_n^\alpha(\theta+l_1\dot\varphi+r\dot\vartheta)\right] \end{aligned} \tag{8}$$

where $C_n^\alpha = \left.\dfrac{dC_n(\alpha)}{d\alpha}\right|_{\alpha=0}$, $C_{\tau0}=C_\tau(0)$, $r_0=r(0)$.

Examine the structure of generalized forces forming right-hand sides of the system (8). First, discuss their part depending only on generalized velocities $\dot\varphi,\dot\vartheta$. Note that the coefficient at $\dot\varphi$ in the second equation is equal to the coefficient at $\dot\vartheta$ in the first equation. Thus, this system does not contain gyroscopic forces.

Now consider the “positional" part of the forces. Evidently, in the second equation the angle φ is absent. The positional forces matrix is asymmetric, that is, non-conservative positional forces are present in the system. In this case the oscillatory (flutter) stability loss is possible.

The necessary and sufficient criterion of stability of the equilibrium position “along the flow” looks as follows:

$$I_1r - I_Gl_1 + ml_1(l_1+l_2)(r_0-l_2) > 0 \tag{9}$$

From the symmetry considerations it can be shown that equilibrium positions (5-7) are statically unstable. For solutions (5-6) the degree of instability is 1. For the solution (7), when both sections are directed “against the flow”, the degree of instability is 2.

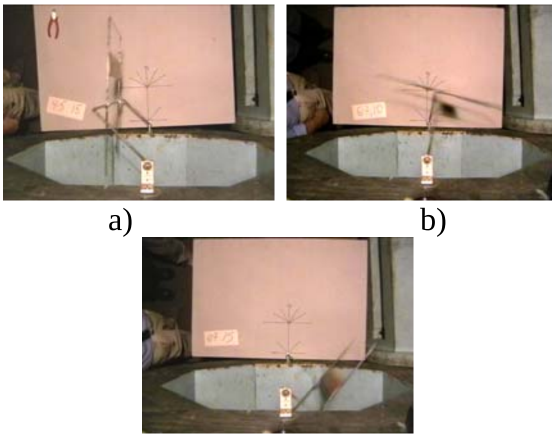
4. Experimental investigations

In the wind tunnel of the Institute of mechanics of the Lomonosov Moscow State University the tests with the double-section pendulum described in [Gertsenstein, Dosaev, Nekrasov, 2004] were performed. The diameter of the wind tunnel is 85cm. Pendulum dimensions are: first section length: 16 cm; second section length: 24 cm; plate chord: 7 cm; plate length: 31,5 cm; plate thickness: 0,9 cm. Moment of inertia of the first section: 5,31 g·m^2, mass of the second section: 0,236 kg.

In these tests, the changeable parameter was the distance L from the center of the plate to the joint O_1. This parameter could take 6 different values.

The experiments were carried out at flow speeds from 5 to 15 mps.

For large enough values of L the equilibrium position «along the flow» was stable. For smaller L equilibrium position along the flow is unstable. There were registered periodic motions where amplitude of φ is much greater than amplitude of ϑ (fig. 2a). For lesser L, another type of periodic motion was registered, where amplitude of oscillations of the second section is large (fig. 2b).



a) b)

c)

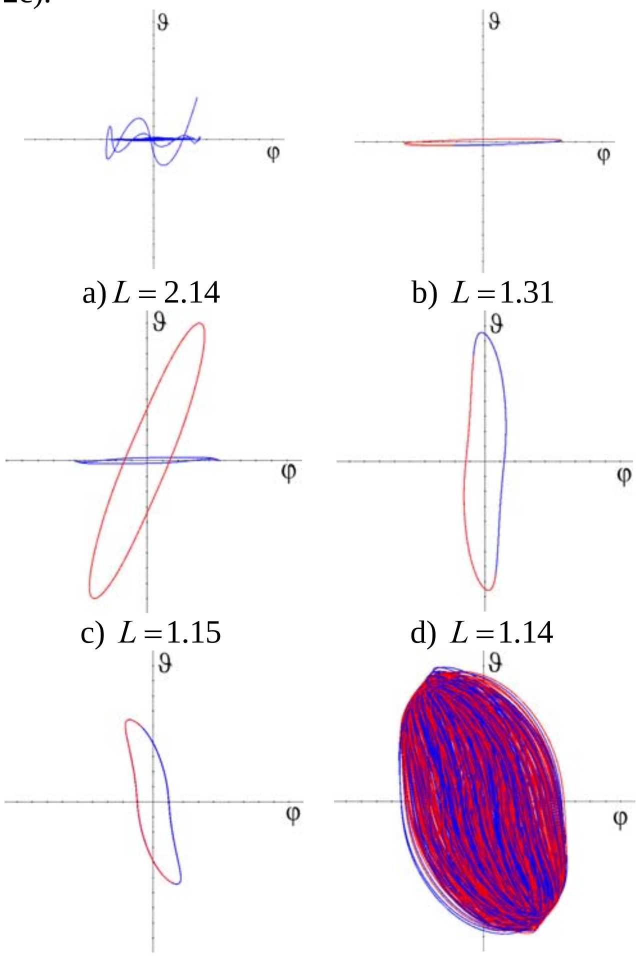
Fig. 2

For still smaller values of the parameter L, there were observed, in particular, long-periodic motions (close to stochastic ones) during which both sections can perform oscillations with large amplitudes (fig. 2c).

5. Numerical integration of motion equations

In parallel, equations (1-2) were integrated numerically. In these calculations the inertial and mass parameters of the pendulum [Gertsenstein, Dosaev, Nekrasov, 2004] were used. For the aerodynamic characteristics for the airfoil were taken experimental data [Tabachnikov, 1974] for NACA0012 wing with aspect ratio 8. Some results of the calculations for different values of the distance L from the center of the plate to the joint O_1 are given in fig. 3.

For large enough L the equilibrium position along the flow is stable (fig. 3a). From (9) it follows that for $L \approx 1.32$ this equilibrium becomes unstable. The Andronov-Hopf bifurcation takes place, and periodic motion appears (fig 3.b). It should be noted, that in this motion amplitude of φ is much greater than amplitude of ϑ (like in experiment). For lesser L there appears another type of periodic motion (fig. 3c). When L decreases further, the first periodic motion disappears (fig. 3d). Evidently, this second type of periodic motion is analogous to the experimental results (fig. 2b). This cycle changes its shape and position with decreasing of L (fig. 3c,d,e). For small L long-periodic motions appear that are close to chaotic ones (fig. 3f). The similar situation was registered in tests (fig. 2c).



a) $L = 2.14$ b) $L = 1.31$

c) $L = 1.15$ d) $L = 1.14$

e) $L = 0.71$ f) $L = 0.51$

Fig. 3

Thus, the calculation results correspond qualitatively to the experimental data.

It should be noted that all these motion types can be easily obtained in experiment by changing of the single design parameter, which makes this system very convenient for using in educational purposes (for example, in practicum for students).

6. Conclusions

Mathematical model of two-section aerodynamic pendulum is developed. All equilibrium positions are determined and their stability conditions are written. Behavior of the pendulum "in the large" is investigated.

Experimental study of pendulum behavior is performed. The results of experiments are in good qualitative agreement with simulation.

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