LOW-FREQUENCY FLUCTUATIONS WITH $1/f$ SPECTRA IN CRITICAL REGIMES WITH PHASE TRANSITIONS

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Abstract

The statistical properties of $1/f$ fluctuations at non-equilibrium phase transitions in a system of two nonlinear stochastic differential equations have been described. It is shown by numerical methods that distributions of duration values of the low-frequency extreme bursts have the power-like form.

Experimental investigation results of statistical characteristics of fluctuation processes at ultrasonic cavitations and explosive boiling of superheated water jets are presented. Results of the experiments carried out fit conclusions of the theoretical model for interacting heterogeneous phase transitions.

1 Introduction

Stationary random processes with power spectrum inversely proportional to the frequency – so called $1/f$ processes - are often observed in natural phenomena. They are characterized by large fluctuations at low frequencies. In some cases such large bursts are evidence of the process catastrophic behavior. Furthermore, the $1/f$ processes attract attention by scale-invariant distribution of the fluctuations. A known example of the scale-invariant fluctuation distribution is the Kolmogorov turbulence, when in fluids emerge streams of various spatial and temporal scales, governed by the law of similarity [Kolmogorov, 1941]. However, not all random $1/f$ processes can be reduced to turbulence. Scale invariance may not include spatial component. Example of such a process is the flicker-noise in electronic devices [Kogan, 1996]. Scale invariance of a random process can be connected with critical behavior or self-organization in complex systems [Jensen, 1998]. Processes with $1/f$ spectrum are observed in different systems which are far from thermodynamic equilibrium, while the fluctuations appear as flashes or indicate avalanche dynamics. The examples can be found not only among current or magnetic fluctuations in solid-state physics [Weissman, 1988], but many random processes in astrophysics, geophysics, biology, informatics have the $1/f$ power spectrum [Klimontovich, 1983].

In search of a general mechanism of $1/f$ processes dynamics a concept of self-organized criticality [Bak, Tang, Wiesenfeld, 1988] was proposed. In essence it is a theory of avalanches, while the random processes are simulated by algorithms of cellular automata. At that, the system is assumed to be spatially distributed, while its behavior is determined by large number of interactions. In the process of evolution there appear fluctuations with fractal spatial and temporal properties which, therefore, are characterized by self-similar distributions of random values. However, the fluctuation power spectrum in the models of self-organized criticality is inversely proportional not to the frequency itself but to the square of frequency $S \sim 1/f^2$ [Jensen, 1998; Kertesz and Kiss, 1990].

This paper is devoted to investigation of statistics of low-frequency extreme pulsations in stationary stochastic processes, which are responsible for increasing part of the power spectrum. The model results are illustrated by experiments with ultrasonic cavitations and explosive boiling of superheated water jets.

2 Statistics of low-frequency large scale pulsations at nonequilibrium phase transitions

Random processes with $1/f$ power spectrum can be obtained in a system of two differential nonlinear stochastic equations, which describe interaction of heterogeneous phase transitions [Koverda, Skokov and Skripov, 1998]. At that, in this case the spatial distribution of the system is not necessary. Fluctuations with $1/f$ power spectrum can be observed in interaction of subcritical and supercritical phase transitions in presence of white noise in the lumped model as well. In the simplest case these equations have the form:

$$\left.\begin{aligned}\frac{d\phi}{dt} &= -\phi\psi^2 + \psi + \Gamma_1(t) \\ \frac{d\psi}{dt} &= -\phi^2\psi + 2\phi + \Gamma_2(t)\end{aligned}\right\} \qquad (1)$$

Here, ϕ and ψ are dynamic variables. They are connected with order parameters for subcritical and supercritical phase transitions. Γ_1, Γ_2 are Gaussian $\delta-$ correlated noises. Coefficient 2 at variable ϕ in the second equation makes this equation driving, while breaking potentiality in the system (1).

In absence of noise, the asymptotic solution for $\phi(t) \to 1/\sqrt{2t}$, i.e. $\phi(t) \to 0$ at $t \to \infty$, while for $\psi(t) \to \sqrt{2t}$, therefore $\psi(t) \to \infty$ at $t \to \infty$. The effect of white noise in system (1) eliminates divergence $\psi(t)$ at $t \to \infty$, and time series of random processes $\phi(t)$ and $\psi(t)$ become stationary. In a certain rather broad range of white noise variance the spectrum of variable $\phi(t)$ is inversely proportional to frequency $S_\phi \sim 1/f$. The spectra of variable $\psi(t)$ are inversely proportional to square of frequency: $S_\psi \sim 1/f^2$. The approximate analytical expressions for distribution functions of the variables ϕ and ψ were obtained [Koverda and Skokov, 2004]:

$$\left.\begin{aligned}P(\psi) &\sim \psi \cdot \exp\left(-\frac{1}{2}\psi^2 \cdot \Delta t\right) \\ P(\phi) &\sim \phi^{-3} \cdot \exp\left(-\frac{1}{2}\Delta t \cdot \phi^{-2}\right)\end{aligned}\right\} \qquad (2)$$

Where Δt is the interval of the numerical integration. The spectra S_ϕ, S_ψ, and the distribution functions $P(\phi)$ and $P(\psi)$ are shown in Figure 1. The solid curves in the inserts of the Fig.1 correspond to the approximations of the distribution functions by dependences (2).

Though the power spectrum of the variable ψ looks like $1/f^2$ behavior, the inverse variable $\chi = 1/\psi$ has the spectrum inversely proportional to the first order of frequency $S_\chi \sim f^{-1}$ coinciding with spectrum S_ϕ. Function $\chi(t)$ is interesting because it is scale-invariant for any intervals of time roughening, like the avalanches in the models of self-organized criticality. Note that function $\phi(t)$ becomes scale-invariant in the limit of large roughening times.

The system is characterized by strongly stretched critical state of transition [Koverda and Skokov, 2002]. In this state in the low-frequency limit fluctuations become self-similar, i.e. characteristic time scale for function of fluctuation distribution disappears. Furthermore, in the low-frequency limit, distribution functions become non-Gaussian. At that, they deviate from behavior of functions of turbulent pulsation distribution, for which scale transformation at roughening leads to Gaussian distribution, while the nonlinearity is mostly expressed for high-frequency part of the spectrum [Carbone, Cavazzana, Antoni, et. al., 2002].

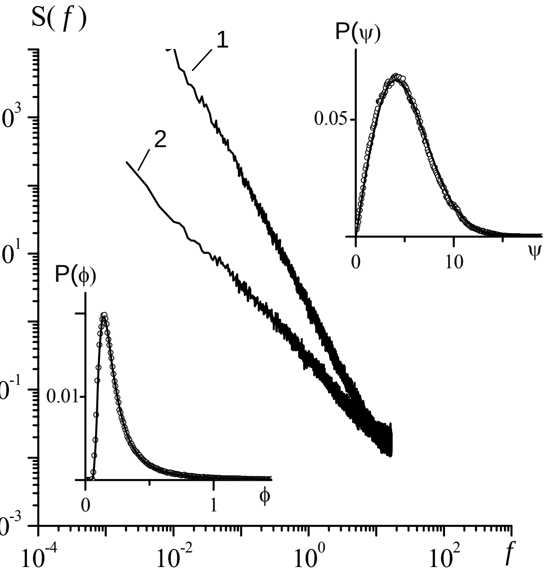

Figure 1. Power spectra S_ψ – (1), S_ϕ – (2) of the variables ψ and ϕ. The distribution functions for these variables are also shown (in inserts), which are obtained by numerically solving the set of Equations (1).

Approach to modeling random processes with 1/fspectrum with help of stochastic equations with deterministic part corresponding to potential of interacting phase transitions of Landau theory does not entirely coincide with results of self-organized criticality theory, where processes are modeled by cellular automata. However, experimental research already carried out for heat and mass transfer accompanied by non-equilibrium phase transitions gives opportunity to hope that this approach will be promising [Skokov, Koverda, Reshetnikov, et al., 2003]. In order to determine an appropriate model in heat mass exchange experiments, one should carry out roughening scale transformation in observed realizations of random processes and investigate statistics of extreme pulsations in details. In theory this problem can be solved with use of the fact that relaxation patterns at steadying of stationary random process in ensemble of realizations with arbitrary initial conditions coincide with statistic patterns of particular large-scale low-frequency bursts.

Relaxation of low-frequency extreme bursts in random processes with 1/*f* spectrum is investigated in the [Koverda, Skokov, 2007]. Relaxation begins as power dependence, then it turns to an exponent with decrease of the parameters. Characteristic relaxation time emerges. With subsequent decrease of parameters relaxation sharply stops, and system "forgets" its initial conditions. Discontinuous "forgetting" of the initial conditions and convergence to a random process determined by white noise only, tells about absence of fine adjustment and self-organization of the system. Investigation of relaxation at steadying of stationary random process under different initial conditions allows one to sort out low-frequency fluctuations in the ensemble of realizations. They are responsible for increasing part of the power spectrum and resemble the avalanches studied in models of self-organized criticality. Since the character of relaxation does not depend on values of the initial conditions, then relaxation patterns at establishing stationary stochastic process in the ensemble of realizations coincide with statistical patterns of particular large-scale low-frequency bursts. Thus, the "avalanches" (non-overlapping low-frequency fluctuations in stochastic process) statistics can be described by the system (1). In order to find statistical distributions on duration and maximal values of bursts by numerical integration of system (1), we considered ensembles of random processes containing 1000 time series with different initial conditions. Figure 2 shows distribution function $P(T)$ of duration T of low-frequency pulsations in the log-log scale. The dash line corresponds to the power dependence $\sim 1/T$.

The set of equations (1) describes fluctuation processes at interaction of heterogeneous phase transitions in a system. Such processes with simultaneous passing and interaction of different non-equilibrium phase transitions are rather common in nature, in particular they are observed in critical regimes with nonequilibrium phase transitions.

Experimental investigation of fluctuation dynamics in critical and transitional modes shows existence of irregular high-energy pulsations with power spectrum inversely proportional to the frequency – so called 1/*f* spectrum. Such regimes are characterized by the fact that an essential part of the pulsations energy is connected with very slow processes and mean that large high-energy bursts are possible in the system. Another characteristic feature of such regimes is scale invariance of the fluctuations distribution function. According to the theory, the 1/*f* fluctuations can emerge in physical systems due to simultaneous phase transitions in presence of sufficiently intensive white noise.

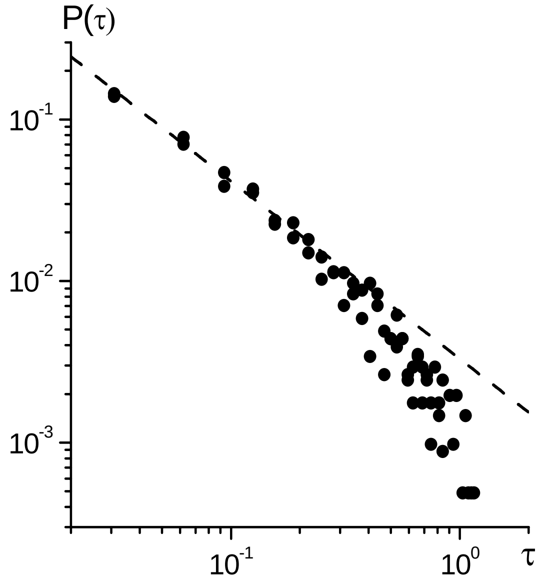

Figure 2. Distribution function $P(T)$ of duration T of low-frequency pulsations.

3 Experimental study of low-frequency pulsations with 1/*f* spectra under nonequilibrium phase transitions

3.1 Acoustic cavitations of liquids

Creation of cavitations clouds at ultrasonic impact can be considered as a non-equilibrium phase transition in complex system of interacting cavitation cavities and acoustic waves. In the acoustic field a stationary random process with non-equilibrium phase transitions, which power spectrum can have 1/*f* form, is formed [Koverda, Skokov, Reshetnikov, and Vinogradov, 2005].

Complex character of interaction between emerging gas-vapor bubbles and with the acoustic field can lead to formation of various spatial structures [Lauterborn, Schmitz, and Judt, 1993; Moussatov, Granger, and

Dubis, 2003; Skokov, Koverda, Reshetnikov, and Vinogradov, 2006]. One of them is shown in the Figure 3.

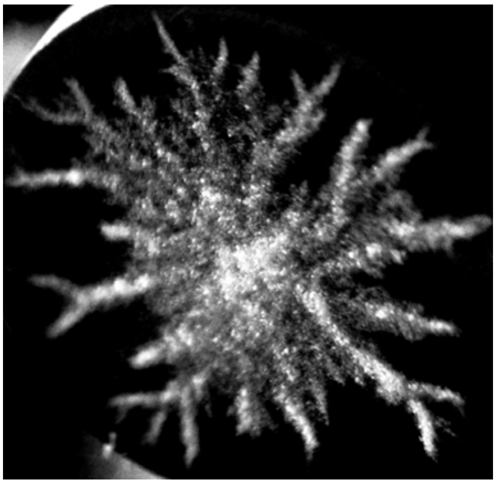

Figure 3. A fractal cluster under acoustic cavitations of water.

In order to investigate fluctuation dynamics a laser beam with tuned intensity was transmitted through the cavitation areas. The fluctuation power spectrum was determined from experimental time series and is shown in Figure 4.

One can see from the figure that $1/f$ behavior is traced through more than 3 orders of the frequency variation.

Distributions functions of fluctuations were determined through experimental time series. From experimentally measured time series roughening realizations were created with help of averaging over a certain time scale. Beginning with a certain scale, functions of fluctuation distribution became scale-invariant. In the region of large fluctuation values, distribution functions differ from the Gaussian ones, that corresponds to the low-frequency diverging part of the power spectrum and means a possibility of large low frequency bursts. The initial realization (65000 points) and two roughened ones with coefficients of scale transformations $\tau = 50$ (1300 points in the realization) and $\tau = 200$ (325 points in the realization), respectively, are shown in Fig. 5. Functions of distributions of the two last time series do not differ from each other, which indicates scale invariance of the fluctuations.

Intervals between intersections of roughened time series and their average values can be considered as durations of the "avalanches".

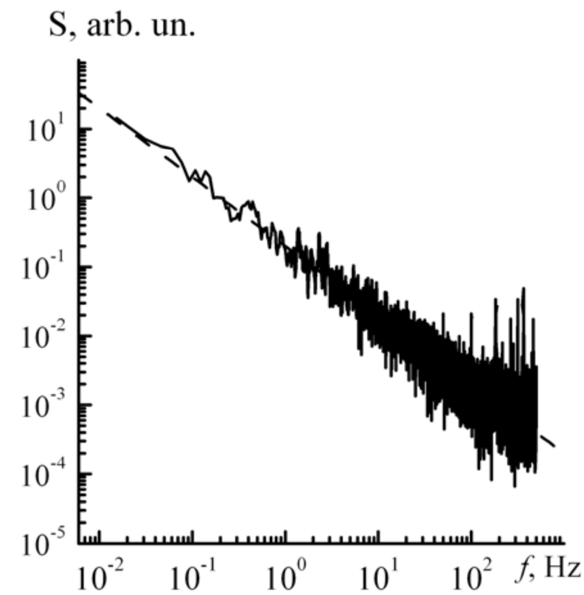

Figure.4. Power spectrum of fluctuations under acoustic cavitations of water. Dash line corresponds to the $1/f$ dependence.

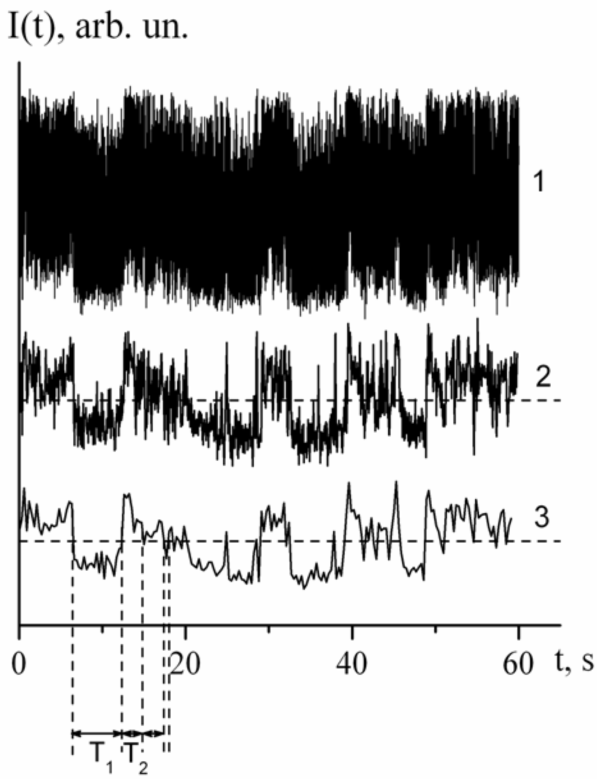

Figure 5. Initial (*1*) and roughened time series of fluctuation at water acoustic cavitations. Durations of low-frequency bursts T_i are shown on the roughened time series.

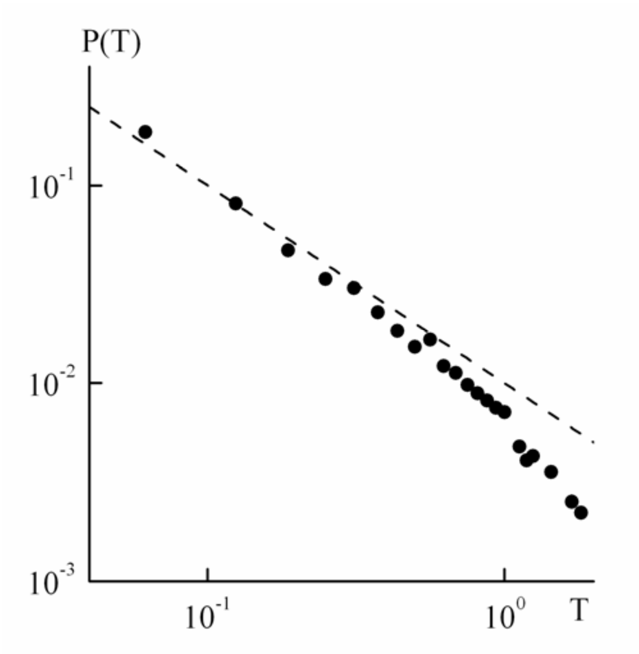

Figure 6. Distribution of duration of low-frequency bursts at water ultrasonic cavitation. The dash line—dependence $P(T) \sim 1/T$.

3.2 Explosive boiling of superheated water jets

Discharge of coming to boil water from volumes with high pressure through a short cylindrical nozzle to the atmosphere is accompanied by strong deviation from thermodynamic equilibrium and deep submersion of liquid into the region of metastable phase states [Skripov, 1974]. If intensive nucleation on heterogeneous centers (explosive boiling) is realized in a jet of overheated liquid, then the effect of complete fanning can be observed, when the jet's cone opening angle amounts to 180^0. and the jet "sticks" to the side surface of the chamber [Reshetnikov, Mazheiko, and Skripov, 2000]. The complete jet fanning, beside realization of liquid explosive boiling regime, requires a surface perpendicular to the nozzle axis behind the nozzle exit section. Figure 7 shows a fluctuations power spectrum. Experimental realization and the corresponding distribution function are shown in the insets.

The distribution function in the region of small fluctuation values is close to Gaussian one, but starts to deviate from the normal distribution in the region of large fluctuation values, that corresponds to low-frequency divergent part of the power spectrum and means possibility of large low-frequency bursts in the flow. The time series roughening over a certain time scale does not change frequency dependence of the power spectra and, beginning with a certain scale, the form of the distribution function, i.e. fluctuations demonstrate scale-invariant properties, like in the case of cavitations. Furthermore, like in the case of cavitations, distribution of low-frequency bursts revealed by the roughened realizations has the power form with index $\alpha \approx 1$.

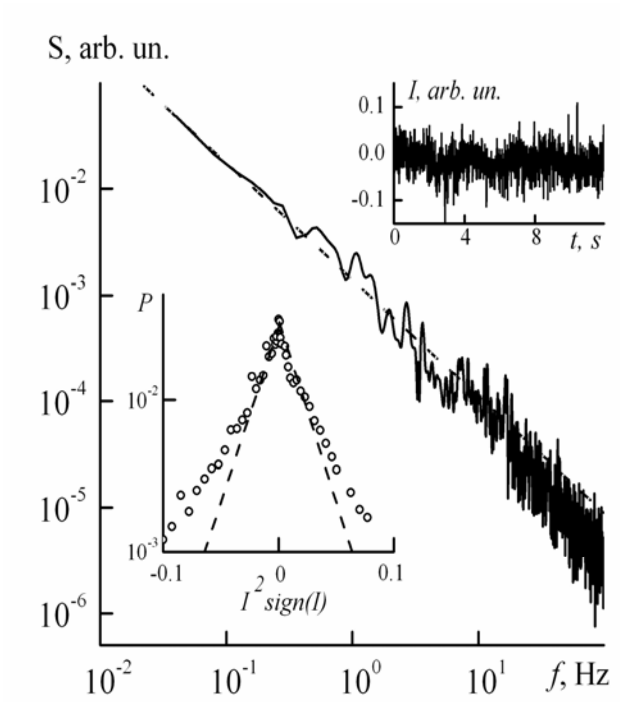

Figure 7. Power spectrum of pulsations at explosive boiling in a jet of superheated water. Experimental time series (above) and pulsations distribution function (below) are shown in the insets.

4 Conclusion

Thus, fluctuations with $1/f$ spectrum arise in a system owing to the interacting nonequilibrium phase transitions in the presence of a white noise of sufficient intensity. Relaxation processes give data on statistical patterns of the low-frequency pulsations. The results obtained on the pulsation duration and their maximal values indicate power dependence in distribution of these values.

The experiments conducted on the ultrasonic cavitations and flash boiling of superheated water show appearance of critical fluctuations with low-frequency divergence of the power spectrums, scale-invariant distribution of fluctuations and power dependences in distribution of low-frequency bursts. Results of the experiments conducted correlate with conclusions of the theoretical model for interacting heterogeneous phase transitions.

Acknowledgements

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References

1. Bak, P., Tang, C., Wiesenfeld, K. (1988) Self - organized criticality. Phys. Rev. **A 38**, pp. 364–374.
2. Carbone, V., Cavazzana, R., Antoni, V., et. al.(2002) To what extent can dynamical models describe statistical features of turbulent flows? Europhysics Lett. **58**, pp. 349–355.
3. Jensen, H.J. (1998) *Self-Organized Criticality*. New York.
4. Kertesz J. and Kiss L.B. (1990) The noise spectrum in the model of self-organized criticality. J. Phys. **A 23**, pp. L433 – L440.
5. Klimontovich, Yu.L.(1996) *Statistical Theory of Open Systems*. New York.
6. Kogan S. (1996) *Electronic Noise and Fluctuations in Solids*. Cambridge.
7. Kolmogorov A.N. (1941) The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. Doklady Akademii Nauk USSR. **30**, pp. 299 – 303.
8. Koverda V.P. and Skokov V.N. (2004) Scaling of 1/*f* noise in nonequilibrium phase transitions. Technical Physics. **49**, pp. 1104 - 1109.
9. Koverda, V.P., Skokov, V.N., and Skripov, V.P. (1998) 1/f Noise in a nonequilibrium phase transition: Experiment and mathematical model J. Exp. Theor. Phys. **86** (5), pp.953 – 958.
10. Koverda, V.P., Skokov, V.N. (2002) Noise-induced transition and 1/f fluctuations while intersecting nonequilibrium phase transitions. Doklady Physics. **47** (9), pp. 654 – 656.
11. Koverda, V.P., Skokov, V.N. (2007) Statistics of extreme low-frequency fluctuations in a stationary stochastic process exhibiting the 1/ *f* spectrum. Doklady Physics. **52** (7), pp. 354 – 358.
12. Koverda, V.P., Skokov, V.N., Reshetnikov, A.V., and Vinogradov, A.V. (2005) Fluctuations with 1/*f* Spectrum under Conditions of Acoustic Cavitation of Water. High Temperature. **43** (4), pp.631 – 636.
13. Lauterborn, W., Schmitz, E., and Judt, A. (1993) Experimental Approach to a Complex Acoustic System. Int. J. Bifurcation Chaos. **3**, pp. 635 – 642.
14. Moussatov, A., Granger, C., and Dubis, B. (2003) Cone-like bubble formation in ultrasonic cavitation field. Ultrasonics Sonochemisty. **10**, pp. 191 – 195.
15. Reshetnikov, A.V., Mazheiko, N.A., and Skripov, V.P. (2000) Jets of Incipient Liquids. Journal of Applied Mechanics and Technical Physics. 2000. **41** (3), pp. 491– 497.
16. Skokov, V.N., Koverda, A.V., Reshetnikov, A.V., et al. (2003) 1/f noise and self-organized criticality in crisis regimes of heat and mass transfer. Int. J. Heat and Mass Transfer. **46**, pp. 1879 – 1883.
17. Skokov, V.N., Koverda, V.P., Reshetnikov, A.V., and Vinogradov, A.V. (2006) 1/f fluctuations under acoustic cavitation of liquids. Physica **A 364**, pp. 63 – 69.
18. Skripov, V.P. (1974) *Metastable Liquid*. Wiley, New York.
19. Weissman, M.B. (1988) 1/*f* noise and other slow, nonexponential kinetics in condensed matter. Rev. Mod, Phys. **60** (2), pp.537 – 571.