IS THE NONLINEAR SLEWING FLEXIBLE BEAM SYSTEM INPUT-STATE LINEARIZABLE ?

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Abstract
The main purpose of this paper is to verify the controllability and the involutivity of a set of governing equations of motion representing a nonlinear dynamic system. The objective of this verification is to check whether this set of equations is input-state linearizable and to prepare the system for the application of a nonlinear control technique named feedback linearization. The system under investigation is the slewing flexible beam. The nonlinearity comes from the coupling between the beam deflection and the angular displacement of the actuator, a DC motor. It is shown that the studied system is controllable and involutive.

Key words
Controllability, involutivity, input-state linearization, nonlinear systems, nonlinear control.

1 Introduction

Feedback linearization is a control technique used for nonlinear systems. It is viewed as a generalization of pole placement for linear systems [Marino and Tomei, 1995; Sheen and Bishop, 1992]. The basic idea of this approach to nonlinear control design is to algebraically transform a more complex nonlinear system dynamics into a simpler and equivalent linear one (completelly or partly), so that well known linear control techniques can be applied [Isidori, 1995].

The existence of an output function h(x) used for feedback is essencial to solving the feedback linearization problem. The necessary and sufficient conditions for the existence of h(x) involves the rank of a controllability matrix whose columns are composed by Lie brackets of vector fields associated to the system to be controled and the concept of involutivity of a distribution which is formed by these same Lie brackets [Sheen and Bishop, 1992; Marquez, 2003] as discussed in this paper.

Feedback linearization is an approach to nonlinear control design that has attracted many researches in different fields [Singh and Yin, 1996; Joo and Seo, 1997; Sheen and Bishop, 1992, for example].

2 Geometric and Mathematical Model

The geometric model of the system investigated in this paper is presented in Figure 1. This system comprises a flexible beam-like structure in slewing motion driven by an actuator (a DC motor).

In this figure, the inertial axis is represented by XY, the moving axis (attached to the slewing axis and moving with it) is represented by xy, the beam deflection (as a space-time variable) is represented by v(x,t) and the slewing angle is represented by θ (t).

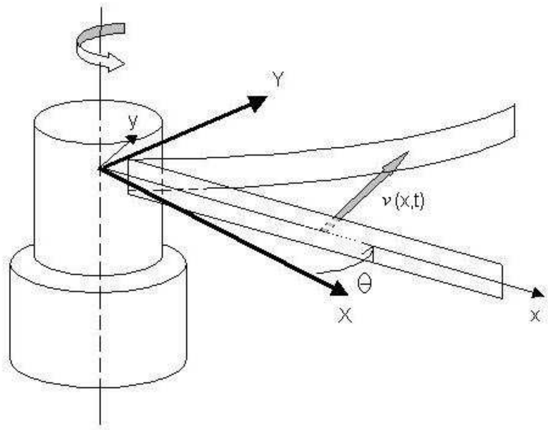

Figure 1. The slewing flexible beam system.

The mathematical model of this system is given by [Fenili, 2000]:

$$\begin{cases} \dot{i}_a + c_1 i_a + c_2\dot{\theta} = c_5\,U \\ \ddot{\theta} + c_3\dot{\theta} - c_4\, i_a = 0 \\ \ddot{q}_j + \mu\dot{q}_j + \omega_j^2 q_j + \alpha_j\ddot{\theta} - \dot{\theta}^2 q_j = 0 \end{cases} \quad (1)$$

plus the boundary conditions given by $\phi_i''(L,t) = 0$ e $\phi_i'''(L,t) = 0$, where $\phi_i(x,t)$ are the mode shapes and

$c_1 = \frac{R_a}{L_m}$, $c_2 = \frac{K_b N_g}{L_m}$, $c_3 = \frac{C_m N_g^2}{I_{shaft} + I_{motor} N_g^2}$, $c_4 = \frac{K_t N_g}{I_{shaft} + I_{motor} N_g^2}$ and $c_5 = \frac{1}{L_m}$.

In the set of governing equations given by (1), the first two equations are related to the DC motor (electric equation and mechanical equation, respectively) and the last one is related to the flexible beam-like structure.

In these equations, i_a represents the electric current in the DC motor, θ represents the angular displacement of the motor axle (also known as the slewing angle) and q_j represents the time component of each one of the vibraton modes of the flexible structure.

In the analysis developed here, the flexible structure is geometrically modeled assuming linear curvature and the Euler-Bernoulli assumptions for a slender beam are considered. Only the first flexural mode of the beam is considered. The last equation in the set (1) is a discretized equation.

Writing Equations (1) in state space form and considering only one mode for the beam deflection, one has:

$$\begin{cases} \dot{x}_1 = -c_1 x_1 - c_2 x_3 + c_5 U \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -c_3 x_3 + c_4 x_1 \\ \dot{x}_4 = x_5 \\ \dot{x}_5 = -\omega_1^2 x_4 - \mu x_5 + x_3^2 x_4 + \alpha_1 c_3 x_3 - \alpha_1 c_4 x_1 \end{cases} \tag{2}$$

In Equations (2), the states considered are: $x_1 = i_a$, $x_2 = \theta$, $x_3 = \dot{\theta}$, $x_4 = q_1$ and $x_5 = \dot{q}_1$.

3 The vector fields f and g

In order to check whether the proposed nonlinear control technique named feedback linearization can be applied to the nonlinear slewing flexible beam system or not, the set of governing equations of motion in state space form as given by Equations (2) must be writen in the form [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$\dot{\mathbf{x}} = \mathbf{f}(x) + \mathbf{g}(x)u \tag{3}$$

Writing Equation (2) in this form results:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{Bmatrix} = \begin{Bmatrix} -c_1 x_1 - c_2 x_3 \\ x_3 \\ -c_3 x_3 + c_4 x_1 \\ x_5 \\ -\omega_1^2 x_4 - \mu x_5 + x_3^2 x_4 + \alpha_1 c_3 x_3 - \alpha_1 c_4 x_1 \end{Bmatrix} + \begin{Bmatrix} c_5 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} U \tag{4}$$

and the vector fields **f** and **g** are given by:

$$\mathbf{f}(x) = \begin{Bmatrix} \mathbf{f}_1(x) \\ \mathbf{f}_2(x) \\ \mathbf{f}_3(x) \\ \mathbf{f}_4(x) \\ \mathbf{f}_5(x) \end{Bmatrix} \tag{5}$$

where:

$$\mathbf{f}_1(x) = -c_1 x_1 - c_2 x_3$$
$$\mathbf{f}_2(x) = x_3$$
$$\mathbf{f}_3(x) = -c_3 x_3 + c_4 x_1$$
$$\mathbf{f}_4(x) = x_5$$
$$\mathbf{f}_5(x) = -\omega_1^2 x_4 - \mu x_5 + x_3^2 x_4 + \alpha_1 c_3 x_3 - \alpha_1 c_4 x_1$$

and

$$\mathbf{g}(\mathbf{x}) = \begin{Bmatrix} c_5 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \\ C_{51} \end{Bmatrix} \tag{6}$$

4 Defining the Lie Brackets

The next step is to build the vetor fields $\mathbf{g}$, $ad_{\mathbf{f}}\,\mathbf{g}$, ... , $ad^{n-1}_{\mathbf{f}}\,\mathbf{g}$ for the system of Equations (4). The notation $ad_{\mathbf{f}}\,\mathbf{g}$ represents the Lie bracket of the vector fields f and g and defines a third vector as given by [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$ad_{\mathbf{f}}\,\mathbf{g} = [\mathbf{f}, \mathbf{g}] = \nabla\mathbf{g}\ \mathbf{f} - \nabla\mathbf{f}\ \mathbf{g} \tag{7}$$

For the case investigated here one has five states. Using the definition given by Equation (7) and the vectors f and g given by Equations (5) and (6) one has:

$$ad_{\mathbf{f}}\,\mathbf{g} = \begin{Bmatrix} c_1 c_5 \\ 0 \\ -c_4 c_5 \\ 0 \\ \alpha_1 c_4 c_5 \end{Bmatrix} = \begin{Bmatrix} C_{12} \\ C_{22} \\ C_{32} \\ C_{42} \\ C_{52} \end{Bmatrix} \tag{8}$$

In the same way, one can show that:

$$ad^2_{\mathbf{f}}\,\mathbf{g} = [\mathbf{f}, ad_{\mathbf{f}}\,\mathbf{g}] = \nabla ad_{\mathbf{f}}\,\mathbf{g}\ \mathbf{f} - \nabla\mathbf{f}\ ad_{\mathbf{f}}\,\mathbf{g}$$

or:

$$ad^2{}_{\mathbf{f}}\mathbf{g} = \begin{Bmatrix} c_6 \\ c_4c_5 \\ -c_7 \\ -\alpha_1c_4c_5 \\ 2x_3x_4c_4c_5 + c_8 \end{Bmatrix} = \begin{Bmatrix} C_{13} \\ C_{23} \\ C_{33} \\ C_{43} \\ C_{53} \end{Bmatrix} \qquad (9)$$

where:

$$c_6 = c_1^2c_5 - c_2c_4c_5$$
$$c_7 = c_1c_4c_5 + c_3c_4c_5$$
$$c_8 = \alpha_1c_4c_5(\mu + c_1 + c_3)$$

and

$$ad^3{}_{\mathbf{f}}\mathbf{g} = [\mathbf{f}, ad^2{}_{\mathbf{f}}\mathbf{g}] = \nabla ad^2{}_{\mathbf{f}}\mathbf{g}\,\mathbf{f} - \nabla\mathbf{f}\; ad^2{}_{\mathbf{f}}\mathbf{g}$$

or:

$$ad^3{}_{\mathbf{f}}\mathbf{g} = \begin{Bmatrix} c_9 \\ c_7 \\ -c_{10} \\ -2x_3x_4c_4c_5 - c_8 \\ 2x_3x_4c_{11} + 2x_1x_4c_4^2c_5 + 2x_3x_5c_4c_5 + \alpha_1x_3^2c_4c_5 + c_{12} \end{Bmatrix} = \begin{Bmatrix} C_{14} \\ C_{24} \\ C_{34} \\ C_{44} \\ C_{54} \end{Bmatrix} \qquad (10)$$

where:

$$c_9 = c_1c_6 - c_2c_7$$
$$c_{10} = c_4c_6 - c_3c_7$$
$$c_{11} = c_7 + c_4c_5\mu - c_3c_4c_5$$
$$c_{12} = \alpha_1c_{10} - \alpha_1\omega_1^2c_4c_5 + c_8\mu$$

and, finally:

$$ad^4{}_{\mathbf{f}}\mathbf{g} = [\mathbf{f}, ad^3{}_{\mathbf{f}}\mathbf{g}] = \nabla ad^3{}_{\mathbf{f}}\mathbf{g}\,\mathbf{f} - \nabla\mathbf{f}\; ad^3{}_{\mathbf{f}}\mathbf{g}$$

or:

$$ad^4{}_{\mathbf{f}}\mathbf{g} = \begin{Bmatrix} c_{23} \\ c_{10} \\ -c_{24} \\ 2x_3x_4c_{22} - 4x_1x_4c_4^2c_5 - 4x_3x_5c_4c_5 - \alpha_1x_3^2c_4c_5 - c_{12} \\ 2x_1x_4c_{19} - 2x_3x_4c_{20} + 2x_3x_5c_{21} + 4x_1x_5c_4^2c_5 + 4x_3^3x_4c_4c_5 + x_3^2c_{17} - c_{18} \end{Bmatrix} = \begin{Bmatrix} C_{15} \\ C_{25} \\ C_{35} \\ C_{45} \\ C_{55} \end{Bmatrix} \qquad (11)$$

where:

$$c_{13} = c_4c_{11} - c_1c_4^2c_5$$
$$c_{14} = c_3c_{11} + c_2c_4^2c_5 + c_4c_5\omega_1^2$$
$$c_{15} = c_{11} - c_3c_4c_5 - c_4c_5\mu$$
$$c_{16} = c_{10} + c_{11}\mu$$
$$c_{17} = c_8 + \alpha_1c_4c_5\mu$$
$$c_{18} = c_8\omega_1^2 - c_{12}\mu - \alpha_1c_4c_9 - \alpha_1c_3c_{10}$$
$$c_{19} = c_{13} + c_4^2c_5\mu$$
$$c_{20} = c_{14} + c_4c_5\omega_1^2 - c_{16}$$
$$c_{21} = c_{15} + c_4c_5\mu$$
$$c_{22} = c_3c_4c_5 - c_{11}$$
$$c_{23} = c_1c_9 - c_2c_{10}$$
$$c_{24} = c_4c_9 + c_3c_{10}$$

4 The controllability matrix

The controllability matrix for nonlinear systems is given by [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$C = [\,\mathbf{g}\;\; ad_{\mathbf{f}}\mathbf{g}\;\; ad^2{}_{\mathbf{f}}\mathbf{g}\;\; ad^3{}_{\mathbf{f}}\mathbf{g}\;\; ad^4{}_{\mathbf{f}}\mathbf{g}\,]$$

All the columns of the controllability matrix C are the Lie brackets given in Section 3.

Substituting each one of these vectors in matrix C one obtains:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \qquad (12)$$

Since for the system analysed in this work the elements of matrix C given by C_{21}, C_{31}, C_{41}, C_{51}, C_{22} and C_{42} are equal to zero, the matrix given in (12) can be rewriten as:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ 0 & 0 & C_{23} & C_{24} & C_{25} \\ 0 & C_{32} & C_{33} & C_{34} & C_{35} \\ 0 & 0 & C_{43} & C_{44} & C_{45} \\ 0 & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \qquad (13)$$

One can note by checking the elements of matrix C (in (12) or (13)) that the controllability matrix does not depend on the state variable x_5 (which is associated to $\dot{q}$, the velocity of deflection of the beam).

It is also important to note that, in this same matrix, only the elements C_{44}, C_{45}, C_{53}, C_{54} and C_{55} are functions of the system states (all the other elements are constants).

5 The rank of the controllability matrix

The analytical verification of the rank of matrix (12) (or (13)) is not an easy task because of the complexity (and the length) of the expression found for the determinant of this matrix.

For this reason, the controllability of the nonlinear system studied here is verified in the neighborhood of a specific set of states. Let this set of states be given by:

$$x_1 = x_{10}$$
$$x_2 = x_{20}$$
$$x_3 = x_{30}$$
$$x_4 = 0$$
$$x_5 = 0$$

This set means that one is interested in analysing the controllability of the system in the situation in which the beam deflection and velocity of deflection in near zero but all the other states are anyone.

Substituting $x_4 = x_5 = 0$ in the controllability matrix given in (13) results:

$$C_0 = \begin{bmatrix} C_{01} & C_{02} & C_{03} & C_{04} & C_{05} \\ 0 & 0 & C_{06} & C_{07} & C_{08} \\ 0 & C_{09} & C_{010} & C_{011} & C_{012} \\ 0 & 0 & C_{013} & C_{014} & C_{015} \\ 0 & C_{016} & C_{017} & C_{018} & C_{019} \end{bmatrix} \quad (14)$$

where

$$C_{01} = c_5$$
$$C_{02} = c_1 c_5$$
$$C_{03} = c_6$$
$$C_{04} = c_9$$
$$C_{05} = c_{23}$$
$$C_{06} = c_4 c_5$$
$$C_{07} = c_7$$
$$C_{08} = c_{10}$$
$$C_{09} = -c_4 c_5$$
$$C_{010} = -c_7$$
$$C_{011} = -c_{10}$$
$$C_{012} = -c_{24}$$
$$C_{013} = -\alpha_1 c_4 c_5$$
$$C_{014} = -c_8$$
$$C_{015} = -\alpha_1 c_4 c_5 x_3^2 - c_{12}$$
$$C_{016} = \alpha_1 c_4 c_5$$
$$C_{017} = c_8$$
$$C_{018} = \alpha_1 c_4 c_5 x_3^2 + c_{12}$$
$$C_{019} = c_{17} x_3^2 - c_{18}$$

and whose determinant depends only on the state variable x_3 and is given by:

$$\begin{aligned} \mathrm{Det}(C_0) = -C_{01} \big[& C_{09} (C_{06} C_{014} C_{019} + \\ & C_{013} C_{018} C_{08} + C_{017} C_{07} C_{015} - \\ & C_{08} C_{014} C_{017} - C_{015} C_{018} C_{06} - \\ & C_{07} C_{013} C_{019}) + \\ & C_{016} (C_{06} C_{011} C_{015} + \\ & C_{010} C_{014} C_{08} + C_{07} C_{012} C_{013} - \\ & C_{08} C_{011} C_{013} - C_{015} C_{07} C_{010} - \\ & C_{06} C_{012} C_{014}) \big] \end{aligned} \quad (15)$$

which is different to zero. This result shows that the rank of the controllability matrix is 5 and, therefore, complete in the considered region.

6 The involutivity condition

Another condition to be satisfied in order to the slewing flexible beam system to be input-state linearizable is that the distribution:

$$\Delta = \mathrm{span}\left\{ \mathbf{g}, ad_{\mathbf{f}} \mathbf{g}, \ldots, ad_{\mathbf{f}}^{n-2} \mathbf{g} \right\} \quad (16)$$

be involutive near some equilibrium state [Slotine e Li, 1991]. This condition is a result of the Frobenius Theorem and guarantee the existence of a diffeomorphic transformation [Isidori, 1995]. The existence of a diffeomorphic transformation implies the existence of a 1-to-1 mapping from a nonlinear vector field to a linear vector field and vice-versa. In other words, if a Lie bracket is formed by two vectors (from a determined set, as the distribution presented in (16), for example) the vector field resulting from this operation can be expressed as a linear combination of the original set of vector fields [Isidori, 1995].

In this work, since there are five states, one must verify the involutivity of:

$$\Delta = \mathrm{span}\left\{ \mathbf{g}, ad_{\mathbf{f}} \mathbf{g}, ad_{\mathbf{f}}^2 \mathbf{g}, ad_{\mathbf{f}}^3 \mathbf{g} \right\} \quad (17)$$

In order to check the involutivity of the distribution given in (17), the following two steps must be followed [Isidori, 1995]:

Step A. The following Lie brackets must be determinated:

A.1. [$\mathbf{g}$, $ad_{\mathbf{f}}\mathbf{g}$]
A.2. [$\mathbf{g}$, $ad^2_{\mathbf{f}}\mathbf{g}$]

A.3. [**g**, ad $^3{}_{\mathbf{f}}$ **g**]
A.4. [ad $_{\mathbf{f}}$ **g**, ad $^2{}_{\mathbf{f}}$ **g**]
A.5. [ad $_{\mathbf{f}}$ **g**, ad $^3{}_{\mathbf{f}}$ **g**]
A.6. [ad $^2{}_{\mathbf{f}}$ **g**, ad $^3{}_{\mathbf{f}}$ **g**]

Step B. The existence of a_i and b_i must be proved such that:

B.1. $a_1\mathbf{g} + b_1 \text{ ad}_{\mathbf{f}}\mathbf{g} = [\mathbf{g}, \text{ad}_{\mathbf{f}}\mathbf{g}]$
B.2. $a_2\mathbf{g} + b_2 \text{ ad}^2_{\mathbf{f}}\mathbf{g} = [\mathbf{g}, \text{ad}^2_{\mathbf{f}}\mathbf{g}]$
B.3. $a_3\mathbf{g} + b_3 \text{ ad}^3_{\mathbf{f}}\mathbf{g} = [\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}]$
B.4. $a_4 \text{ ad}_{\mathbf{f}}\mathbf{g} + b_4 \text{ ad}^2_{\mathbf{f}}\mathbf{g} = [\text{ad}_{\mathbf{f}}\mathbf{g}, \text{ad}^2_{\mathbf{f}}\mathbf{g}]$
B.5. $a_5 \text{ ad}_{\mathbf{f}}\mathbf{g} + b_5 \text{ ad}^3_{\mathbf{f}}\mathbf{g} = [\text{ad}_{\mathbf{f}}\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}]$
B.6. $a_6 \text{ ad}^2_{\mathbf{f}}\mathbf{g} + b_6 \text{ ad}^3_{\mathbf{f}}\mathbf{g} = [\text{ad}^2_{\mathbf{f}}\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}]$

If any of these conditions do not exist, the system of governing equations under investigation is not involutive and, therefore, it is also not input-state linearizable [Slotine e Li, 1991].

6.1 Determination of the Lie brackets (given in step A)

The Lie brackets given i n Step A are:

$$[\mathbf{g}, \text{ad}_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}_{\mathbf{f}}\mathbf{g})\,\mathbf{g} - (\nabla \mathbf{g})\,\text{ad}_{\mathbf{f}}\mathbf{g} = \{0 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathbf{T}} \tag{18}$$

$$[\mathbf{g}, \text{ad}^2_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}^2_{\mathbf{f}}\mathbf{g})\,\mathbf{g} - (\nabla \mathbf{g})\,\text{ad}^2_{\mathbf{f}}\mathbf{g} = \{0 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathbf{T}} \tag{19}$$

$$[\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}^3_{\mathbf{f}}\mathbf{g})\,\mathbf{g} - (\nabla \mathbf{g})\,\text{ad}^3_{\mathbf{f}}\mathbf{g} = \{0 \quad 0 \quad 0 \quad 0 \quad 2x_4c_4^2c_5^2\}^{\mathbf{T}} \tag{20}$$

$$[\text{ad}_{\mathbf{f}}\mathbf{g}, \text{ad}^2_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}^2_{\mathbf{f}}\mathbf{g})\,\text{ad}_{\mathbf{f}}\mathbf{g} - (\nabla \text{ad}_{\mathbf{f}}\mathbf{g})\,\text{ad}^2_{\mathbf{f}}\mathbf{g} = \{0 \quad 0 \quad 0 \quad 0 \quad -2x_4c_4^2c_5^2\}^{\mathbf{T}} \tag{21}$$

$$[\text{ad}_{\mathbf{f}}\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}^3_{\mathbf{f}}\mathbf{g})\,\text{ad}_{\mathbf{f}}\mathbf{g} - (\nabla \text{ad}_{\mathbf{f}}\mathbf{g})\,\text{ad}^3_{\mathbf{f}}\mathbf{g}$$

$$= \left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ 2x_4c_4^2c_5^2 \\ 2x_4c_1c_4^2c_5^2 - 2(x_4c_4c_5c_{11} + x_5c_4^2c_5^2) + 4\alpha_1x_3c_4^2c_5^2 \end{array}\right\} \tag{22}$$

$$[\text{ad}^2_{\mathbf{f}}\mathbf{g}, \text{ad}^3_{\mathbf{f}}\mathbf{g}] = (\nabla \text{ad}^3_{\mathbf{f}}\mathbf{g})\,\text{ad}^2_{\mathbf{f}}\mathbf{g} - (\nabla \text{ad}^2_{\mathbf{f}}\mathbf{g})\,\text{ad}^3_{\mathbf{f}}\mathbf{g}$$

$$= \left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ 2(x_4c_4c_5c_7 + \alpha_1x_3c_4^2c_5^2) \\ 2x_4c_4^2c_5c_6 - 2c_7(x_4c_{11} + x_5c_4c_5 + \alpha_1x_3c_4c_5) - 2\alpha_1c_4c_5(x_3c_{11} + x_1c_4^2c_5) + \\ 2x_4c_4c_5c_{10} + 8x_3^2x_4c_4^2c_5^2 - 2x_3c_4c_5c_8 \end{array}\right\} \tag{23}$$

6.2 Verifying the existence of a_i and b_i (in Step B)

The idea now, in order to prove the involutivity of the distribution (16), is to prove that the coefficients a_i and b_i in B.1 to B.6 do exist.

The Equation B.1 can be written as:

$$\begin{bmatrix} c_5 & c_1c_5 \\ 0 & 0 \\ 0 & -c_4c_5 \\ 0 & 0 \\ 0 & \alpha_1c_4c_5 \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{24}$$

Solving the system of equations with two unknows given by (24) using the least squares method one obtains:

$$\begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{25}$$

and the condition for involutivity is satisfied for Equation B.1.

The vector fields **g** and ad $_{\mathbf{f}}$ **g** are constant. The Lie bracket of two constant vectors is simply the vector zero [Slotine e Li, 1991], as concluded above. Therefore, it can be trivially expressed as a linear combination of these vector fields.

Following the same steps, the solution of B.2 is given by:

$$\begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{26}$$

and the condition for involutivity is satisfied for Equation B.2.

The solution of B.3 is given by:

$$\begin{Bmatrix} a_3 \\ b_3 \end{Bmatrix} = \begin{Bmatrix} \dfrac{a_{3a}}{a_{3b}} \\ \dfrac{a_{3c}}{a_{3b}} \end{Bmatrix} \tag{27}$$

where:

$$a_{3a} = -2c_9c_4^2c_5^2x_4d_1$$

$$a_{3b} = c_5^2\,(c_{10}^2 + c_7^2 + c_9^2 + (-c_8 - 2c_4c_5x_3x_4)^2 + d_1^2) - c_5^2c_9^2$$

$$a_{3c} = 2c_4^2 c_5^2 x_4 d_1$$
$$d_1 = c_{12} + 2c_4^2 c_5 x_1 x_4 + 2c_{11} x_3 x_4 + 2c_4 c_5 x_3 x_5 + \alpha_1 c_4 c_5 x_3^2$$

and the condition for involutivity is satisfied for Equation B.3.

The solution of B.4 is given by:

$$\begin{Bmatrix} a_4 \\ b_4 \end{Bmatrix} = \begin{Bmatrix} \dfrac{a_{4a}}{a_{4b}} \\ \dfrac{a_{4c}}{a_{4b}} \end{Bmatrix} \qquad (28)$$

where:

$$a_{4a} = -2c_4^2 c_5 x_4 (\alpha_1^3 c_4^3 c_5^2 + \alpha_1 c_4 (c_6^2 + c_7^2 + c_4^2 c_5^2) - d_4 (c_8 + 2c_4 c_5 x_3 x_4))$$
$$a_{4b} = c_4^2 d_5 + d_3 + c_1^2 (c_8^2 + c_7^2) + 4c_4 c_5 x_3 x_4 d_6 + 4c_4^2 c_5^2 x_3^2 x_4^2 (c_1^2 + c_4^2)$$
$$a_{4c} = -2c_4^2 c_5^2 x_4 (-\alpha_1 c_1 c_4 c_6 - \alpha_1 c_4^2 c_7 + (c_8 + 2c_4 c_5 x_3 x_4)(c_1^2 + c_4^2))$$
$$d_2 = c_1^2 + \alpha_1^2 c_1^2 + c_4^2 + 2\alpha_1^2 c_4^2 + \alpha_1^4 c_4^2$$
$$d_3 = -2c_4 (c_1 c_6 c_7 + \alpha_1 c_1 c_6 c_8 + \alpha_1 c_4 c_7 c_8)$$
$$d_4 = c_1 c_6 + c_4 c_7$$
$$d_5 = c_5^2 d_2 + c_4^2 c_6^2 + \alpha_1^2 c_6^2 + \alpha_1^2 c_7^2 + c_8^2$$
$$d_6 = c_1^2 c_8 - \alpha_1 c_4 d_4 + c_4^2 c_8$$

and the condition for involutivity is satisfied for Equation B.4.

The solution of B.5 is given by:

$$\begin{Bmatrix} a_5 \\ b_5 \end{Bmatrix} = \begin{Bmatrix} \dfrac{a_{5a}}{a_{5b}} \\ \dfrac{a_{5c}}{a_{5d}} \end{Bmatrix} \qquad (29)$$

where:

$$a_{5a} = \alpha_1 c_4 d_{10} d_{11} - (d_8 d_{10} + d_7 d_9)(c_4 c_{10} + c_1 c_9) - \alpha_1 c_4 d_7 d_8 d_9$$
$$a_{5b} = c_5 (d_{12} - 2c_1 c_4 c_9 c_{10} + d_7^2 (c_1^2 + c_4^2 + \alpha_1^2 c_4^2) - d_{13} + d_{14} + c_1^2 d_8^2)$$
$$a_{5c} = -\alpha_1 c_4^2 c_{10} d_{10} - \alpha_1 c_1 c_4 c_9 c_{10} + (d_8 d_{10} + d_7 d_9)(c_1^2 + c_4^2) + \alpha_1^2 c_4^2 d_7 d_9$$
$$a_{5d} = d_{12} - 2c_1 c_4 c_9 c_{10} + d_7^2 (c_1^2 + c_4^2 + \alpha_1^2 c_4^2) - d_{13} + d_{14} + c_1^2 d_8^2$$
$$d_7 = -2x_3 x_4 c_4 c_5 - c_8$$
$$d_8 = 2(x_3 x_4 c_{11} + x_1 x_4 c_4^2 c_5 + x_3 x_5 c_4 c_5) + \alpha_1 x_3^2 c_4 c_5 + c_{12}$$
$$d_9 = 2x_4 c_4^2 c_5^2$$
$$d_{10} = 2x_4 c_1 c_4^2 c_5^2 - 2(x_4 c_4 c_5 c_{11} + x_5 c_4^2 c_5^2) + 4\alpha_1 x_3 c_4^2 c_5^2$$
$$d_{11} = c_7^2 + c_9^2 + c_{10}^2 + d_7^2$$
$$d_{12} = (c_7^2 + c_{10}^2)(c_1^2 + \alpha_1^2 c_4^2)$$
$$d_{13} = 2\alpha_1 c_4 d_8 (c_4 c_{10} + c_1 c_9)$$
$$d_{14} = c_4^2 (c_7^2 + c_9^2 + \alpha_1^2 c_9^2 + d_8^2)$$

and the condition for involutivity is satisfied for Equation B.5.

Finally, the solution of B.6 is given by:

$$\begin{Bmatrix} a_6 \\ b_6 \end{Bmatrix} = \begin{Bmatrix} \dfrac{a_{6a}}{a_{6b}} \\ \dfrac{a_{6c}}{a_{6b}} \end{Bmatrix} \qquad (30)$$

where:

$$a_{6a} = -\alpha_1 c_4 c_5 d_{17} d_{19} + d_{15} d_{17} d_{20} + d_{15} d_{18} d_{21} - d_{16} d_{18} d_{22}$$
$$a_{6b} = c_{10}^2 d_{25} + c_7^2 d_{26} + d_{16}^2 d_{24} + d_{15}^2 d_{27} - 2\alpha_1 c_4 c_5 d_{15} d_{20} - 2d_{15} d_{16} d_{28} + d_{29}$$
$$a_{6c} = \alpha_1 c_4 c_5 d_{17} d_{20} - d_{15} d_{17} d_{23} - d_{15} d_{18} d_{22} + d_{16} d_{18} d_{24}$$
$$d_{15} = 2x_3 x_4 c_4 c_5 + c_8$$
$$d_{16} = 2(x_3 x_4 c_{11} + x_1 x_4 c_4^2 c_5 + x_3 x_5 c_4 c_5) + \alpha_1 x_3^2 c_4 c_5 + c_{12}$$
$$d_{17} = 2(x_4 c_4 c_5 c_7 + \alpha_1 x_3 c_4^2 c_5^2)$$
$$d_{18} = 2x_4 c_4^2 c_5 c_6 - 2c_7 (x_4 c_{11} + x_5 c_4 c_5 + \alpha_1 x_3 c_4 c_5) - 2\alpha_1 c_4 c_5 (x_3 c_{11} + x_1 c_4^2 c_5) + 2x_4 c_4 c_5 c_{10} + 8x_3^2 x_4 c_4^2 c_5^2 - 2x_3 c_4 c_5 c_8$$
$$d_{19} = c_7^2 + c_9^2 + c_{10}^2 + d_{16}^2$$
$$d_{20} = c_7 c_{10} + c_4 c_5 c_7 + c_6 c_9 + d_{15} d_{16}$$
$$d_{21} = c_7^2 + c_9^2 + c_{10}^2 + d_{15}^2$$
$$d_{22} = c_7 c_{10} + c_4 c_5 c_7 + c_6 c_9 + \alpha_1 c_4 c_5 d_{15}$$
$$d_{23} = c_4^2 c_5^2 + c_6^2 + c_7^2 + d_{15}^2$$
$$d_{24} = c_4^2 c_5^2 + c_6^2 + c_7^2 + \alpha_1^2 c_4^2 c_5^2$$
$$d_{25} = c_4^2 c_5^2 + c_6^2 + \alpha_1^2 c_4^2 c_5^2 + d_{15}^2$$
$$d_{26} = -2c_{10} c_4 c_5 + \alpha_1^2 c_4^2 c_5^2 + c_6^2 + c_7^2 + c_9^2$$
$$d_{27} = c_4^2 c_5^2 + c_6^2 + 2c_7^2 + c_9^2 + d_{15}^2$$
$$d_{28} = c_7 c_{10} + c_4 c_5 c_7 + c_6 c_9$$
$$d_{29} = -2c_6 c_7 c_9 (c_{10} + c_4 c_5) + c_4^2 c_5^2 c_9^2 (1 + \alpha_1^2)$$

and the condition for involutivity is satisfied for Equation B.6.

Equations (25) to (30) prove that all the conditions given in Step B are satisfied. Therefore, the set of vector fields given by the distribution (17) is involutive.

7 Conclusion

It is well known that it is not possible to apply any nonlinear control technique in any nonlinear dynamic system.

The possibility of applying a specific nonlinear control technique named feedback linearization in a specific dynamic system composed by a flexible beam-like structure in slewing motion is investigated in this work. This dynamic system is representative of a lightweight robotic manipulator used in industry or space applications, for instance.

Feedback linearization is a technique to transform original system models into equivalent models of a simpler form. The central idea is to transform nonlinear systems dynamics into fully or partly linear ones.

The necessary and sufficient conditions to be investigated in order to verify the applicability of this control technique are controllability and involutivity in a region in the neighborhood of an equilibrium solution. Feedback linearization uses mathematical tools from differential geometry, as the concept of Lie derivatives.

According to the results presented here, the necessary and sufficient conditions to apply this technique are completely satisfied for the system investigated.

The next step in this research is to apply this technique and control the studied system.

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