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PHYTOPLANKTON-ZOOPLANKTON SYSTEM WITH BOUNDED RANDOM PARAMETER*

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Abstract

The aim of this work is to use the Chebyshev orthogonal polynomial approximation method to analyze the stochastic nonlinear dynamical systems with bounded random parameter. Consider a prey – predator model describing the time evolution of phytoplankton-zooplankton populations in an open environment. This stochastic phytoplankton zooplankton system (SPZ system for short) is transformed by the Chebyshev orthogonal polynomial approximation method into an equivalent deterministic system, whose responses can be readily obtained by conventional numerical methods. In this way we can explore plenty of stochastic dynamical phenomenons of the SPZ system by some deterministic nonlinear dynamical methods.

Key words

phytoplankton-zooplankton, Chebyshev polynomial approximation, stochastic system, bounded random parameter

1 Introduction

Among famous models of the same kind related to three trophic ecological system in a chemostat, let us quote those of Rosenzweig-MacArthur [Rosenzweig and MacArthur, 1963], Hastings-Powell [Hastings and Powell, 1991] and Volterra- Gause [Ginoux, Rossetto and Jamet, 2005]. The last paper [Ginoux, Rossetto and Jamet, 2005] shows similarities between these three models: they have the same number of equilibrium points and attractional sinks, and one slow invariant manifold. The shape of their chaotic attractor is a Moebius strip for the Rosenzweig-MacArthur model, a teacup for Hastings-Powell and a snail-shell for Volterra-Gause.

In this paper, we study the time evolution of phytoplankton and zooplankton populations in an open environment. However, in an open environment, the influence of external physical parameters, like the annual variations of temperature, photoperiod and nutrition, must be taken in account. This is performed by using the phytoplankton-zooplankton populations system (PZ system for short) below

$$\begin{cases} \dot{p}(t) = \alpha(t)\,p(t) - \beta_1 p(t)z(t) + n(t) \\ \dot{z}(t) = \beta_2\, p(t)z(t) + \gamma z(t) \end{cases} \tag{1}$$

where $p(t)$ and $z(t)$ is phytoplankton and zooplankton; the variable coefficient $\alpha(t)$ is a periodic mapping $t \mapsto \alpha(t)$; $n(t)$ is the time evolution of nutrition; β_1, β_2 and γ are all parameters.

The system with random parameters which is usually called a stochastic system. The modifier 'stochastic' here implies dependent on some random parameter. There are several mathematical methods available for the stochastic structure analysis: one is the Monte-Carlo method, which is simple and universal, but usually involved with a quite amount of computational effort. Another method is the stochastic perturbation method, which is involved with the least computational effort, but usually restricted to system with random variables of small fluctuations only. The third one is the orthogonal polynomial approximation method, which was introduced in [Spanos and Ghanem, 1989] and [Jensen and Iwan, 1992], and improved by Li [Li, 1996]. This method does not require the small random perturbation assumption, thus providing more applicability. Chebyshev polynomial approximation and Gegenbauer polynomial approximation were firstly applied to the evolutionary random response problem of a stochastic structure [Fang, Leng and Song, 2003] and [Fang, Leng, Ma, et al, 2003], and then further applied to some dynamical problems of the stochastic nonlinear dynamical systems [Zhang, Xu, Fang and Xu, 2007], [Zhang, Xu and Fang, 2007] and [Zhang, Xu, Sun and Fang, 2007]. In this paper the Chebyshev orthogonal polynomial approximation method will be used to explore the stochastic dynamical behaviors in the SPZ system here.

2 Chebyshev polynomial approximation for SPZ system

In the open environment, there must be several chemical and biological variations influencing on the PZ system (1). Therefore, we can consider the stochastic system has a random parameter and the SPZ system can be expressed as

$$\begin{cases} \dot{p}(t) = \alpha(t)p(t) - \beta_1 p(t)z(t) + n(t) \\ \dot{z}(t) = \beta_2 p(t)z(t) + (\bar{\gamma} + \sigma\xi)z(t) \end{cases} \tag{2}$$

where $\gamma = \bar{\gamma} + \sigma\xi$ is the random parameter, $\bar{\gamma}$ is the mean value of γ, ξ is a bounded random variable defined on [-1, 1] with a given arch-like probability distribution function (PDF), and σ is the intensity of ξ. Since the system itself is stochastic, so are the responses. Thus the responses of (2) should be a function of time t and the random variable ξ, namely $p = p(t,\xi)$ and $z = z(t,\xi)$. According to the orthogonal polynomial approximation, the responses of (2) can be expressed by the following series:

$$\begin{cases} p(t,\xi) = \sum_{i=0}^{N} p_i(t)H_i(\xi) \\ z(t,\xi) = \sum_{i=0}^{N} z_i(t)H_i(\xi) \end{cases} \tag{3}$$

In this paper we take $N = 4$, and then the responses (3) are approximate solutions with a minimum mean square residual [Wang and Guo, 2000].

$$\begin{cases} \dot{p}_0(t) = \alpha(t)p_0(t) - \beta A_0 + n(t) \\ \dot{z}_0(t) = \beta A_0 + \bar{\gamma}z_0(t) + \frac{1}{2}\sigma z_1(t) \\ \dot{p}_1(t) = \alpha(t)p_1(t) - \beta A_1(t) \\ \dot{z}_1(t) = \beta A_1 + \bar{\gamma}z_1(t) + \frac{1}{2}\sigma[z_0(t) + z_2(t)] \\ \dot{p}_2(t) = \alpha(t)p_2(t) - \beta A_2(t) \\ \dot{z}_2(t) = \beta A_2 + \bar{\gamma}z_2(t) + \frac{1}{2}\sigma[z_1(t) + z_3(t)] \\ \dot{p}_3(t) = \alpha(t)p_3(t) - \beta A_3(t) \\ \dot{z}_3(t) = \beta A_3 + \bar{\gamma}z_3(t) + \frac{1}{2}\sigma[z_2(t) + z_4(t)] \\ \dot{p}_4(t) = \alpha(t)p_4(t) - \beta A_4(t) \\ \dot{z}_4(t) = \beta A_4 + \bar{\gamma}z_4(t) + \frac{1}{2}\sigma z_3(t) \end{cases} \tag{4}$$

Substituting (3) into (2) and by using the Chebyshev orthogonal polynomials, the recurrent formulas and the orthogonal relationship of the Chebyshev orthogonal polynomials, we finally have equation (4), which is just the equivalent deterministic nonlinear system (DM-SPZ system for short) of SPZ system (2), which plays an important role in this paper. Since in the course of deriving DM-SPZ system, we have taken some expectation with respect to ξ, the obtained DM-SPZ system is a kind of weighted average system. So the responses of this system must reflect some averaged characteristics of the original stochastic dynamic system (2).

3 Analysis of the stochastic dynamical behaviors of the SPZ system

In order to study the stochastic dynamical behaviors of the SPZ systems comparing with the PZ system, firstly we need to choose the useful responses of the two systems.

If we sample the random parameter as $\xi = 0$, then the system is a sample of mean parameter system, of which the response may be expressed as (SR for short)

$$\begin{cases} p_{SR}(t) = p(t,0) = \sum_{i=0}^{4} p_i(t)H_i(0) = p_0(t) - p_2(t) + p_4(t) \\ z_{SR}(t) = z(t,0) = \sum_{i=0}^{4} z_i(t)H_i(0) = z_0(t) - z_2(t) + z_4(t) \end{cases} \tag{5}$$

And the ensemble mean response of the SPZ system can be obtained as (EMR for short)

$$\begin{cases} p_{EMR}(t) = E[p(t,\xi)] = \sum_{i=0}^{4} p_i(t)E[H_i(\xi)] = p_0(t) \\ z_{EMR}(t) = E[z(t,\xi)] = \sum_{i=0}^{4} z_i(t)E[H_i(\xi)] = z_0(t) \end{cases} \tag{6}$$

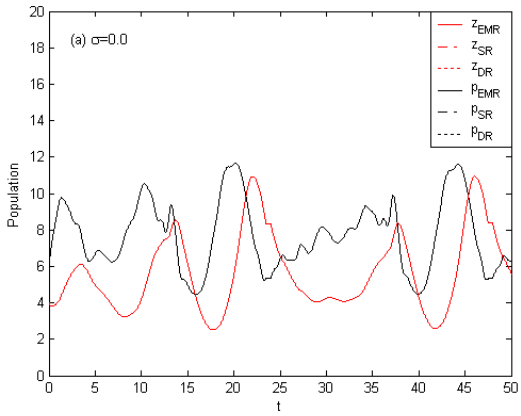
If $\sigma = 0$ or $\xi \equiv 0$, system (2) is reduced to the mean-parameter system (7) which is a deterministic system.

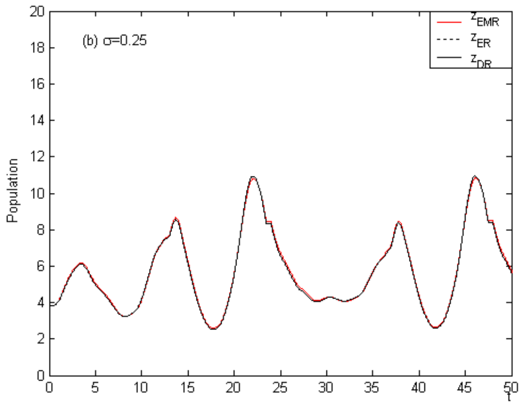
$$\begin{cases} \dot{p}(t) = \alpha(t)p - \beta_1 p(t)z(t) + n(t) \\ \dot{z}(t) = \beta_2 p(t)z(t) + \bar{\gamma}(t)z(t) \end{cases} \tag{7}$$

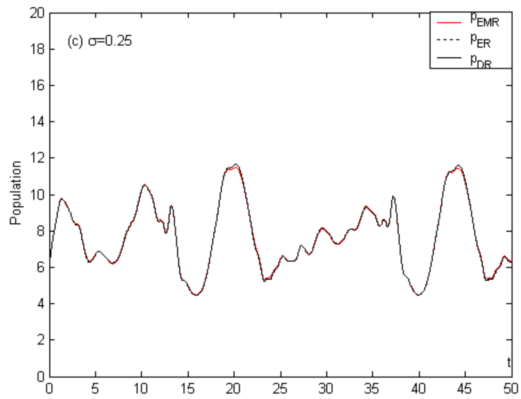
Applying the Runge-Kutta method to (4) and (7), we obtain the responses $p_i(t)$ and $z_i(t)$ ($i = 1,2,3,4$) of the equivalent system and the deterministic responses (DR for short) $p_{DR}(t)$ and $z_{DR}(t)$ of the mean parameter system respectively. Then the responses (5) and (6) can be worked out easily. We can use EMR to explore basic nonlinear phenomena in SPZ system, and verify the feasibility of Chebyshev orthogonal polynomial approximation method by comparing DR with SR, and see more different characteristics of SPZ system by comparing DR and EMR.

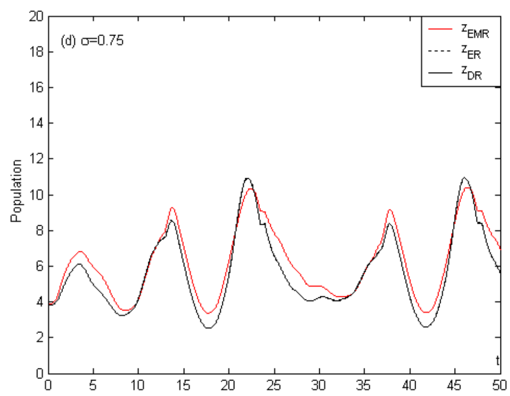
By all the analysis above, now we can investigate the stochastic phenomenon by comparing with the PZ system, the following are some results of the research.

From Fig.1 (a), we can see that without the influence of the random factor, the three kinds of responses DR, SR and EMR are coincide with each other very well. The evident coherence between the time series illuminates that the Chebyshev orthogonal polynomial approximate method works very well, and the equivalent deterministic nonlinear system preserves the similar nonlinear phenomena of the deterministic PZ system. Then choosing $\sigma = 0.25$ and









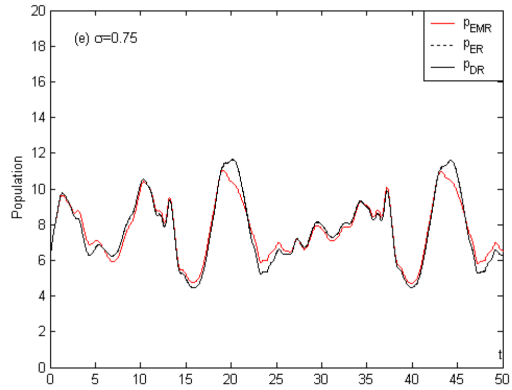
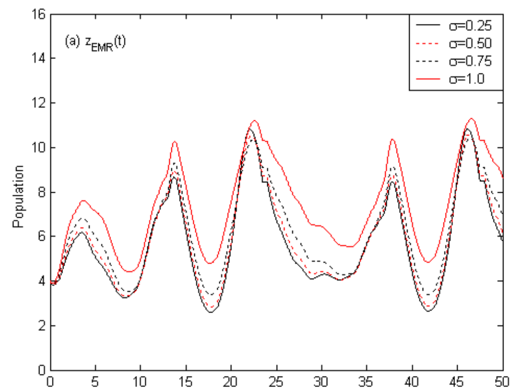


Figure 1. The time series of EMR, SR and DR of the populations of phytoplankton and zooplankton:
(a) $\sigma = 0.0$; (b) and (c) $\sigma = 0.25$; (d) and (e) $\sigma = 0.75$.



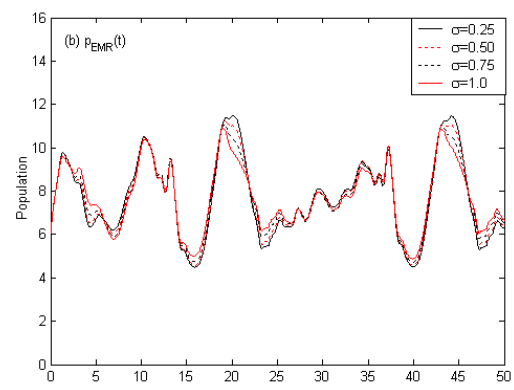


Figure 2. When $\sigma = 0.25$, $\sigma = 0.50$, $\sigma = 0.75$ and $\sigma = 1.0$, the time series of EMR of:
(a) the population of phytoplankton; (b) the population of zooplankton.

$\sigma = 0.75$ respectively, the time series are shown in Fig.1 (b)-(e). The coherence of DR and SR means that with the presence of the random variable, the Chebyshev orthogonal polynomial approximate method is still credible; with the increasing of the intensity of the random variable σ, there are some special variations of stochastic responses. In Fig.2, we can see that obviously. In the some environment, the intensity σ is stronger, the response $z_{EMR}(t)$ is larger which can be seen in Fig.2 (a); and with the changing of the intensity, we can discover the changes of $p_{EMR}(t)$ in Fig.2 (b), which has some differences from the changing of $z_{EMR}(t)$.

4 Conclusions

In this paper, we use the Chebyshev orthogonal polynomial approximation method to analyze a prey-predator model describing the time evolution of phytoplankton and zooplankton populations, which has a bounded random parameter because of some physical or chemical influences in an open environment. By the investigation, firstly, we show the dynamical behaviors of the deterministic model (PZ model) and illuminate that the Chebyshev orthogonal polynomial method is effective in this research. Secondly, we find that with the effect of the stochastic factors, the changes of the populations of phytoplankton and zooplankton follow the intensity of the random variable; and the sensitivities to the effect of the random parameter of the phytoplankton population and the zooplankton population are not the same.

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