Functionalization method in motion image analysis

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Astract**:** In the report it is developed a straight method of motion parameters calculation of an observed object based on analysis of image sequence of the object.

Key words: optical flow, velocity of moving object, object image, gradient motion estimation, functionalization method

1. INTRODUCTION

At the present time there exists a class of high performance registration image methods based on optical flow computation. Optical flow is a vector field of apparent velocity of moving object images. A brightness constancy constraint equation (BCCE) gives one of the ways to calculate optical flow [Black, M. J., *et al*], [Lucas B.D., *et al*], [Horn B.K.P., *et al*]. The BCCE equation determines a connection between time-spatial variations of image intensity and image motion parameters, BCCE appears as

$$\frac{\partial E(x,y)}{\partial t} + \upsilon_x \frac{\partial E(x,y)}{\partial x} + \upsilon_y \frac{\partial E(x,y)}{\partial y} = 0, \quad (1)$$

v_x, v_y – components of an apparent velocity vector,

$E(x,y)$ – image intensity function.

Methods based upon the equation (1) are referred to as gradient-based motion estimation methods (GM methods).

Comprehensive surveys of the GM methods are performed in [J. Barron *et al.*] and [M. Irrani. *et al.*]. The GM methods are efficient in solving a parametric image registration problem: global translation, rotation, affine and projective motions and some others.

The GM method uses spatio-temporal derivatives of image intensity functions, but these functions are non-differentiable and noncontinuous in a general case. That makes the GM method non rigorous and as a consequence of that there arise errors in results of image motion parameters calculations. Considerable efforts were undertaken to eliminate this shortcoming of the GM method [Black, M. J. *et al.*], [Horn B.K.P. *et al.*], [Lucas B.D. *et al.*], [Schalkov R.J.], [J. Barron *et al.*]. Nevertheless implementations of all the known versions of the method require preliminary image smoothing through filtration with using of convolution. In a general case convolution is a non-linear operation in respect of velocity vector components. Nonlinearity takes place, for example, when image rotates. So, smoothing yields additional errors to results of calculations. In addition, the GM method gives estimations not of real motion parameters of an observed object but of apparent image velocity vectors.

In this report we develop a straight method of motion parameters estimation of moving object via analysis of it image series. The method is called as the functionalization method [Абакумов А.М., *et al.*], [П.К.Кузнецов, (1990) *et al.*], [П.К.Кузнецов, (1994) *et al.*], [А.с. №1742729 СССР].

The functionalization method is free from the main shortcomings of the GM method as it does not uses derivatives of an image intensity function. It uses a fundamental functional connection equation (FC equation), that determines a functional dependence of measurable characteristics of an observed object image on the object motion parameters. A measurable image characteristic is one that can be calculated as a particular definite integral on any of image regions of non-nil square. Spatio-temporal derivatives of image intensity are not measurable image characteristics in this sense as well as an image intensity function itself. The functionalization method gives a generalization of the class of GM methods.

2. FUNCTIONALIZATION METHOD

The functionalization method differs from GM methods in that it operates not upon an image intensity functions but upon a special functionals, determined on a set of image intensity functions. That brings considerably new features of universality and accuracy to a technique of image motion parameters estimation. I this section we develop the method for a case, when an observed object is a flat underlying surface and a platform, carrying an imaging device or a video camera, executes, a motion with six degrees of freedom relative to an underlying surface.

2.1 The model of moving underlying surface imaging

We assume that the carrying platform (CP) executes a translation and free rotation motion relative to it's own center of mass, and that an underlying surface is flat, rigid, possess plane albedo and is uniformly illuminated with an off-site light source, distance between CP and underlying surface considerably more then focal length of an imaging device in use, as well as an optical system (OS) of the imaging device has no distortions (feature of isoplanatism takes place).

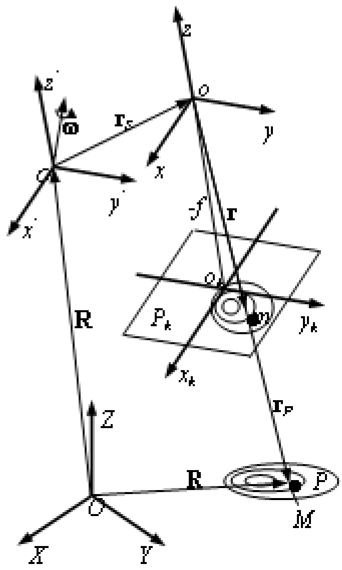


Fig. 1 Coordinate systems used for image modeling

Coordinate systems (Fig1) used for image modeling are the next

- *OXYZ* with *OX* and *OY* axes being immovable and belonging to a plane *P;*

- *P*- underlying surface;

- *o'x'y'z'* , axes of it are fixed to the platform, an origin of coordinates *o'* coincides with CP center of mass;

- *oxyz*, axes of it are fixed to the platform and parallel to similar axes of a coordinate system *o'x'y'z'*,an origin *o* of coordinates coincides with the main front focal point of OS, the main axis of OS coincides with the axis *oz*;

- $o_k x_k y_k$, axes of it are parallel to similar axes of a coordinate system *oxy*, an origin o_k belongs to the main axis of OS and is placed on an image plane (P_k) of OS at a distance of $-f$ from an origin *o* of coordinate system *oxyz* ($-f$ – a rear focal point of OS).

Position of coordinate system *oxyz* relative to cordinate system *o'x'y'z'* is identified with a vector $\mathbf{r}_S=[x_S,y_S,z_S]^T$

($[]^T$ - symbol of transposition).

Position of coordinate system *o'x'y'z'* relative to coordinate system *OXYZ* is identified with a vector $\mathbf{R}_H=\mathbf{R}_H(t)= =[X_H(t),Y_H(t),Z_H(t)]^T$ and with a transformation matrix (matrix of direction cosiness) $\mathbf{A}=\mathbf{A}(t)=[a_{i,j}(t)]$ (i, j = 1,2,3). It is well known that matrix $\mathbf{A}$ is orthogonal and $\mathbf{A}^{-1}=\mathbf{A}^T$.

Each point belonging to P maps it`s image at the plane $P_k$ (for example, on Fig. 1 a point $M$ maps it`s own image at a point m).

Position of a point $M \in P$ in a coordinate system *OXYZ* is identified on Fig.1 with a vector $\mathbf{R}=[X,Y,0]^T$, and in a mobile coordinate system *oxyz*– with a vector

$\mathbf{r}_P=\mathbf{r}_P(t)[x_P(t),y_P(t),z_P(t)]^T$.

Position of a point $m \in P_k$ on Fig.1 in a coordinate system *oxyz* is identified with a vector

$\mathbf{r}=\mathbf{r}(t)=[x(t),\ y(t),\ -f]$

From Fig.1 follows vector equality:

$$\mathbf{r} = \mathbf{r}_P = k(\mathbf{A}(\mathbf{R} - \mathbf{R}_H) - \mathbf{r}_S), \qquad (2)$$

where $k_m=k_m(\mathbf{r})=|\mathbf{r}|/|\mathbf{r}_P|= - f/(\mathbf{A}_3(\mathbf{R}-\mathbf{R}_H) - z_S)= - \mathbf{A}_3^T r/(Z_H+\mathbf{A}_3^T \mathbf{r}_S)$; $|\mathbf{r}|$ и $|\mathbf{r}_M|$ – modulus of vectors $\mathbf{r}$ and $\mathbf{r}_M$; $\mathbf{A}_3=[a_{31},a_{32},a_{33}]$ – row-matrix that is the third row of matrix $\mathbf{A}$; $\mathbf{A}_3^T=[a_{13},a_{23},a_{33}]$ – row-matrix that is the third row of matrix $\mathbf{A}^T$.

2.2 Image motion equation

To obtain image motion equation of underlying surface we derive (2) in respect of time t and then get:

$$\dot{\mathbf{r}} = -\left(\frac{1}{f}\mathbf{r}\Omega_3 + \Omega\right)\left(\mathbf{r} + k_m\mathbf{r}_s\right) - k_m\left(\frac{1}{f}\mathbf{r}A_3 + A\right)\mathbf{V}_H. \qquad (3)$$

where $\dot{\mathbf{r}} = [\upsilon_x, \upsilon_y, 0]^T$ - image velocity vector ;

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix},$$

where elements $\omega_1,\omega_2,\omega_3$ of matrix $\mathbf{\Omega}$ are projections of vector $\boldsymbol{\omega}=\boldsymbol{\omega}(t)=[\omega_1(t),\ \omega_2(t),\ \omega_3(t)]$ of angular velocity of CP on axes of coordinate system *o'x'y'z';* $\boldsymbol{\omega}_3=[-\ \omega_2,\ \omega_1,\ 0]$ – row-matrix, that is the third row of the matrix $\mathbf{\Omega}$;

$\mathbf{V}_H = \dot{\mathbf{R}}_H = [V_{XH}, V_{YH}, V_{ZH}]^T$ – velocity vector of transient motion of CP relative to underlying surface.

This equation (3) allows getting estimations of an apparent velocity vector of moving image. We will use this expression below to solve a problem of motion vector velocity estimation of underlying surface.

2.3 Functional connection equation (FC equation)

Let it be allocated singly connected regular region of analysis (analyzing window) D with boundary $B(D)$ at a plane P_k. Then we will define a functional $\Phi\ (\mathbf{r},t)$ on set of image intensity functions $\{E(r,t)\}$ in the window D:

$$\Phi = \iint_D K(\mathbf{r})E(\mathbf{r}(t),t)\, ds; \qquad (4)$$

$K(\mathbf{r})$ – weight function that is continuous, uniformly bounded and differentiable one time almost everywhere in respect of all the arguments; we assume that $K(\mathbf{r}) = 0$, if $\mathbf{r} \in B(D)$; $E(\mathbf{r}(t),t)$ – image intensity function, uniformly bounded.

Let us calculate a total derivative of functional (4) with respect to time t under the assumption that image motion equation (3) is fulfilled. Then we get:

$$\dot{\Phi}(t) = \Phi_t'(t) + \iint_D K(\mathbf{r})E(\mathbf{r}(t),t)'_{\mathbf{r}}\dot{\mathbf{r}}\, ds, \qquad (5)$$

where

$$\Phi_1(t) = \iint_D [K(r)]'_{\mathbf{r}} E(\mathbf{r}(t),t)\{[(\mathbf{r}\Omega_3/f + \Omega)(\mathbf{r} + k_m\mathbf{r}_s)] + k_m(\mathbf{r}A_3/f + \mathbf{A})\mathbf{V}_H\}ds;$$

With the Green formula, connecting double and curvilinear integrals, transformation of (5) yields:

$$\dot{\Phi}(t) = \Phi_t'(t) + \Phi_1(t) + \Phi_2(t), \qquad (6)$$

where

$$\Phi_1(t) = \iint_D [K(r)]'_{\mathbf{r}} E(\mathbf{r}(t),t)\{[(\mathbf{r}\Omega_3/f + \Omega)(\mathbf{r} + k_m\mathbf{r}_s)] + k_m(\mathbf{r}A_3/f + \mathbf{A})\mathbf{V}_i\}ds;$$

$$\Phi_2(t) = -\iint_D K(\mathbf{r})E(\mathbf{r}(t),t)\{V_{ZH}/(\mathbf{A}_{3T}\mathbf{r}_s + Z_H) - 3k_m\mathbf{A}_3\mathbf{V}_H/f - 3\Omega_3(\mathbf{r} + k_m\mathbf{r}_s)/f + \mathbf{A}_{3T}\Omega\mathbf{r}_s/(\mathbf{A}_{3T}\mathbf{r}_s + Z_i)\}ds;$$

where $K(\mathbf{r})'_{\mathbf{r}} = [K(\mathbf{r})'_x, K(\mathbf{r})'_y, 0]$ – vector of partial derivatives of weight function K with respect to spatial variables x and y correspondingly.

In the sequel we will admit $\Phi_t'(t) = 0$, i.e. the off-site light source has constant intensity in time.

The equation (6) is the desired FC equation. It connects object motion parameters $\boldsymbol{V_H}$ and $\boldsymbol{\Omega}$ and measurable characteristics of the object image; the characteristics are double integrals defined on the analysing window D.

An example of a suitable weight function K gives a pyramid like function presented at Fig.2, where coordinates are count off the diagonals cross point.

$\forall(x,y)$: $x\in[-a\text{-}a]$, $y\in[-kx, kx]$, $\rightarrow f(x,y)=u(1-|x|/a)$;

$\forall(x,y)$: $y\in[-b,b]$, $x\in[-y/k, y/k]$, $\rightarrow f(x,y)=u(1-|y|/b)$,

where $u=a/b$.

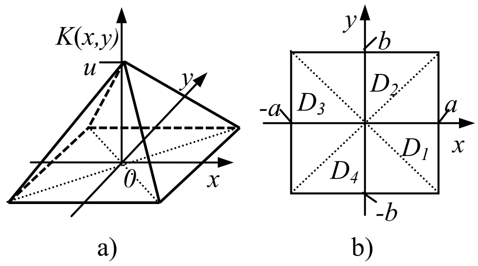


Fig. 2. Pyramid like weight function (a) and location of subdomains $D_1 \ldots D_4$ (b) on the analysing window

Analysis of the FC equation (6) shows, that it con be transformed to an algebraic equation, that is linear with respect to components of the full motion vector:

$$b_1V_{XH} + b_2V_{YH} + b_3V_{ZH} + b_4\omega_1 + b_5\omega_2 + b_6\omega_3 = h, \quad (7)$$

where V_{XH}, V_{YH}, V_{ZH}, u ω_1, ω_2 u ω_3 – the full velocity vector components of the carrying platform;

h=dΦ/dt – total derivative of functional Φ with respect to time t.

b_1 - b_6 – coefficients, given by magnitude of integrals in (6).

The set of equations needed to calculate components of the full velocity vector $\Lambda=[V_X, Vy, V_Z, \omega_1, \omega_2, \omega_3]^T$ composes by parameterization of functional Φ. There is a vast option for the parameter. It can be time t, position, number and shape of the analysing windows as well as a form of weight function $K(\mathbf{r})$ *ets*. Then we get a set of parameterized equations:

$$\mathbf{B}\Lambda = \mathbf{H}, \qquad (8)$$

where $\mathbf{B}=[b_{i,j}]$ – matrix of coefficients b_i; $\mathbf{H}=[h_i]^T$ – column-vector; $i=1\ldots N$ ($N\geq$M), $j=M$ (M=1,…,6) – number of freedom degrees of CP motion.

We over define the system (7) ($N>>M$) to decrease errors in velocity estimations and applying generalized inverse method we , arrive at a solution:

$$\mathbf{\Lambda}=(\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{H}. \qquad (9)$$

In a simple case, when CP makes only transient motion relative to underlying surface, and with the main exis of OS directed to nadir then FC equation (7) takes form:

$$\dot{\Phi} = \omega_3 \left\{ \begin{array}{l} \Phi_0^1(y) - \Phi_0^3(y) - \Phi_0^2(x) + \Phi_0^4(x) + \\ \dfrac{f}{z_S + Z_H}\left[y_S\left(\Phi_0^1 - \Phi_0^3\right) + x_S\left(-\Phi_0^2 - \Phi_0^4\right)\right] \end{array} \right\} -$$

$$- V_{XH}\frac{2f}{a(z_S+Z_H)}\left\{\Phi_0^1 - \Phi_0^3\right\} - V_{YH}\frac{2f}{a(z_S+Z_H)}\left\{\Phi_0^2 - \Phi_0^4\right\} -$$

$$- V_{ZH}\frac{1}{z_S+Z_H}\left\{\left[\Phi_0^1(x) - \Phi_0^3(x) + \Phi_0^2(y) - \Phi_0^4(y)\right] - 2\Phi\right\}, \qquad (10)$$

where $\Phi_0^i(\xi) = \iint_{D^i}\xi E(\mathbf{r},t)ds$; $\Phi_0^i = \iint_{D^i} E(\mathbf{r},t)ds$;

$$\hat{O} = \iint_{D^i} K(\mathbf{r})E(\mathbf{r},t)ds$$

$\xi=x, y$; i=1…4 – number of subdomain D^i in the analyzing window (Fig. 2).

In a case, when $z_s = 0$, $\omega_3 = 0$, $V_{ZH} = 0$ the equation (10) takes the simplest form:

$$\hat{O} = -V_{XH}\frac{2f}{(aZ_H)}\left\{\hat{O}_0^1 - \hat{O}_0^3\right\} - V_{YH}\frac{2f}{(aZ_H)}\left\{\hat{O}_0^2 - \hat{O}_0^4\right\}$$

Comparing the last as well as (10) with BCCE (1) we see, that right hand sight of FC equation contains only measurable characteristics.

The functionalization method affords to estimate not only velocity vector $\mathbf{\Lambda}$ but attitude of the optical axis at any time $t > t_0$, if only there are known initial values of elements of matrix $\mathbf{A}$ ($\mathbf{A}(t_0)=\mathbf{A}_0$).

To realize this ability it is necessary to add to FC equation (9) a cinematic equation of angular motion of CP:

$$\dot{\mathbf{A}}(t) = -\mathbf{\Omega}\,\mathbf{A}(t), \qquad \mathbf{A}(t_0)=\mathbf{A}_0. \qquad (11)$$

3. IMPACT OF ADDITIVE NOISE

In this section we investigate an additive noise impact on the displacement of motion vector estimates derived with the use of functionalization method. We examine a case of plane-parallel motion of CP relative to underlying surface. In the case the main functional (4) will have a form:

$$\tilde{\Phi}(t) = \iint_D K(x,y,-f)\,\tilde{E}(x(t),y(t),-f,t)ds, \qquad (12)$$

where

$$\tilde{E}(x(t),y(t),-f,t) = \\ = E(x(t),y(t),-f,t) + n(x,y,-f,t)$$

- video signal, available for instrumental measurement; $n(x,y,-f,t)$ – noise (stochastic function). We assume that noise and image intensity function are not mutually correlated and indispensable conditions for differentiability of realizations of noise $n(x,y,-f,t)$ in respect of time are fulfiled. Derive functional (12) in respect of time t, then we get

$$\dot{\tilde{\Phi}} = -\upsilon_x\Phi_x^{/} - \upsilon_y\Phi_y^{/} + H_t^{/}, \qquad (13)$$

where $\upsilon_x = V_{XH}f/Z_H$; $\upsilon_y = V_{YH}f/Z_H$;

$$\Phi_x^{/} = \iint_D K_x^{/}(x,y,-f)E(x(t),y(t),-f)ds;$$

$$\Phi_y^{/} = \iint_D K_y^{/}(x,y,-f)E(x(t),y(t),-f)ds;$$

$$H_t^{/} = \iint_D K(x,y,-f)\frac{\partial n(x,y,-f,t)}{\partial t}ds.$$

As the stochastic function $H_t^{/}$ cannot be practically measured it is impossible to use equation (13) for calculation of image velocity vector $\dot{\mathbf{r}} = [\upsilon_x, \upsilon_y, 0]^T$. Then we will use for this aim an approximate form (14) deduced from (13) by eliminating the term $H_t^{/}$ from right-hand side of equation (13):

$$\dot{\tilde{\Phi}}(t) = -\tilde{\upsilon}_x\tilde{\Phi}_x^{/} - \tilde{\upsilon}_y\tilde{\Phi}_y^{/}, \qquad (14)$$

where $\tilde{\Phi}_x^{/} = \iint_{\mathrm{D}} K_x^{/}(x,y,-f)\tilde{E}(x(t),y(t),-f,t)ds;$

$$\tilde{\Phi}_y^{/} = \iint_D K_y^{/}(x,y,-f)\tilde{E}(x(t),y(t),-f,t)ds;$$

$\tilde{\upsilon}_x = \upsilon_x + \Delta\upsilon_x$ и $\tilde{\upsilon}_y = \upsilon_y + \Delta\upsilon_y$ – estimations of components of image motion velocity vector; $\Delta\upsilon_x$ and $\Delta\upsilon_y$ – estimation errors. All of the coefficients in right-hand side of (14) are measurable in defined above sense.

To reduce errors $\Delta\upsilon_x$ and $\Delta\upsilon_y$ we will over define equation (14) taking D_i ($i\in\{1\ldots l\}$) , l >>2 and thus compose the next set of equations:

$$
\begin{cases}
\widetilde{\upsilon}_x \sum_{i=1}^{n} (\widetilde{\hat{O}}'_{xi})^2 + \widetilde{\upsilon}_y \sum_{i=1}^{n} (\hat{O}'_{xi}\hat{O}'_{yi} + \hat{O}'_{xi}H'_{yi} + H'_{xi}\hat{O}'_{yi} + H'_{xi}H'_{yi}) \\
= -\sum_{i=1}^{n} (\dot{\widetilde{\hat{O}}}_i \hat{O}'_{xi} + \dot{\widetilde{\hat{O}}}_i H'_{xi}); \\
\widetilde{\upsilon}_x \sum_{i=1}^{n} (\hat{O}'_{xi}\hat{O}'_{yi} + \hat{O}'_{yi}H'_{xi} + H'_{xi}\hat{O}'_{yi} + H'_{xi}H'_{yi}) + \widetilde{\upsilon}_y \sum_{i=1}^{n} (\widetilde{\hat{O}}'_{yi})^2 \\
= -\sum_{i=1}^{n} (\dot{\widetilde{\hat{O}}}_i \hat{O}'_{\acute{o}i} + \dot{\widetilde{\hat{O}}}_i H'_{\acute{\imath}i}).
\end{cases}
$$

If noise and analyzing image are such that functioals $\varPhi'_x$, $\varPhi'_y$, H'_x, H'_y are not mutually correlated, then asymptotic forms when $l \to \infty$ for $\Delta\upsilon_x$ and $\Delta\upsilon_y$ will be:

$$
\begin{cases}
\Delta\overline{\upsilon}_x = -\upsilon_x \dfrac{1}{1 + \sum_{i=1}^{n} (\varPhi'_{xi})^2 / \sum_{i=1}^{n} (H'_{xi})^2}, \\
\Delta\overline{\upsilon}_y = -\upsilon_y \dfrac{1}{1 + \sum_{i=1}^{n} (\varPhi'_{yi})^2 / \sum_{i=1}^{n} (H'_{yi})^2}.
\end{cases} \qquad (15)
$$

It can be seen from (15) that velocity estimations obtained with the functionalization method have negative sign shifts directly proportional to image velocity and reciprocally proportional to a ratio of signal power to noise power.

Moreover, it follows from (15) that velocity estimation shifts can be eliminated with the use of compensating measurement method that can be implemented with the use of mechanical or electronil image tracking.

3 ITERATIVE TECHNIQUE OF IMAGE MOTION PARAMETERS ESTIMANION

One of the ways to implement compensating measurement method is overlapping images in two sequential in time image frames.

We suppose that an analyzing window ($OA1_0$) is placed on the first image frame that was registered at time moment t_0. Position of it is demonstrated at Fig.3a. The second image frame is registered in time interval Δt at time $t_1=t_0+\Delta$. Let it another analyzing window be positioned at any arbitrary place $OA2_0$ on the second frame as it is shown at fig. 3b. Our aim is to overlap the windows in these two frames so that the images in the windows should matched each other in the best way.

The iterative scheme we put forward is the next. On every i-th step of the procedure we use information obtained from windows $OA1_0$ and $OA2_i$ to calculate estimations of mutual displacement ($\widetilde{s}_{x1} = \widetilde{\upsilon}_{x1}\Delta t$ and $\widetilde{s}_{y1} = \widetilde{\upsilon}_{y1}\Delta t$) of the two windows with the use of functionalization method. Forming FC equation in that case we use the first difference of main functional (12) volumes in the windows $OA1_0$ and $OA2_i$, instead of the first temporal derivative of the functional.

In order to form over defined set of FC equations we will use additional analyzing windows covering up the main windows $OA1_0$ and $OA2_i$ with sets of identical windows of fewer dimensions. An example of such covering is shown at Fig.3. At the example each of the coverings consists of five windows and consequently we get a set of five FC equations. A solution of over defined set of FC equations is arrived by applying generalized inverse method.

At the next step of the iterative scheme the analyzing window $OA2_i$ is warped towards $OA1_0$ with the use of current estimates $(\widetilde{s}_{y1}$ and $\widetilde{s}_{x1})$ of the windows displacement and we get a new position of the window $OA2_i$ denoted at Fig.3 as $OA2_{i+1}$. The procedure iterates till displacements $\widetilde{s}_{y1}$ and $\widetilde{s}_{x1}$ become less than a preliminarily specified radius ρ of accessibility tube and will not leave the tube in the sequel. This procedure can be written as

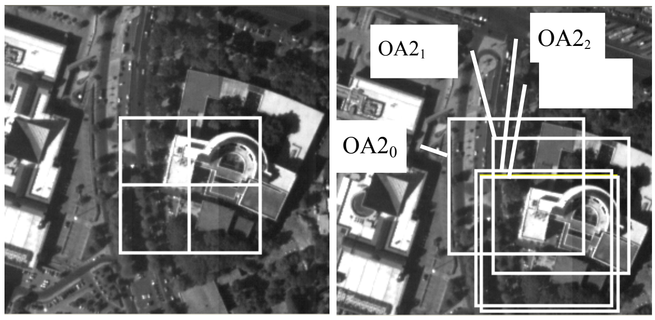
$$s(k+1)= s(k)+ \Delta s(k), \qquad (16)$$

where $s=(s_x, s_y)$; $\Delta s=(\Delta s_x, \Delta s_y)$; $s(0)=s_0$; k=1, 2,… - number of an iteration. At every iteration we get increments of displacement estimations $\Delta s(k) = (\Delta s_x (k), \Delta s_y(k))$ by solving set of FC equations, coefficients of which are formed with the use of information obtained from windows OA_0 and OA_i.

$$\Delta s(k) = -(\varPhi'_x(k))^T \cdot (\varPhi'_x(k))^{-1}((\varPhi'_x(k))^T \Delta\varPhi(k)),$$

where $\Delta\hat{O}(k) = [\hat{O}2_i - \hat{O}1_i]$; i=1,2,…;

$\varPhi1_i$ $\varPhi2_i$ – volumes of the main functionals defined in the windows OA_0 and OA_i; $\varPhi'_x(k) = \frac{1}{2}[\widetilde{\varPhi}1'_x + \widetilde{\varPhi}2'_x]$.



a) b)

Fig.3 Iterative technique of image motion parameters estimanion

4. COMPUTER ANALYSIS OF THE ITERATIVE TECHNIQUE ERRORS

We used computer modeling to estimate the iterative technique errors. The aim of the modeling was to approbate the iterative technique of functionalization method and to study the influence of stochastic noise on results obtained with this method. A modeling system encloses three modules: a module of video signal modeling, a module of elaborating estimations of image displacement of two image frames and a module for overlapping two frames of image. A computational model of a video signal corresponded a mixture of pure video signal and additive random noise. Random noise had a uniform frequency distribution function. Amplitude of noise was specified in lower order bits (l.o.b). The main functional, used at modeling, had a pyramid-like weight function. Results of computer modeling are represented at Fig.4. Experiments were undertaken to search out relationship of mathematical expectation of a radius (ρ) of accessibility tube and a parameter A_n (an amplitude) of noise. A dynamic range of video signal belonged to an interval [0-127 l.o.b.] Initial displacement of images varied from 2 to 50 pixels. Sufficient number (no more the 4 – 6, as a fact) of iterations were undertaken to get reliable estimations for radius (ρ) of accessibility tube.

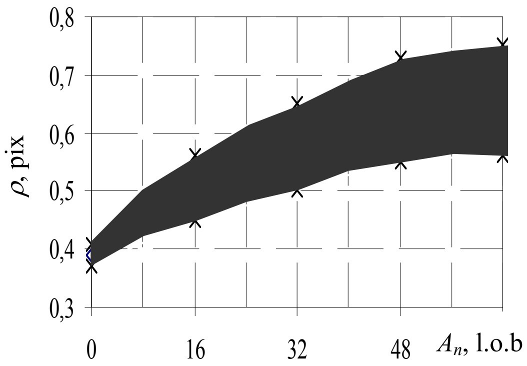


Fig. 4. Experimental data. Relationship of mathematical expectation M of a radius (ρ) of an accessibility tube and a parameter A_n (an amplitude) of noise, $D_{0,96}$ - (gray bacground) confidence interval

5 CONCLUSIONS

Computational experiments show that functionalization method introduced in the report allows to eliminate the main shortcomings of gradient motion estimation method. The functionalization method does not involve calculation of spatial derivatives of image dense function. More over the iterative technique of the method eludes calculation of temporal derivatives of image dense. That brings essentially new features of universality and accuracy to the image motion estimation methods.

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