ADAPTIVE OUTPUT REGULATION OF NONLINEAR SYSTEMS DESCRIBED BY MULTIPLE LINEAR MODELS

Mai Bando *,[1] **Akira Ichikawa** *

* *Kyoto University, Department of Aeronautics and Astronautics, Yoshidahonmachi, Sakyo-ku, Kyoto, 606-8501, Japan*

Abstract: In this paper the output regulation problem for nonlinear systems described by multiple linear models with unknown parameters is considered. Based on the Lyapunov stability theory, an adaptive controller which stabilize the system is derived. Then sufficient conditions for the output regulation problem with full information to be solvable are established. Simulation results are given to illustrate the theory.

Keywords: Fuzzy System, Output Regulation, Adaptive Control, Stabilization

1. INTRODUCTION

The nonlinear system described by multiple linear models is a convex combination of systems with state-dependent nonlinear weights. This class contain the Takagi-Sugeno Fuzzy systems and is in applications (Feng, 2006),(Yoneyama *et al.*, 2000). In this paper adaptive output regulation is considered under the assumption that the underlying linear systems are of controllable canonical form with the same structure and contain unknown parameters. As preliminaries, linear systems are assumed known, and stabilization and output regulation is considered. A feedback control is obtained by solving an algebraic Riccati equation depending on the state variable. Using Lyapunov theory, local asymptotic stability is proved. Then assuming that the norm of the reference signal is small, sufficient conditions for local output regulation are obtained.

[1] Partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for JSPS Fellows and Center of Excellence for Research and Education on Complex Functional Mechanical Systems (COE program of Ministry of Education, Culture, Sports, Science, and Technology Japan).

For the unknown system, the state estimator and adaptive laws for unknown matrix are introduced, and sufficient conditions for convergence to zero of the state estimation error are given. To design feedback gains, a Riccati equation involving the state of the estimator and estimated matrices is introduced, and the local asymptotic stability of the closed loop system is shown. Using this feedback and the solution of the regulator equation for the nonlinear system, local output regulation is fulfilled.

For numerical simulations, a two-dimensional system is introduced and step- and sine-tracking problems are considered. In the case of adaptive stabilization and step tracking, the solution of the Riccati equation converges to constant matrices, while for sine-tracking they become periodic.

2. OUTPUT REGULATION OF NONLINEAR SYSTEMS DESCRIBED BY MULTIPLE LINEAR MODELS

Consider the nonlinear system described by multiple linear models

$$
\begin{aligned}
&\dot{x} = \sum_{i=1}^{r} \lambda_i(x) A_i x + B_1 w + \sum_{i=1}^{r} \lambda_i(x) B_{2i} u, \\
&\sum_{i=1}^{r} \lambda_i(x) = 1, \\
&z = C_1 x + D_{11} w + D_{12} u
\end{aligned} \tag{1}
$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control and $z \in \mathbb{R}^q$ is the output to be regulated and r denotes the number of local models. The matrices A_i, B_1, B_{2i}, C_1 and D_{11} are constant and of appropriate dimensions. $\lambda_i(x)$ are continuously differentiable functions of state x.

The signal $w \in \mathbb{R}^s$ denotes disturbances or reference signals generated by an anti-stable exosystem

$$
\dot{w} = Sw. \tag{2}
$$

We assume that the state x is accessible and consider the regulation problem for (1) under the following conditions.

Assumption 2.1. $D_{12} = 0$ and (A_i, B_{2i}) is controllable canonical form of the same structure each i and C_1 has the structure

$$
C_1 = \left[\, C^1 \;\, C^2 \;\, \cdots \;\, C^m \,\right], \tag{3}
$$

$$
C^i = \begin{bmatrix} 0 & \\ \vdots & \\ 1 & 0 \\ \vdots & \\ 0 & \end{bmatrix} \in \mathbb{R}^{q \times n_i}, \tag{4}
$$

where the i-th elements of the first column of C^i are one.

Note that if each (A_i, B_{2i}) is in the controllable canonical form of the same structure then $(\sum_{i=1}^{r} \lambda_i(x) A_i, \sum_{i=1}^{r} \lambda_i(x) B_{2i})$ is also in the controllable canonical form for any x.

2.1 Preliminaries

We consider the algebraic Riccati equation:

$$
A^T(x) X + X A(x) + C^T C - X B(x) B(x)^T X = 0. \tag{5}
$$

Lemma 2.1. Let (A(x),B(x),C) be stabilizable and detectable for x in an open set N. Then there exists a unique solution $X = X(x)$ of (5) which is continuous and continuously differentiable with respect to $x \in N$. Moreover, if x stays in a compact domain, $\dfrac{\partial X}{\partial x}$ is bounded.

We consider the regulator equation (Saberi *et al.*, 2000):

$$
\begin{aligned}
&A\Pi - \Pi S + B_1 + B_2 \Gamma = 0, \\
&C_1 \Pi + D_{11} + D_{12} \Gamma = 0.
\end{aligned} \tag{6}
$$

Lemma 2.2. Let $(A,\ B_2)$ be in the controllable canonical form. Then the regulator equation exists. Moreover Π does not depend on the parameters of A and B_2.

2.2 Stabilization

Consider the regulation problem of (1) with $w = 0$. Let Q be positive-definite. Then $(C, \sum_{i=1}^{r} \lambda_i(x) A_i)$ is observable where $C = \sqrt{Q}$. Since $(\sum_{i=1}^{r} \lambda_i(x) A_i, \sum_{i=1}^{r} \lambda_i(x) B_{2i})$ is stabilizable by Assumption 2.1, there exists a positive stabilizing solution X of the algebraic Riccati equation

$$
\begin{aligned}
&(\sum_{i=1}^{r} \lambda_i(x) A_i)^T X + X (\sum_{i=1}^{r} \lambda_i(x) A_i) + Q \\
&- X (\sum_{i=1}^{r} \lambda_i B_{2i}(x)) (\sum_{i=1}^{r} \lambda_i B_{2i}(x))^T X = 0.
\end{aligned} \tag{7}
$$

Now we set

$$
\sum_{i=1}^{r} \lambda_i A_i \triangleq A, \quad \sum_{i=1}^{r} \lambda_i B_{2i} \triangleq B_2.
$$

and introduce the control law

$$
u = -B_2^T X(x) x \tag{8}
$$

and consider the stability of the control system.

Theorem 2.1. The equilibrium $x_e = 0$ of the system (1) is locally asymptotically stable.

Proof 2.1. Substituting (8) into (1) , we have

$$
\dot{x} = (A - B_2 B_2^T X) x \tag{9}
$$

Consider the Lyapunov function candidate

$$
V(x) = x^T X(x) x. \tag{10}
$$

The time derivative of (10) along the solutions of (9) is given by

$$
\begin{aligned}
\dot{V}(x) &= [(Xx)^T + x^T X \\
&\quad + x^T (\nabla_x \otimes X)(I_n \otimes x)](A - B_2 B_2^T X) x \\
&\leq -x^T [Q - (\nabla_x \otimes X)(I_n \otimes x)(A - B_2 B_2^T X)] x
\end{aligned}
$$

where $\nabla_x = \left[\dfrac{\partial}{\partial x_1}, \cdots, \dfrac{\partial}{\partial x_n}\right]$ and $\otimes$ denotes Kronecker product. We set $\Omega = \{x \in \mathbb{R}^n |\ Q - (\nabla_x \otimes X)(I_n \otimes x)(A - B_2 B_2^T X) > 0\}$. If $x \in \Omega$ then $\dot{V} \leq 0$. Hence the origin is locally asymptotically stable. □

2.3 Output regulation

Consider the output regulation problem associated with (1). By Assumption 2.1 and Lemma 2.2, there exists a solution (Π, Γ) of the regulator equation

$$(\sum_{i=1}^{r}\lambda_i(x)A_i)\Pi - \Pi S + B_1 + (\sum_{i=1}^{r}\lambda_i(x)B_{2i})\Gamma = 0,$$
$$C_1\Pi + D_{11} + D_{12}\Gamma = 0, \tag{11}$$

such that Π is a constant matrix. We choose the controller

$$u = -B_2^T(x)X(x)x + (\Gamma(x) + B_2^T(x)X(x)\Pi)w, \tag{12}$$

where $X(x)$ is the solution of the Riccati equation (7).

Theorem 2.2. Under Assumption 2.1, the local output regulation is fulfilled i.e., $\lim_{t\to\infty} z(t) = 0$.

Proof 2.2. Substituting (12) into (1), we have

$$\begin{aligned}\dot{x} &= (A - B_2^T B_2^T X)x + (B_1 + \Gamma + B_2^T X)w \\ &\triangleq A_f x + Bw,\end{aligned} \tag{13}$$

Consider $V_1(x) = x^T X(x)x$. The time derivative of $V_1(x)$ along the solution of (13) is given by

$$\begin{aligned}\dot{V}_1 \le &-x^T[Q - (\nabla_x \otimes X)(I_n \otimes x)(A_f x + Bw)]x \\ &+2x^T XBw.\end{aligned} \tag{14}$$

There exist ϵ, $\alpha > 0$ such that $|w| \le \epsilon$ and $V_1(x) \le \alpha$ imply $\dot{V}_1 \le -\delta V_1 + \beta|w|^2$ and $\frac{\beta\epsilon^2}{\delta} < \alpha$ for some β. By integrating (14) we have

$$V_1(x(t)) \le e^{-\delta t}V_1(x_0) + \frac{\beta}{\delta}|w|^2.$$

Now choose α_0 such that $\alpha_0 + \frac{\beta\epsilon^2}{\delta} < \alpha$ and x_0 such that $V_1(x_0) \le \alpha_0$. The solution of (13) starting from x_0 stays in $\Omega = \{x|V_1(x) \le \alpha\}$ for all $t \ge 0$. Consider

$$\tilde{x} = x - \Pi w$$

then

$$\dot{\tilde{x}} = (A - B_2 B_2^T X)\tilde{x} = A_f\tilde{x}. \tag{15}$$

Consider $V_2(x, \tilde{x}) = \tilde{x}^T X(x)\tilde{x}$. The time derivative of $V_2(x)$ along the solutions of (15) is given by

$$\begin{aligned}\dot{V} &\le -\tilde{x}^T[Q - (\nabla_x \otimes X)(I_n \otimes x)(A_f x + Bw)]\tilde{x} \\ &< -\delta'|\tilde{x}|^2,\end{aligned} \tag{16}$$

where the inequality holds since $x \in \Omega$ and $|w| \le \epsilon$ for all $t \ge 0$. Then the origin of (15) is locally asymptoticaly stable. Now

$$\begin{aligned}z &= C_1x + D_{11}w + D_{12}u \\ &= (C_1 - D_{12}B_2^T X)\tilde{x} + (C_1\Pi + D_{11} + D_{12}\Gamma)w \\ &= (C_1 - D_{12}B_2^T X)\tilde{x} \to 0.\end{aligned}$$

Hence local output regulation is achieved. □

3. ADAPTIVE REGULATION AND ADAPTIVE OUTPUT REGULATION

Consider the nonlinear system described by multiple linear models

$$\begin{aligned}\dot{x} &= \sum_{i=1}^{r}\lambda_i(x)A_i x + B_1 w + \sum_{i=1}^{r}\lambda_i(x)B_{2i}u, \\ &\sum_{i=1}^{r}\lambda_i(x) = 1, \\ z &= C_1x + D_{11}w + D_{12}u,\end{aligned} \tag{17}$$

where the constant matrices A_i,B_{2i} contain unknown parameters and the other matrices are assumed to be known. We assume A_i, B_{2i} and C_1 satisfy Assumption 2.1 Note that if each (A_i, B_{2i}) is in the same controllable canonical form then $(\sum_{i=1}^{r}\lambda_i(x)A_i, \sum_{i=1}^{r}\lambda_i(x)B_{2i})$ is also controllable canonical form for any unknown parameters of (A_i, B_{2i}) and for any x.

3.1 Adaptive regulation

First we consider the stabilization problem. Introduce an estimator and adaptive laws of the form

$$\dot{\hat{x}} = A_m\hat{x} + \sum_{i=1}^{r}\lambda_i(x)(\hat{A}_i - A_m)x + B_1w + \sum_{i=1}^{r}\lambda_i(x)\hat{B}_{2i}u, \tag{18}$$

$$\begin{aligned}\dot{\hat{A}}_i &= \dot{\Phi}_i = -\lambda_i(x)Pex^T, \\ \dot{\hat{B}}_{2i} &= \dot{\Psi}_{2i} = -\lambda_i(x)Peu^T,\end{aligned} \tag{19}$$

where A_m is an $n \times n$ stable matrix, P is the solution of the following matrix equation

$$A_m^T P + PA_m = -Q_0$$

for some positive-definite matrix Q_0 and

$$e = \hat{x} - x, \Phi_i = A_i - A, \Psi_{2i} = \hat{B}_{2i} - B_2.$$

Then the error equation is given by

$$\dot{e} = A_m e + \sum_{i=1}^{r}\lambda_i(x)(\Phi_i x + \Psi_{2i}u). \tag{20}$$

If some elements of A_i and B_{2i} are known, we can omit their adaptive laws in (19), but for notational convenience we use (19).

Theorem 3.1. If x(t) and u(t) is bounded for all $t \geq 0$, then (20) and (19) is globally stable.

Proof 3.1. Consider the Lyapunov function candidate

$$V(e, \Phi_i, \Psi_{2i}) = e^T P e + \sum_{i=1}^{r} tr(\Phi_i^T \Phi_i + \Psi_{2i}^T \Psi_{2i}), \quad (21)$$

where trA denotes the trace of the matrix A. The time derivative of (21) along the solutions of (20) is given by

$$\dot{V}(e, \Phi_i, \Psi_{2i}) = -e^T Q_0 e \leq 0.$$

Hence the origin of (19) and (20) is globally stable. It follows that e, Φ_i and Ψ_{2i} are bounded for all $t \geq 0$ and $e \in \mathcal{L}_2$. □

Let Q be positive-definite. Then $(C, \hat{A})$ is observable where $C = \sqrt{Q}$. Since $((\sum_{i=1}^{r} \lambda_i(x(t))A_i(t), \sum_{i=1}^{r} \lambda_i(x(t))B_i(t)))$ is stabilizable by Assumption 2.1 for each t there exists a positive stabilizing solution X of the algebraic Riccati equation

$$(\sum_{i=1}^{r} \lambda_i(x)\hat{A}_i)^T X + X(\sum_{i=1}^{r} \lambda_i(x)\hat{A}_i) + Q - X(\sum_{i=1}^{r} \lambda_i(x)\hat{B}_{2i})(\sum_{i=1}^{r} \lambda_i(x)\hat{B}_{2i})^T X = 0. \quad (22)$$

Now we set

$$\sum_{i=1}^{r} \lambda_i \hat{A}_i \triangleq \hat{A}, \sum_{i=1}^{r} \lambda_i \hat{B}_{2i} \triangleq \hat{B}_2,$$
$$\theta = [vec\ \Phi_1, \cdots, vec\ \Phi_r, vec\ \Psi_1, \cdots vec\ \Psi_r]^T$$

and introduce the control law

$$u = -\hat{B}_2^T X(x, \theta)\hat{x} \quad (23)$$

and consider the stability of the adaptive control system.

Theorem 3.2. For sufficiently small $x(0)$, $\hat{x}(0)$, $e(0)$ and $\theta(0)$, x, $\hat{x}$, e and θ is bounded and $\lim_{t\to\infty} e(t) = 0$. Moreover, if $w = 0$ then $\lim_{t\to\infty} x(t) = 0$.

Proof 3.2. Substituting (23) into (17) and (18) , we have

$$\begin{aligned}\dot{x} &= (\hat{A} - \hat{B}_2\hat{B}_2^T X)x + \sum_{i=1}^{r} \lambda_i(x)(\Phi_i x - \Psi_i \hat{B}_2^T X\hat{x}) \\ &\quad + (B_1 + \hat{B}_2\Gamma + \hat{B}_2\hat{B}_2^T \Pi)w. \\ &\triangleq \hat{A}_f x + \Phi x + \Psi\hat{x} + Bw \quad (24)\\ \dot{\hat{x}} &= (\hat{A} - \hat{B}_2\hat{B}_2^T X)\hat{x} + (A_m - \hat{A})e + B_1 w \\ &\triangleq \hat{A}_f \hat{x} + Ee + B_1 w. \quad (25)\end{aligned}$$

Consider $V_1 = x^T X(x, \theta)x$. The time derivative of $V_1(x)$ along the solution of (25) is given by

$$\begin{aligned}\dot{V}_1 \leq -x^T[&Q \\ &-(\nabla_x \otimes X)(I_n \otimes x)(\hat{A}_f x + \Phi x + \Psi\hat{x} + Bw) \\ &-(\nabla_\theta \otimes X)(I_{2r} \otimes \theta)\dot{\theta}]x \\ &+2x^T X\Phi x + x^T X\Psi\hat{x} + 2x^T XBw.\end{aligned} \quad (26)$$

There exist ϵ_e, ϵ_w, $\alpha > 0$ such that $|e|, |\theta| \leq \epsilon_e$, $|w| \leq \epsilon_w$ and $V_1(x) \leq \alpha$ imply $\dot{V}_1 \leq -\delta V_1 + \beta_1|w|^2 + \beta_2\hat{x}^T X\hat{x}$ and $\frac{\beta_1\epsilon_w^2}{\delta} + \frac{\beta_2\rho}{\delta} < \alpha$ for some β_1, β_2 and ρ. Now choose α_0 such that $\alpha_0 + \frac{\beta_1\epsilon_w^2}{\delta} + \frac{\beta_2\rho}{\delta} < \alpha$ and x_0 such that $V_1(x_0) \leq \alpha_0$. Now consider $V_2 = \hat{x}^T X(x, \theta)\hat{x}$. The time derivative of $V_2(\hat{x})$ along the solution of (25) is given by

$$\begin{aligned}\dot{V}_2 \leq -\hat{x}^T[&Q \\ &-(\nabla_x \otimes X)(I_n \otimes x)(\hat{A}_f x + \Phi x + \Psi\hat{x} + \hat{B}_1 w) \\ &-(\nabla_\theta \otimes X)(I_{2r} \otimes \theta)\dot{\theta}]\hat{x} \\ &+2x^T X\Phi x + \hat{x}^T XEe + 2\hat{x}^T XBw. \\ &\leq -\delta V_2 + \beta_3|e|^2 + \beta_4|w|^2.\end{aligned} \quad (27)$$

Choose ρ_0 such that $\rho_0 + \frac{\beta_3\epsilon_w^2}{\delta} + \frac{\beta_4\epsilon_e^2}{\delta} < \rho$ and $\hat{x}_0$ such that $V_2(\hat{x}_0) \leq \rho_0$. Then the solution starting from x_0, $\hat{x}_0$ stays in $\Omega = \{x, \hat{x}|V_1(x) \leq \alpha, V_2(\hat{x}) \leq \rho\}$.

Now, we conclude from (20) that $\dot{e}$ is bounded. Therefore by Barbalat's lemma (Narendra & Annaswam, 1989) we obtain $\lim_{t\to\infty} e(t) = 0$. If in particular $w = 0$ then $\lim_{t\to\infty} \hat{x}(t) = 0$ and hence $\lim_{t\to\infty} x(t) = 0$. □

3.2 Adaptive output regulation

Now we consider the adaptive output regulation problem associated with (17), (18) and (19). In this case regulator equation is given by

$$\begin{aligned}\hat{A}\Pi - \Pi S + B_1 + \hat{B}_2\Gamma = 0, \\ C_1\Pi + D_{11} + D_{12}\Gamma = 0.\end{aligned} \quad (28)$$

By Assumption 2.1 and Lemma 2.2, there exists a solution (Π, Γ) of (28) such that Π is an constant matrix. We choose the controller

$$u = -\hat{B}_2^T X(x, \theta)\hat{x} + (\Gamma(x, \theta) + \hat{B}_2^T X(x, \theta)\Pi)w, \quad (29)$$

where $X(x,\theta)$ is the solution of the Riccati equation (22) corresponding to (17), (18) and (19). The following result is obtained.

Theorem 3.3. For sufficiently small $x(0)$, $\hat{x}(0)$, $e(0)$ and $\theta(0)$, the adaptive local output regulation is fulfilled i.e., $\lim_{t\to\infty} z(t) = 0$.

Proof 3.3. Choose $x(0)$, $\hat{x}(0)$, $e(0)$ and $\theta(0)$ as Theorem 3.2. Consider

$$\tilde{x} = \hat{x} - \Pi w$$

then

$$\dot{\tilde{x}} = (\hat{A} - \hat{B}_2\hat{B}_2^T X)\tilde{x} + (A_m - \hat{A})e.$$

Since homogeneous part is asymptotically stable and $\lim_{t\to\infty} e(t) = 0$ by Theorem 3.2, $\lim_{t\to\infty} \tilde{x}(t) = 0$. Now

$$\begin{aligned} z &= C_1 x + D_{11} w + D_{12} u \\ &= (C_1 - D_{12}\hat{B}_2^T X)\tilde{x} + C_1 e \to 0. \end{aligned}$$

Hence local output regulation is achieved. □

4. SIMULATION RESULTS

Example 4.1

Consider the nonlinear mass-spring system

$$\ddot{\xi} = -0.01\xi - 0.67\xi^3 + u, \qquad (30)$$

The nonlinear term satisfies the following conditions for $\xi \in [-1 \quad 1]$:

$$\begin{aligned} -0.67\xi \leqslant -0.67\xi^3 \leqslant 0\xi, \qquad \xi \geqslant 0, \\ 0\xi \leqslant -0.67\xi^3 \leqslant -0.67\xi, \qquad \xi \leqslant 0. \end{aligned}$$

Hence it can be represented by the following nonlinear system:

$$\dot{x} = \sum_{i=1}^{2} \lambda_i(x)(A_i x + B_i u), \qquad (31)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ a_3 & a_4 \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$x = \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix}, \lambda_1(x) = 1 - \xi^2, \lambda_2(x) = \xi^2,$$
$$a_1 = -0.01,\ a_2 = 0,\ a_3 = -0.68,\ a_4 = 0.$$

Here a_i regarded unknown, but λ_i are given functions, and state x is accesible. This is an example of the nonlinear system described by multiple linear models (17). We consider the adaptive regulation problem for this system. The simulation result with $x(0) = [0.8 \quad 0]^T$ is given in Figure 1. The solution of the Riccati equation (22) is also shown in Figure 2.

Example 4.2

For the system (30) we design a state feedback controller such that $x_1(t) \to 0.5$. For this purpose we set $C_1 = 1$, $D_{11} = -1$ and take the following exosystem

$$S = 0,\ w(0) = 0.5.$$

In this case $\Pi = [1\ 0]^T$. The simulation result with $x(0) = [0.8 \quad 0]^T$, $\hat{a}_i(0) = 0$ and $\hat{b}_1(0) = 0$ is shown in Figure 3.

Example 4.3

Here (30) we design a state feedback controller such that $x_1(t) \to 0.3sin(t)$. In this case we set $C_1 = [1 \quad 0]$, $D_{11} = [-1 \quad 0]$ and take the following exosystem

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Then $\Pi = I_2$. The simulation result with $x(0) = [0.1 \quad 0]^T$, $w(0) = [0 \quad 0.3]^T$, Figures 5 and 6 show the simulation results when all the matrices are assumed to be known. Figures 7 and 8 show the simulation results with small initial conditions and parameter errors. In either case the output z goes to zero. In this example $\dot{X}$ does not go to zero but assumption in the Theorem 2.2 and Theorem 3.3 hold.

REFERENCES

G. Feng (2006). A Survey on Analysis and Design of Model-Based Fuzzy Control Systems *Fuzzy Systems, IEEE Transactions on* Volume 14, Issue 5, pp. 676–697

J. Yoneyama, M. Nishikawa, H. Katayama, and A. Ichikawa (2000). *Output stabilization of Takagi-Sugeno fuzzy systems. Fuzzy Sets and Systems* 121, pp. 127–148

A. Saberi, A. A. Stoorvogel and P. Sannuti (2000) *Control of Linear Systems with Regulation and Input Constraints.* Springer.

B. Zhou and G. -R. Duan (2006) A new solution to the generalized Sylvester matrix equation $AV - EVF = BW$, *Systems & Control Letters.* volume 55, pp. 193–198.

P. A. Ioannou and J. Sun (1996) *Robust Adaptive Control* Prentice Hall.

K. S. Narendra, A. M. Annaswamy (1989). *Stable Adaptive Systems*, Prentice Hall.

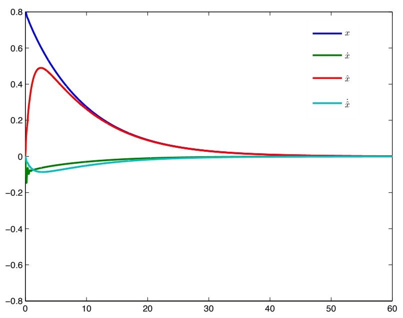

Fig. 1. The trajectories of the state and its estimate.

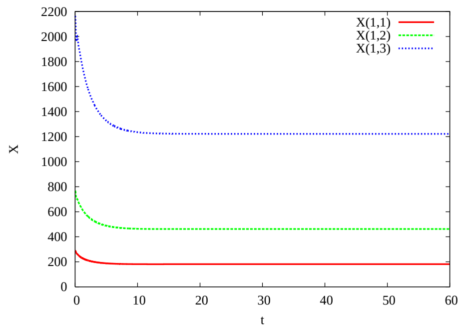

Fig. 2. Solution of the Riccati equation.

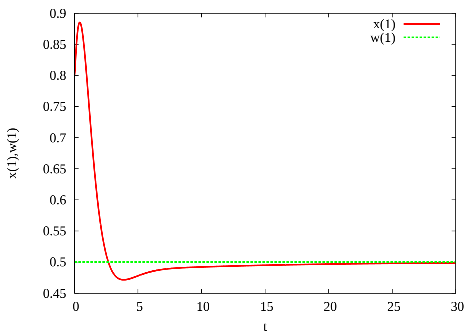

Fig. 3. Step tracking.

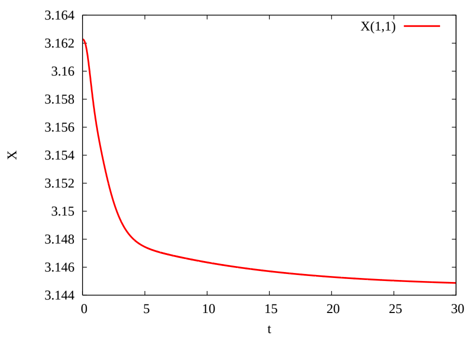

Fig. 4. Solution of the Riccati equation.

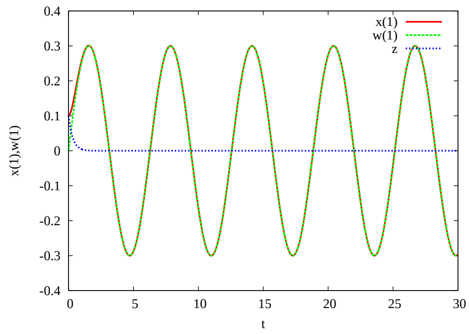

Fig. 5. Sine tracking(known model).

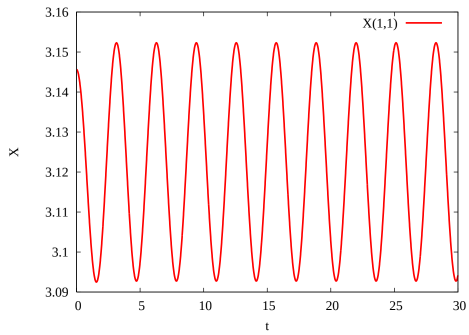

Fig. 6. Solution of the Riccati equation.

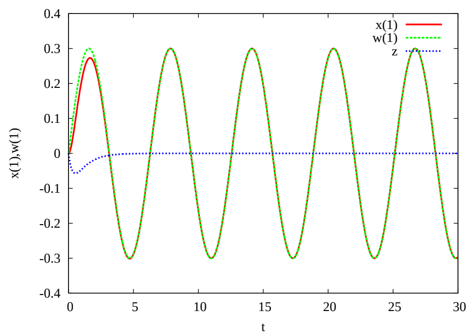

Fig. 7. Sine tracking(adaptive).

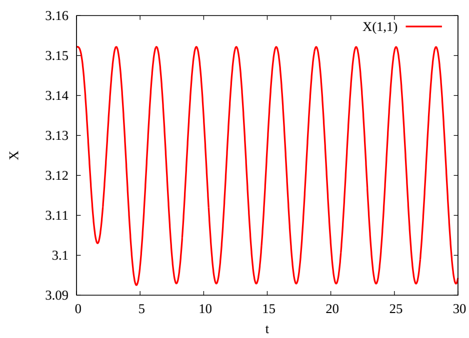

Fig. 8. Solution of the Riccati equation(known model).