LEARNING A CONTROLLER FOR A COUPLED DRIVES APPARATUS USING VRFT STRATEGY

František Gazdoš, Petr Dostál

Department of Process Control, Faculty of Applied Informatics,
Tomas Bata University in Zlín, Nad stráněmi 4511, 760 05 Zlín, Czech Republic
phone: +420-576-035-199, fax:+420-576-032-719, e-mail: gazdos@fai.utb.cz

Abstract: The paper utilizes the Virtual Reference Feedback Tuning methodology for the iterative way of controller design and fine-tuning. It uses a series of experiments with no restriction on data generation to design an optimal controller of desired structure without the intermediate plant identification step. The approach is shown to be successful for the design and fine-tuning of a controller for a coupled drives system by means of adjusting a desired settling-time of a controlled variable.

Keywords: iterative methods, controller tuning, direct methods, virtual reference feedback tuning, coupled drives apparatus.

1. INTRODUCTION

Principles of adaptation and learning are very common phenomena of many processes in nature. Various species, from the simplest to extremely complex ones including mankind, possess the ability of adaptation to new environmental conditions and the quality of learning. Therefore, when the development of automated systems started, it was natural to try to incorporate also these concepts. In control engineering, adaptive control was the answer, starting in the fifties of the last century mainly in the aeronautics research area and booming in the eighties and nineties thanks to the rapid expansion of microprocessor technology. Consequently, the complexity of realizable algorithms could increase which led to the development of more sophisticated methods of *Model Reference Adaptive Control* (MRAC), *Self-Tuning Control* (STC) and many *auto-tuning* techniques (e.g. Åström, 1983; Åström and Wittenmark, 1989). In the next decade, the research revealed new areas of control systems adaptation and learning based on neural networks, fuzzy sets and artificial intelligence. Besides this, from the nineties, there has been a boom of iteration methods (e.g. Albertos and Sala, 2002; Gevers, 2002; Xu and Tan, 2003) which was a natural result of the efforts to connect identification and controller design together. The iterative way of controller tuning turned to be a solution for the problem of optimizing simultaneously both, a criterion for identification and controller design criterion (Schrama, 1992). Next step, in order to guarantee convergence of the methods, was towards *direct* optimization of controller parameters. This has led to the development of new powerful methods for direct controller tuning – data-based methods, omitting the intermediate plant identification step (Hjalmarsson, 2002; Soma *et al*., 2004). The direct approach seems more natural as real input-output (I/O) data of a plant include fruitful information about the dynamics of the system more directly than mathematical models obtained in system identification. Therefore, it is expected that such direct approaches provide effective controllers reflecting the dynamics of the plant (Kaneko *et al*., 2005).

One of the recently developed techniques for direct optimization of controller parameters is the *virtual reference feedback tuning* methodology (VRFT). It is based on the idea of constructing a virtual reference signal and of the model reference control (Guardbassi, 2000; Campi *et al*., 2000, 2002). It uses only a single set of experimental data to design a controller with defined structure. In other words, originally it is a one-shot method without the need for iterations nor specific inputs. Consequently, it can be implemented easily.

In this contribution, the VRFT method is utilized in a new way for the iterative approach to controller-tuning. It uses a gradual way of tightening performance specifications in a series of repeated closed-loop experiments to design and fine-tune a controller of defined structure. The paper is divided into 6 main parts. After some background information in this section, the next part introduces VRFT methodology basics. The third section presents the new suggested strategy of iterative learning followed by plant description in the fourth chapter. The fifth part presents results of real-time experiments on the controlled system – coupled drives apparatus and the final section summarizes main points and gives a conclusion.

2. VRFT METHODOLOGY

The basics of the virtual reference feedback tuning methodology (VRFT) presented here are based on the recent works of Campi *et al.* (2000, 2002, 2003).

2.1 Problem formulation

Suppose a *linear* single input – single output plant to be controlled described by a discrete-time rational transfer function $P(z)$. Assume that this transfer function (t.f.) is *unknown* and only a set of I/O data collected during an experiment on the plant is available. The control specifications are given using a reference model $M(z)$ describing a desired t.f. of the closed-loop system outlined in Fig. 1.

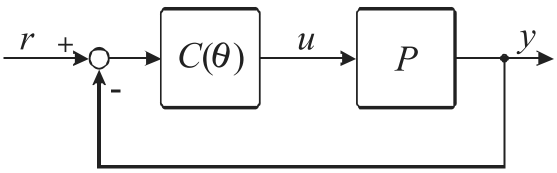

Fig. 1. The control system.

Further, suppose a class of linearly parameterized controllers $\{C(z;\boldsymbol{\theta})\}$ with $C(z;\boldsymbol{\theta}) = \boldsymbol{\beta}^T(z)\boldsymbol{\theta}$, where $\boldsymbol{\beta}(z) = [\beta_1(z) \quad \beta_2(z) \quad \cdots \quad \beta_n(z)]^T$ is a known vector of linear discrete-time t.f. and $\boldsymbol{\theta} = [\theta_1 \quad \theta_2 \quad \cdots \quad \theta_n]^T \in \Re^n$ is n-dimensional vector of parameters to be optimized. Then, the control objective is the minimization of the following model-reference criterion:

$$J_{MR}(\boldsymbol{\theta}) = \left\| \left(\frac{P(z)C(z;\boldsymbol{\theta})}{1+P(z)C(z;\boldsymbol{\theta})} - M(z) \right) W(z) \right\|_2^2 \quad (1)$$

with $W(z)$ - the weighting function chosen by a user. In other words, the goal is to utilize the information contained in the I/O data of the plant to optimize the controller parameters according to the model-reference criterion with both, structure of the controller and the reference model chosen by a user.

2.2 The basic idea

Assume that a controller $C(z;\boldsymbol{\theta})$ implemented in the closed-loop system of Fig. 1 results in a t.f. from r to y equal to $M(z)$. Then, if the closed-lop system is fed by any reference signal $r(t)$, its output is $M(z)r(t)$. Therefore, a necessary condition for the closed-loop system to have the same t.f. as the reference model $M(z)$ is that the output of the systems is the same for a given $\bar{r}(t)$. While standard model-reference design methods start selecting a reference $\bar{r}(t)$, followed by a choice of $C(z;\boldsymbol{\theta})$ so that the condition holds, which is rather difficult without a model of the plant, the VFRT method is based on a *wise* selection of $\bar{r}(t)$ in order to ease determination of the controller (Campi *et al.*, 2002).

Suppose we have two files of data collected from measurements on the plant $\{u(t), y(t)\}$ with no specific restrictions on the data generation. Next, suppose that the plant is not affected by a noise signal, which is discussed later in the paper. Consider a reference signal $\bar{r}(t)$ such that $M(z)\bar{r}(t) = y(t)$ where $M(z)$ is the desired closed-loop response. This reference signal is called *virtual* since it was not used to generate the output $y(t)$. Further, compute the corresponding tracking error $e(t) = \bar{r}(t) - y(t)$ and notice that when the unknown plant $P(z)$ is fed by the actually measured $u(t)$, it generates $y(t)$ as the output. Consequently, a good controller is one that generates $u(t)$ when fed by $e(t)$. Now, the only task is to search for such a controller. As both signals $u(t)$ and $e(t)$ are known quantities, the problem reduces to identification of the relationship between these signals. Given a set of the measured I/O data $\{u(t), y(t)\}_{t=1,\ldots,N}$, the whole algorithm can be implemented using the following 3-step procedure:

1) calculate $\bar{r}(t)$ such that $M(z)\bar{r}(t) = y(t)$ and the corresponding tracking error $e(t) = \bar{r}(t) - y(t)$;

2) filter the signals with a suitable filter $L(z)$:

$$e_L(t) = L(z)e(t), \; u_L(t) = L(z)u(t); \quad (2)$$

3) find the controller parameter vector $\hat{\boldsymbol{\theta}}_{\boldsymbol{N}}$ which minimizes the criterion

$$J_{VR}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) - C(z;\boldsymbol{\theta})e_L(t)\right)^2. \quad (3)$$

This criterion represents a new data-based control cost computable without the knowledge of $P(z)$ and moreover, it is quadratic in $\boldsymbol{\theta}$ which ease the minimization. For a controller in the form $C(z;\boldsymbol{\theta}) = \boldsymbol{\beta}^T(z)\boldsymbol{\theta}$, it can be rewritten as:

$$J_{VR}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{t=1}^{N}\left(u_L(t) - \varphi_L^T(t)\boldsymbol{\theta}\right)^2 \quad (4)$$

with $\varphi_L(t) = \boldsymbol{\beta}(z)e_L(t)$. Then, the optimal parameter vector can be computed using the formula:

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{N}} = \left[\sum_{t=1}^{N}\varphi_L(t)\varphi_L^T(t)\right]^{-1}\sum_{t=1}^{N}\varphi_L(t)u_L(t). \quad (5)$$

Note that the model-reference criterion $J_{MR}(\boldsymbol{\theta})$ (1) has been replaced with a new control cost $J_{VR}^{N}(\boldsymbol{\theta})$ (3), which is an explicit function of the data computable without the knowledge of $P(z)$. In addition, it is quadratic in $\boldsymbol{\theta}$ which makes the minimization easier. Now the task is to ensure that minimum arguments of these two criteria are close to each other. This can be solved by a suitable filtration of the data using the pre-filter $L(z)$ appearing in the eq. (2), which is addressed in the following section.

2.3 Design of the filter

It can be shown (Campi *et al.*, 2000) that the VRFT approach can be used to solve the model-reference control problem stated in the introduction (1) using a suitable selection of the filter $L(z)$ appearing in (2).

Suppose that $C_0(z)$ is the controller that solves exactly the model-reference problem and the number of available data $N \to \infty$. Then, it can be proved (Campi *et al.*, 2002) that if $C_0(z)$ belongs to the controller class $\{C(z;\boldsymbol{\theta})\}$ and both $J_{MR}(\boldsymbol{\theta})$ and $J_{VR}(\boldsymbol{\theta})$ have a unique minimum, minimizing $J_{VR}(\boldsymbol{\theta})$ yields $C_0(z)$ no matter what $L(z)$, $W(z)$, $M(z)$ and $P(z)$ are. In other words, minimum of the $J_{VR}(\boldsymbol{\theta})$ criterion coincides with the minimum of $J_{MR}(\boldsymbol{\theta})$. However, generally $C_0(z) \notin \{C(z;\boldsymbol{\theta})\}$ as it may not be a proper t.f. or it may result in an unstable closed-loop (or it is just too complex). Then, in order to equalize minimum arguments of the two criteria, the filter $L(z)$ needs to be chosen as:

$$|L|^2 = \frac{|M|^2 |W|^2}{|1+PC(\boldsymbol{\theta})|^2} \frac{1}{\Phi_u}, \ \forall \omega \in [-\pi;\pi] \quad (6)$$

where Φ_u is the power spectral density of the signal $u(t)$. Obviously, this choice is not possible because only a set of input-output data from the plant is at our disposal. Moreover, in this equation $L(z)$ depends on the $\boldsymbol{\theta}$ which is to be optimized. Instead, it is suggested (Campi *et al.*, 2002) to choose the filter as

$$|L|^2 = |1-M|^2 |M|^2 |W|^2 \frac{1}{\Phi_u}, \ \forall \omega \in [-\pi;\pi] \ (7)$$

which leads to the substitution of the term $|1+PC(\boldsymbol{\theta})|^2$ in (6) with $|1+PC_0|^2$. This choice seems to be sensible as we expect that $|1+PC(\boldsymbol{\theta})|^2 \approx |1+PC_0|^2$ for $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}}$, where $\bar{\boldsymbol{\theta}}$ is the minimum of $J_{MR}(\boldsymbol{\theta})$.

It can be summarized that if the class of optimized controllers contains the one giving perfect matching between the closed-loop t.f. and $M(z)$, the obtained $C(z;\hat{\boldsymbol{\theta}}_N)$ using the VRFT methodology is the optimal one. Provided that $\{C(z;\boldsymbol{\theta})\}$ is only slightly under-parameterized, which can be the case of restricted complexity controllers, the resultant $C(z;\hat{\boldsymbol{\theta}}_N)$ is *nearly optimal*, representing a good approximation of the optimal controller; for proof, see e.g. the appendix in (Campi *et al.*, 2002).

2.4 Open problems

The concept of an *optimal* filter $L(z)$ presented above is useful when the designer can select the input signal. Then it is not difficult to compute the power spectral density $\Phi_u(\omega)$ appearing in (7). Practically, however, this case is rarely common. Consequently, it has to be estimated using e.g. a high-order AR or ARX model (Ljung, 1999), which becomes more complicated when the plant is operating in the closed-loop.

Next problems can be caused by an additive noise signal affecting the open or closed loop. The noise results in a bias in the resultant controller's parameter vector, which leads to significant performance deterioration. So far, to the authors' knowledge, two approaches solving the task appeared in the literature. Both methods utilize an instrumental variable technique (Ljung, 1999) and differ in the way of constructing the instrumental signal. While the first one requires an additional experiment using the same input sequence, the second approach requires only one set of I/O data but plant identification is necessary. Then, however, the method cannot be stated as *fully direct*, even though the identification is used only for generation of the instrumental variable, not directly linked to the controller design. For details of the approaches, an interested reader is referred e.g. to the paper (Campi *et al.*, 2002).

Another possible problem is stability of the designed loop. Generally, it depends on the choice of the reference model. When chosen inappropriately, it may result in a destabilizing controller. Therefore, a controller validation test should be performed before applying it to the real plant. So far, there are only few works on this subject, e.g. (Campi *et al.*, 2000).

3. LEARNING A CONTROLLER

The goal here is to utilize the VRFT methodology to design and fine-tune the control loop gradually – in the iterative way by a series of closed-loop experiments on a real plant described in the next section. Control specifications are given by the desired closed-loop response $M(z)$ as required by the VRFT method. In the experiments, it is chosen simply as a first-order proportional system, in the continuous-time form expressed as

$$M(s) = \frac{1}{Ts+1} \tag{8}$$

where T is a time-constant controlling speed of the response. In order to make control specifications setting more transparent, a desired settling time t_s together with a required range δ is being set instead. The settling time t_s describes the time required for the controlled variable to first enter and then remain within a band whose width is δ percentage of the total change of $y(t)$. Then, based on a given t_s and δ, the time-constant T of (8) can be derived as

$$T = \frac{-t_s}{\ln\left(\frac{\delta}{100}\right)} \tag{9}$$

Hence, the main tuning parameter is the settling-time t_s with a given range δ. An initial controller for the process was obtained by an open-loop experiment using the VRFT method and the goal was to improve performance of this controller. The procedure of tuning the feedback loop was performed as follows: if the controller response is poor, extend the settling time or change structure of the controller and compute a new one using a new set of measured I/O data; otherwise, try to tighten the performance specifications by shortening the settling time in the next experiments. Using this iterative way of repeating experiments it is possible to find an *optimal* controller for the process. In practice, however, the designed controller should be tested for stabilization of the controlled plant first. This was done by simulation means using a mathematical model of the process based on works of Wellstead and his colleagues (Wellstead, 1979; Hagadoorn and Readman, 2004; Readman and Hagadoorn, 2004).

4. CONTROLLED SYSTEM

The methodology was tested using the CE108 Coupled Drives Apparatus sketched in Figure 2.

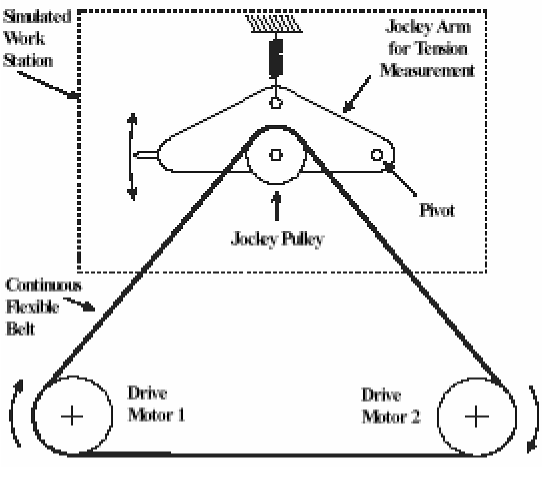

Fig. 2. Scheme of a coupled drives system.

The system relates to industrial material transport problems as they occur in magnetic tape drives, textile machines, paper mills, strip metal production plants, etc. where the material is processed in continuous lengths, it is transported through work stations by drive systems and the material speed and tension have to be controlled within defined limits at all times. The system has two drive motors operating together to control the speed of a continuous flexible belt that goes round pulleys on the drive motor shafts and so called *jockey pulley*. The jockey pulley is mounted on a swinging arm that is supported by a spring. The deflection of the arm is a measure of the tension in the drive belt. The pulley and arm assembly represents a *work station* where material that the belt represents can be processed. The control problem is to regulate the belt speed and tension by varying the motor torques. The CE108 coupled drive apparatus is a product of TecQuipment Inc. and it is designed to have characteristics seen in industrial drives, but it is not any particular industrial application – it is a prototype for all industrial coupled drive applications. Detailed description of the system together with derivation of a mathematical model can be found in the works (Wellstead, 1979; Hagadoorn and Readman, 2004; Readman and Hagadoorn, 2004). From the control theory point of view, the system is a multivariable plant with two inputs (power supply into the two drive motors) and two outputs (pulley speed and belt tension). There is a strong coupling between the inputs and outputs given by the fact that both motors change both outputs (due to the drive belt). In addition, the system behaves quite well only when speed of both motors is relatively close to each other. When there is a significant difference between torques of the motors, the system starts to oscillate (the belt slips) and becomes nearly unstable. For static properties and more details, see e.g. (Gazdoš and Dostál, 2005).

5. EXPERIMENTS

The experiments were performed using the plant described above and the goal was to control speed of the pulley (speed of the belt), which was realized by varying torques of both drive motors simultaneously. A sampling time T_0 was set to 0.1 sec. and initial structure of the designed controller with two tuning parameters ($n = 2$) was chosen, i.e. $\boldsymbol{\theta} = [q_0 \quad q_1]$, resulting in a PI controller of the discrete form:

$$C(z;\boldsymbol{\theta}) = \frac{q_0 + q_1 z^{-1}}{1 - z^{-1}} \tag{10}$$

The desired closed-loop response was controlled by the settling time t_s and range δ as suggested in sec. 3 with the initial values: $t_s = 5\,\text{sec}$, $\delta = 2.5\,\%$. For controller design, the VRFT methodology was utilized exactly as described in section 2. Here, the *optimal* setting of the filter (7) was used together with the instrumental variable technique to cope with the measurement noise, as discussed in 2.4. The algorithm was realized with the help of the *VRFT toolbox*, a free product designed by Prof. Campi and Prof. Savaresi, downloadable from the source: http://bsing.ing.unibs.it/~campi/VRFTwebsite.

5.1 First series of experiments

The first experiment was an open-loop one, however, this need not to be a rule. A response of the plant to a series of step changes of the motors power supply was measured and used for the controller design as outlined above. The resultant controller parameter vector was of the form: $\boldsymbol{\theta} = [0.7481 \quad -0.609]$. A closed-loop response of this controller compared to the desired one is outlined in Figure 3.

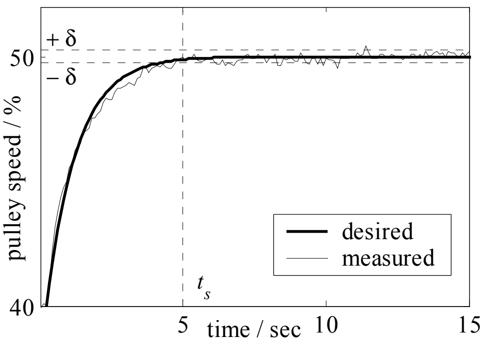

Fig. 3. Closed-loop response for 1st experiment ($n = 2$, $t_s = 5$).

From the graph above it is clear that the feedback loop satisfies the prescribed behaviour – the controlled variable reaches the prescribed range $\pm\delta$ of the reference signal in the given settling time t_s and then stays within. Therefore, it was suggested to try shortening of the settling time in the next experiment to the half of the previous value, i.e. $t_s = 2.5\,\text{sec}$. The measured data from the 1st experiment were used for the new controller design with the resultant vector of controller parameters $\boldsymbol{\theta} = [0.7685 \quad -0.601]$, and the response of Figure 4.

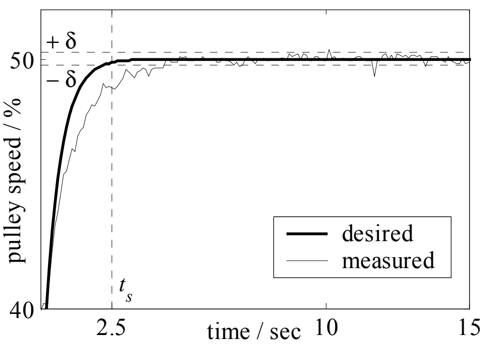

Fig. 4. Closed-loop response for 2nd experiment ($n = 2$, $t_s = 2.5$).

The graph shows that in this case, the controller was not able to fulfill the given requirements – the controlled output neither tracks the reference model nor reaches the desired value in the prescribed settling time. Therefore, the settling time was extended a little bit and the procedure continued this way of repeating experiments to fine-tune the feedback loop. Finally, after 5 iterations, t_s was tuned to its *optimal value* $t_s = 4\,\text{sec}$ with controller parameters $\boldsymbol{\theta} = [0.7406 \quad -0.5767]$. A response of this controller is presented in the next graph, Figure 5. This controller setting represents a satisfactory trade-off between the closed-loop reference-model tracking and a relatively short settling time.

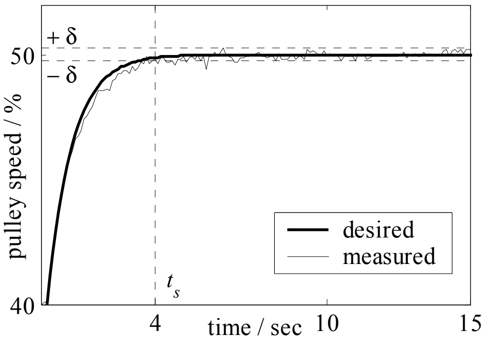

Fig. 5. Closed-loop response for 5th experiment ($n = 2$, $t_s = 4$).

5.2 Second series of experiments

Next experiments were performed with a different controller, having 3 tuning parameters and a discrete transfer function of the form:

$$C(z;\boldsymbol{\theta}) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}}. \qquad (11)$$

Hence, the controller was of the classical PID structure with the optimized vector of parameters $\boldsymbol{\theta} = [q_0 \quad q_1 \quad q_2]$. The procedure was similar to the one of tuning the PI-controller: first, longer settling-time was prescribed, resulting in a safer controller, followed by tightening the performance specifications in order to find an *optimal* setting. The resultant response of the *optimal* controller, obtained after 4 iterations with the parameter vector $\boldsymbol{\theta} = [0.572 \quad -0.3268 \quad -0.0796]$, is shown in Fig. 6.

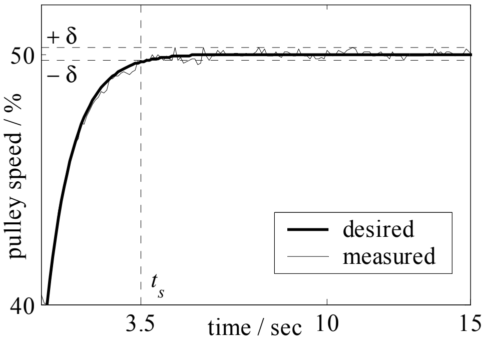

Fig. 6. Closed-loop response for 4th experiment ($n = 3$, $t_s = 3.5$).

From the figure, it can be seen that this controller with one more tuning parameter reaches similar results faster, with the settling time $t_s = 3.5$ sec .

A series of extra experiments was performed with more complex controllers. In this limited space, it can just be added that, e.g. a controller with 4 tuning parameters achieved similar results to the previous simpler *optimal* controllers even faster, with approx. $t_s = 3$ sec and also with better tracking of the desired closed-loop response.

6. CONCLUSION

This contribution was focused on utilization of a recently developed methodology called *virtual reference feedback tuning*. The method is a direct one, i.e. it uses only a set of measured input/output data for design of a controller with desired structure, with no restriction on the data generation. In this paper, the technique was utilized in a new way for the iterative approach to controller design and tuning. The control specifications were assigned simply by a desired settling-time of the closed-loop response, resulting in an appropriate reference model used by the VRFT algorithm. It was shown, by a series of experimental results, that the suggested approach can be successfully applied for controller tuning of a non-linear plant in a given operating point. The way of repeated closed-loop experiments in fully working conditions of the plant allows finding an *optimal* controller for the system gradually, by slowly tightening the performance specifications, which should be safer. However, in order to apply designed controllers safely, there should be a closed-loop stability validation test before the implementation. In this work, this was realized by simulation means only, as discussed at the end of section 3.

In conclusion, it can be stated that if the settling-time is adjusted reasonably, with a longer interval at first, and then followed by its gradual shortening, the presented approach seems to be a relatively safe way of tuning a controller for a given process.

ACKNOWLEDGEMENTS

This work was supported by the Grant Agency of the Czech Republic under the grant no. 102/07/P148 and by the Ministry of Education of the Czech Republic under the grant no. MSM 7088352101.

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