

PREScribed-TIME FIXED-TIME INTEGRAL SLIDING MODE CONTROL OF MAGNETIC LEVITATION SYSTEMS UNDER MATCHED AND MISMATCHED UNCERTAINTIES

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Abstract

This paper proposes a prescribed-time fixed-time integral sliding mode controller for a nonlinear magnetic levitation system subject to matched and mismatched uncertainties. The proposed method combines nonlinear feedback linearization, an integral sliding manifold, and a fixed-time reaching law with an explicit designer-adjustable upper convergence bound. Compared with conventional proportional-integral sliding mode control, the proposed reaching law employs two homogeneous nonlinear terms to accelerate convergence both far from and near the sliding manifold. Compared with discrete quasi-sliding approaches, the continuous-time formulation preserves exact sliding motion in the ideal case, while a saturation-based implementation is introduced to attenuate chattering. New lemmas and theorems are developed to prove fixed-time convergence of the sliding variable, exponential stability of the nominal sliding dynamics, and input-to-state stability under bounded mismatched disturbances. Comparative simulations against feedback linearization control, proportional-integral sliding mode control, adaptive fast terminal sliding mode control, fractional-order sliding mode control, and multirate output-feedback discrete-time sliding mode control demonstrate faster convergence, lower tracking error, smaller control energy, and improved robustness.

Key words

Magnetic levitation system, prescribed-time convergence, fixed-time control, integral sliding mode control, mismatched uncertainty, nonlinear feedback linearization.

1 Introduction

Magnetic levitation systems are representative nonlinear electromechanical systems with important applications in high-speed trains, magnetic bearings, precision positioning, vibration isolation, and contactless transportation. Their main advantages include contact-free motion, low mechanical wear, reduced maintenance, and the absence of lubrication. However, the control of magnetic levitation systems remains challenging because the plant is open-loop unstable, strongly nonlinear, sensitive to parameter variations, and affected by external disturbances. In particular, the uncertainties may enter not only through the control channel but also through other state channels, producing both matched and mismatched disturbance effects [Vernekar and Bandal, 2021].

Recent studies show that magnetic levitation control has moved from classical linearized proportional-integral-derivative and state-feedback methods toward robust nonlinear, finite-time, fixed-time, prescribed-performance, disturbance-observer-based, fuzzy, neural-network, and model-predictive control approaches. Zhu et al. [Zhu et al., 2024] reviewed levitation control methods for low- and medium-speed maglev systems and emphasized that open-loop instability, nonlinear dynamics, and operating uncertainties remain major difficulties in practical maglev control. Vo et al. [Vo et al., 2024] developed a fixed-time sliding mode controller with prescribed performance and anti-saturation compensation for uncertain magnetic levitation systems. Jastrzebski and Kabzinski [Jastrzebski and Kabzinski, 2024] proposed an adaptive fuzzy-inversion control method and verified its effectiveness on a digital signal processor platform. Han et al. [Han et al., 2024] introduced a disturbance-rejection tube model predictive levitation controller for maglev trains under state and input constraints.

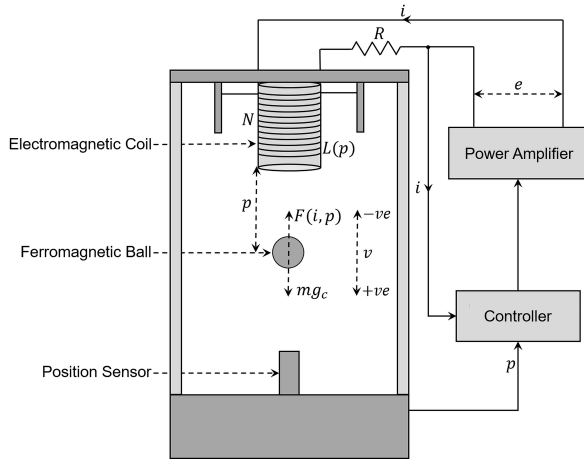


Figure 1. Schematic of the magnetic levitation system.

Sliding mode control remains attractive for magnetic levitation systems because of its robustness against parameter uncertainty and external disturbance. Nevertheless, conventional sliding mode control is often accompanied by high-frequency chattering, current ripple, actuator wear, and sensitivity to measurement noise. To address these limitations, several recent methods have been reported. Hu et al. [Hu et al., 2025] proposed an improved extended-state-observer-based global fast terminal integral sliding mode controller for strongly nonlinear maglev systems. Zhang et al. [Zhang et al., 2025] developed a neural-network-based adaptive super-twisting sliding mode controller for hybrid magnetic levitation systems. Chen et al. [Chen et al., 2025] proposed a fixed-time observer-based sliding mode tracking controller that estimates unmeasurable states and compensates uncertainties. Lan et al. [Lan et al., 2025] further studied finite-time position tracking control for magnetic levitation systems with suspension air-gap constraints.

Vernekar and Bandal [Vernekar and Bandal, 2021] proposed continuous-time and discrete-time sliding mode controllers for a magnetic levitation system with matched and mismatched uncertainties. Their continuous proportional-integral sliding mode controller improves robustness but suffers from chattering. Their state-feedback and multirate output-feedback discrete-time controllers reduce chattering through quasi-sliding motion, but this introduces a finite quasi-sliding band and weakens the exact invariance property of ideal continuous sliding mode control. Therefore, a controller that simultaneously provides integral steady-state regulation, designer-adjustable fixed-time convergence, robustness against matched and mismatched uncertainties, and a transparent stability proof remains desirable.

Related contributions recently published in Cybernetics and Physics also emphasize cybernetic regulation of physical systems under uncertainty and constraints. Imangazieva [Imangazieva, 2024] considered robust control of nonlinear systems affected by dis-

tributed delay and disturbances. Nguyen et al. [Nguyen et al., 2025] addressed guaranteed-bound speed regulation of a direct-current motor, providing a physically motivated electromechanical control application. Sharma et al. [Sharma et al., 2025] investigated trajectory control of a quadcopter in the presence of disturbances and parametric uncertainties. These studies reinforce the relevance of robust nonlinear control to physical systems and motivate the present investigation of unstable electromechanical suspension dynamics.

Motivated by these observations, this paper proposes a prescribed-time fixed-time integral sliding mode controller for the nonlinear magnetic levitation system. The proposed controller differs from existing proportional-integral sliding mode and discrete quasi-sliding approaches in three main aspects. First, a fixed-time reaching law with two homogeneous nonlinear terms is introduced, allowing the upper bound of the reaching time to be explicitly adjusted by the designer. Second, an integral sliding manifold is constructed so that the reduced-order sliding dynamics are governed by a Hurwitz polynomial, which provides clear gain-selection conditions. Third, a saturation-based boundary-layer implementation is introduced to attenuate chattering while preserving practical stability.

The main contributions of this paper are summarized as follows. First, a prescribed-time fixed-time integral sliding mode controller is developed for a feedback-linearized magnetic levitation model subject to matched and mismatched disturbances. Second, new lemmas and theorems are established to prove fixed-time reachability of the sliding variable, exponential stability of the nominal sliding dynamics, and input-to-state stability under bounded mismatched disturbances. Third, comparative simulations are conducted against feedback linearization control, proportional-integral sliding mode control, adaptive fast terminal sliding mode control, fractional-order sliding mode control, and multirate output-feedback discrete-time sliding mode control to verify the advantages of the proposed method in terms of tracking accuracy, settling time, control energy, and robustness.

The remainder of this paper is organized as follows. Section 2 presents the nonlinear magnetic levitation model and the feedback-linearized uncertain system. Section 3 develops the proposed controller. Section 4 gives the stability analysis. Section 5 presents comparative simulation results. Finally, Section 6 concludes the paper.

2 Magnetic Levitation System Model

The magnetic levitation system consists of an electromagnet, a ferromagnetic ball, a position sensor, a power amplifier, and a feedback controller, as shown in Fig. 1.

Following the magnetic levitation model in [Vernekar and Bandal, 2021], the nonlinear electromechanical dy-

namics are described as

$$\begin{aligned} \frac{dp(t)}{dt} &= v(t), \\ Ri(t) + \frac{d(L(p(t))i(t))}{dt} &= e(t), \\ m \frac{dv(t)}{dt} &= mg_c - F(i(t), p(t)) \end{aligned} \quad (1)$$

where $p(t)$ is the ball position, $v(t)$ is the ball velocity, $i(t)$ is the coil current, $e(t)$ is the input voltage, R is the coil resistance, $L(p(t))$ is the position-dependent coil inductance, m is the ball mass, g_c is the gravitational acceleration, and $F(i(t), p(t))$ is the electromagnetic attractive force.

The magnetic force and inductance are

$$\begin{aligned} F(i(t), p(t)) &= Q \left(\frac{i(t)}{p(t)} \right)^2, \quad Q = \frac{\mu_0 AN^2}{4}, \\ L(p(t)) &= L_1 + \frac{2Q}{p(t)} \end{aligned} \quad (2)$$

where Q is the magnetic force constant, μ_0 is the vacuum permeability, A is the effective magnetic area, N is the coil turn number, and L_1 is the constant component of the coil inductance.

Define the state vector and input as

$$\begin{aligned} x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} p(t) \\ v(t) \\ i(t) \end{bmatrix}, \\ u(t) &= e(t), \quad y(t) = x_1(t) \end{aligned} \quad (3)$$

where $x_1(t)$ is the ball position, $x_2(t)$ is the ball velocity, $x_3(t)$ is the coil current, $u(t)$ is the voltage input, and $y(t)$ is the measured output.

Using the state definitions in (3), the uncertain nonlinear state-space representation can be written as [Vernekar and Bandal, 2021]

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) + d(x(t), t), \\ y(t) &= h(x(t)) \end{aligned} \quad (4)$$

where $f(x(t))$ is the drift vector field, $g(x(t))$ is the input vector field, $d(x(t), t)$ is the total uncertainty vector, and $h(x(t)) = x_1(t)$ is the output map.

The functions in (4) are

$$\begin{aligned} f(x) &= \begin{bmatrix} x_2 \\ g_c - \frac{Q}{m} \left(\frac{x_3}{x_1} \right)^2 \\ -\frac{R}{L(x_1)} x_3 + \frac{2Q}{L(x_1)} \left(\frac{x_2 x_3}{x_1^2} \right) \end{bmatrix} \\ g(x) &= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L(x_1)} \end{bmatrix}, \quad d(x, t) = \begin{bmatrix} d_1(x, t) \\ d_2(x, t) \\ d_3(x, t) \end{bmatrix} \end{aligned} \quad (5)$$

As discussed in [Vernekar and Bandal, 2021], the disturbance vector contains both matched and mismatched

components. Here, $d_1(x, t)$ and $d_2(x, t)$ are mismatched disturbances because they do not enter through the control channel, while $d_3(x, t)$ is a matched disturbance because it enters the current dynamics through the same channel as the input.

The desired equilibrium is

$$x_d = \begin{bmatrix} x_{1d} \\ 0 \\ x_{3d} \end{bmatrix}, \quad x_{3d} = x_{1d} \sqrt{\frac{mg_c}{Q}} \quad (6)$$

where x_{1d} is the desired levitation height, 0 is the desired ball velocity, and x_{3d} is the steady-state coil current required to balance gravity.

Assumptions:

A1 (Smoothness and feasibility). The functions $f(x)$, $g(x)$, and $h(x)$ are smooth on a compact domain

$$\Omega_x = \{x \in \mathbb{R}^3 : x_1 > 0, g_c - z_3 > 0, |\beta(z)| \geq \beta_{\min} > 0\} \quad (7)$$

where Ω_x is the admissible state domain and $\beta_{\min}$ is a positive constant lower bound on $|\beta(z)|$. This assumption avoids singularity in the feedback linearization.

A2 (Disturbance boundedness). There exist known positive constants D_1 , D_2 , and D_3 such that

$$|d_1(x, t)| \leq D_1, |d_2(x, t)| \leq D_2, |d_3(x, t)| \leq D_3 \quad (8)$$

where D_1 , D_2 , and D_3 are the disturbance bounds for the first, second, and third transformed channels.

A3 (Mismatched-disturbance regularity). The mismatched disturbances $d_1(x, t)$ and $d_2(x, t)$ are bounded and piecewise continuous. They are not required to vanish asymptotically. Under persistent bounded mismatched disturbances, the reduced-order sliding dynamics remain input-to-state stable as established in Theorem 3. Exact asymptotic convergence to the origin is recovered under the additional condition

$$\lim_{t \rightarrow \infty} d_1(z, t) = 0, \quad \lim_{t \rightarrow \infty} d_2(z, t) = 0 \quad (9)$$

A4 (Initial condition). The initial state $x(0)$ belongs to Ω_x , and the desired position x_{1d} satisfies $x_{1d} > 0$.

Physical interpretation of the assumptions. The controlled variable x_1 represents the physical air gap between the electromagnet and the levitated ball, while x_3 is the coil current generating the attractive force $F = Q(i/p)^2$. Consequently, the condition $x_1 > 0$ ensures a physically admissible positive air gap, whereas $g_c - z_3 > 0$ guarantees a real-valued reconstruction of the electromagnetic current from the transformed coordinates. The condition $|\beta(z)| \geq \beta_{\min} > 0$ expresses nonvanishing electromechanical control authority, i.e., a finite variation of coil voltage must retain the ability

to modify the magnetic acceleration. The bounded disturbances represent mechanical load variations, electrical/model uncertainties, and measurement-induced perturbations within the operating region. Thus, the contribution does not introduce a new electromagnetic law; rather, it establishes how the nonlinear electromagnetic force-gap coupling and available actuator authority can be systematically exploited to achieve robust fixed-time regulation of an intrinsically unstable physical suspension system.

3 Proposed Prescribed-Time Fixed-Time Integral Sliding Mode Controller

To facilitate controller design, the nonlinear magnetic levitation dynamics are transformed into a feedback-linearized form, as commonly used for this class of systems [Vernekar and Bandal, 2021]. Define the transformed state

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - x_{1d} \\ x_2(t) \\ g_c - \frac{Q}{m} \left(\frac{x_3(t)}{x_1(t)} \right)^2 \end{bmatrix} \quad (10)$$

where $z_1(t)$ is the position error, $z_2(t)$ is the velocity error, and $z_3(t)$ is the transformed acceleration-related state.

Using (10), the system can be represented as

$$\begin{aligned} \dot{z}_1(t) &= z_2(t) + d_1(z, t) \\ \dot{z}_2(t) &= z_3(t) + d_2(z, t) \\ \dot{z}_3(t) &= \alpha(z(t)) + \beta(z(t)) u(t) + d_3(z, t) \end{aligned} \quad (11)$$

where $d_i(z, t)$, $i = 1, 2, 3$ are the transformed disturbances, $\alpha(z(t))$ and $\beta(z(t))$ are nonlinear functions produced by the coordinate transformation.

The nonlinear functions are

$$\begin{aligned} \alpha(z) &= 2(g_c - z_3) \left(\left(1 - \frac{2Q}{L(z_1 + x_{1d})} \right) \frac{z_2}{z_1 + x_{1d}} + \frac{1}{L(z_1 + x_{1d})} \right), \\ \beta(z) &= -\frac{2}{L(z_1 + x_{1d})} \sqrt{\frac{Q}{m}} (g_c - z_3) \end{aligned} \quad (12)$$

where $L(z_1 + x_{1d})$ is the coil inductance evaluated at the actual position $x_1 = z_1 + x_{1d}$.

Inspired by integral sliding mode control, which improves robustness and steady-state regulation in uncertain systems [Shtessel et al., 2014; Vernekar and Bandal, 2021], the sliding variable is defined as

$$s(t) = z_3(t) + c_2 z_2(t) + c_1 z_1(t) + c_0 r(t) \quad (13)$$

where $s(t)$ is the integral sliding variable, and c_0 , c_1 , and c_2 are positive sliding-surface gains. The gains are chosen $c_0 > 0$, $c_1 > 0$, $c_2 > 0$, $c_1 c_2 > c_0$ that guarantee the sliding dynamics are Hurwitz stable.

Define the integral state

$$r(t) = \int_0^t z_1(\tau) d\tau \quad (14)$$

where $r(t)$ is the accumulated position error and τ is the integration variable.

Motivated by fixed-time stability theory [Polyakov, 2012] and robust sliding mode design [Shtessel et al., 2014], the proposed inner-loop controller is selected as

$$\begin{aligned} w(t) &= -c_0 z_1(t) - c_1 z_2(t) - c_2 z_3(t) \\ &\quad - a|s(t)|^p \text{sgn}(s(t)) - b|s(t)|^q \text{sgn}(s(t)) \\ &\quad - K \text{sgn}(s(t)) \end{aligned} \quad (15)$$

where $a > 0$ and $b > 0$ are fixed-time reaching gains, p and q are homogeneous exponents satisfying $0 < p < 1 < q$, $K > 0$ is the robust switching gain, and $\text{sgn}(\cdot)$ is the sign function.

Let the outer-loop voltage be

$$u(t) = \beta^{-1}(z(t)) (-\alpha(z(t)) + w(t)) \quad (16)$$

Differentiating the sliding manifold (13) and using (11) give

$$\dot{s} = w + c_2 z_3 + c_1 z_2 + c_0 z_1 + \Delta(t) \quad (17)$$

where $\Delta(t) = d_3 + c_2 d_2 + c_1 d_1$ is the lumped disturbance term.

Substituting the proposed inner-loop controller (15) into the above equation yields

$$\begin{aligned} \dot{s} &= -a|s|^p \text{sgn}(s) - b|s|^q \text{sgn}(s) \\ &\quad - K \text{sgn}(s) + \Delta(t) \end{aligned} \quad (18)$$

Suppose Assumption A2 holds. Then the lumped disturbance satisfies

$$|\Delta(t)| \leq \bar{\Delta} \quad (19)$$

where $\bar{\Delta} = c_1 D_1 + c_2 D_2 + D_3$

Substituting (16) into (11) gives

$$\dot{z}(t) = Az(t) + Bw(t) + d(z, t) \quad (20)$$

where $A = [0, 1, 0; 0, 0, 1; 0, 0, 0]$, $B = [0, 0, 1]^T$, $d(z, t) = [d_1(x, t), d_2(x, t), d_3(x, t)]^T$.

In (16) and (20), A is the Brunovsky chain matrix, B is the input matrix, and $d(z, t)$ is the transformed uncertainty vector.

In the numerical implementation, $\text{sgn}(s)$ in the switching terms of (15) is replaced by $\text{sat}_\phi(s)$ [Shtessel et al., 2014]

$$\text{sat}_\phi(s) = \begin{cases} \text{sgn}(s), & |s| > \phi, \\ \frac{s}{\phi}, & |s| \leq \phi, \end{cases} \quad (21)$$

where $\text{sat}_\phi(s)$ is the saturation function and $\phi > 0$ is the boundary-layer thickness.

The boundary-layer thickness ϕ determines the compromise between tracking precision and switching attenuation. A smaller ϕ provides a closer approximation to the ideal sign function and a narrower quasi-sliding region, but increases sensitivity to measurement noise and high-frequency switching. Conversely, a larger ϕ produces a smoother control signal at the expense of a wider steady-state neighborhood around the sliding manifold. In practice, ϕ should be selected slightly above the equivalent sliding-variable variation induced by measurement noise while satisfying the prescribed steady-state tracking tolerance. With the boundary-layer implementation, this result is interpreted as practical convergence to a neighborhood of the sliding manifold. In the simulations, $\phi = 5 \times 10^{-4}$ is adopted to provide this compromise.

4 Stability Analysis

The stability analysis is carried out in three steps. First, the fixed-time convergence of the sliding variable is established. Second, the stability of the nominal reduced-order sliding dynamics is examined. Third, input-to-state stability under bounded mismatched disturbances is established. These results show that the proposed controller guarantees fixed-time reachability and asymptotic stabilization of the transformed magnetic levitation system.

The following lemma recalls a standard scalar fixed-time convergence result, which follows from finite-time Lyapunov arguments and fixed-time stability theory [Bhat and Bernstein, 2000; Polyakov, 2012].

Lemma 1 (Fixed-Time Scalar Convergence). Consider a nonnegative scalar $V(t)$ satisfying

$$\dot{V}(t) \leq -aV^p(t) - bV^q(t) \quad (22)$$

where $a > 0$, $b > 0$, $0 < p < 1$, and $q > 1$. Then $V(t)$ converges to zero within

$$T_{\max} \leq \frac{1}{a(1-p)} + \frac{1}{b(q-1)} \quad (23)$$

Based on Lemma 1 and the fixed-time reaching-law structure in [Polyakov, 2012], the next theorem proves that the proposed controller drives the sliding variable to zero within an upper time bound independent of the initial condition.

Theorem 1 (Prescribed-Time Fixed-Time Convergence). Consider the uncertain magnetic levitation system (20) controlled by (16).

Suppose that the robust gain is selected as

$$K > \bar{\Delta} \quad (24)$$

Then the sliding variable $s(t)$ converges to zero within a fixed upper bound independent of the initial condition.

Proof.

Choose the Lyapunov candidate

$$V_s(t) = \frac{1}{2}s^2 \quad (25)$$

Differentiating along system trajectories gives

$$\dot{V}_s(s) = s\dot{s} \quad (26)$$

Substituting (18) into (26) and using $|s\Delta| \leq |s||\Delta| \leq \bar{\Delta}|s|$. It obtains

$$\dot{V}_s \leq -a|s|^{p+1} - b|s|^{q+1} - (K - \bar{\Delta})|s| \quad (27)$$

From (24) and using the relation (25), equation (27) becomes

$$\dot{V}_s \leq -a_1 V_s^\mu - b_1 V_s^\nu \quad (28)$$

where $0 < \mu = \frac{p+1}{2} < 1$, $\nu = \frac{q+1}{2} > 1$, $a_1 = 2^\mu a$, $b_1 = 2^\nu b$.

From Lemma 1, $s(t)$ reaches zero within a prescribed upper bound independent of the initial condition

$$T_s \leq \frac{1}{a_1(1-\mu)} + \frac{1}{b_1(\nu-1)} \quad (29)$$

After the reaching phase, the closed-loop behavior is determined by the sliding-manifold dynamics. The stability of these reduced-order dynamics is analyzed using the Hurwitz polynomial condition, which is standard in linear systems theory.

Theorem 2 (Stability of Sliding Motion). Assume the sliding condition $s(t) = 0$ is achieved and that the mismatched disturbances vanish $d_1(z, t) = 0$, $d_2(z, t) = 0$. Then the reduced-order dynamics are exponentially stable if the polynomial

$$\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 \quad (30)$$

is Hurwitz.

Proof.

When $s(t) = 0$, the sliding manifold (13) gives

$$z_3(t) + c_2z_2(t) + c_1z_1(t) + c_0r(t) = 0 \quad (31)$$

Therefore,

$$\dot{z}_3 + c_2\dot{z}_2 + c_1\dot{z}_1 + c_0\dot{r} = 0 \quad (32)$$

For the nominal sliding dynamics, $d_1(z, t) = 0$ and $d_2(z, t) = 0$. Hence, $\dot{z}_1 = z_2$, $\dot{z}_2 = z_3$, $\dot{r} = z_1$, equation (32) becomes

$$\ddot{z}_1 + c_2\ddot{z}_1 + c_1\dot{z}_1 + c_0z_1 = 0 \quad (33)$$

The characteristic polynomial is therefore

$$\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0 \quad (34)$$

If this polynomial is Hurwitz, then all roots lie in the open left-half complex plane. Consequently,

$$\lim_{t \rightarrow \infty} z_1(t) = 0, \lim_{t \rightarrow \infty} z_2(t) = 0, \lim_{t \rightarrow \infty} z_3(t) = 0 \quad (35)$$

After the exponential stability of the nominal sliding motion has been established, it remains necessary to analyze the effect of bounded mismatched disturbances. In practical magnetic levitation systems, the disturbances $d_1(z, t)$ and $d_2(z, t)$ may not vanish completely. Therefore, the following theorem proves that the reduced-order sliding dynamics are input-to-state stable with respect to the mismatched disturbance vector.

Theorem 3 (Input-to-State Stability Under Bounded Mismatched Disturbances). Assume that the sliding condition $s(t) = 0$ is achieved for $t \geq T_s$, and suppose that the polynomial (30) is Hurwitz. If the mismatched disturbances $d_1(z, t)$ and $d_2(z, t)$ are bounded, there exist positive constants M , ρ , and γ such that

$$\begin{aligned} \|\chi(t)\| &\leq M e^{-\rho(t-T_s)} \|\chi(T_s)\| \\ &+ \gamma \sup_{T_s \leq \tau \leq t} \|d_m(\tau)\|, t \geq T_s \end{aligned} \quad (36)$$

where $d_m(t) = [d_1(z, t), d_2(z, t)]^T$, $\chi(t) = [r(t), z_1(t), z_2(t)]^T$.

Consequently, bounded mismatched disturbances imply bounded reduced-order sliding states. Moreover, if $\lim_{t \rightarrow \infty} d_m(t) = 0$, then

$$\lim_{t \rightarrow \infty} \chi(t) = 0 \quad (37)$$

Proof.

When $s(t) = 0$, from (13)

$$z_3(t) = -c_0 r(t) - c_1 z_1(t) - c_2 z_2(t) \quad (38)$$

From the transformed magnetic levitation model (11) and (14), one obtains

$$\begin{aligned} \dot{r}(t) &= z_1(t), \\ \dot{z}_1(t) &= z_2(t) + d_1(z, t), \\ \dot{z}_2(t) &= z_3(t) + d_2(z, t). \end{aligned} \quad (39)$$

Therefore,

$$\dot{z}_2(t) = -c_0 r(t) - c_1 z_1(t) - c_2 z_2(t) + d_2(z, t) \quad (40)$$

Then, the reduced-order sliding dynamics can be written as

$$\dot{\chi}(t) = A_s \chi(t) + E_s d_m(t) \quad (41)$$

$$\text{where } A_s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c_0 & -c_1 & -c_2 \end{bmatrix}, E_s = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The characteristic polynomial of A_s is

$$\det(\lambda I - A_s) = \lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 \quad (42)$$

By assumption, the polynomial in equation (42) is Hurwitz. Therefore, A_s is Hurwitz. Hence, for any symmetric positive-definite matrix $Q_s = Q_s^T > 0$, there exists a symmetric positive-definite matrix $P_s = P_s^T > 0$ satisfying the Lyapunov equation

$$A_s^T P_s + P_s A_s = -Q_s \quad (43)$$

Choose the Lyapunov function candidate

$$V_\chi(t) = \chi^T(t) P_s \chi(t) \quad (44)$$

Since $P_s > 0$, the following inequalities hold

$$\lambda_{\min}(P_s) \|\chi(t)\|^2 \leq V_\chi(t) \leq \lambda_{\max}(P_s) \|\chi(t)\|^2 \quad (45)$$

where $\lambda_{\min}(P_s)$ and $\lambda_{\max}(P_s)$ denote the minimum and maximum eigenvalues of P_s , respectively.

Differentiating equation (45) along equation (41) gives

$$\dot{V}_\chi = \dot{\chi}^T P_s \chi + \chi^T P_s \dot{\chi} \quad (46)$$

Substituting equation (41) into equation (46), one obtains

$$\dot{V}_\chi = \chi^T (A_s^T P_s + P_s A_s) \chi + 2\chi^T P_s E_s d_m \quad (47)$$

Using equation (43), equation (47) becomes

$$\dot{V}_\chi = -\chi^T Q_s \chi + 2\chi^T P_s E_s d_m \quad (48)$$

Applying the norm inequality gives

$$\dot{V}_\chi \leq -\lambda_{\min}(Q_s) \|\chi\|^2 + 2\|P_s E_s\| \|\chi\| \|d_m\| \quad (49)$$

Using Young's inequality,

$$\begin{aligned} 2\|P_s E_s\| \|\chi\| \|d_m\| &\leq \frac{\lambda_{\min}(Q_s)}{2} \|\chi\|^2 \\ &+ \frac{2\|P_s E_s\|^2}{\lambda_{\min}(Q_s)} \|d_m\|^2 \end{aligned} \quad (50)$$

Substituting equation (50) into equation (49) yields

$$\dot{V}_\chi \leq -\frac{\lambda_{\min}(Q_s)}{2} \|\chi\|^2 + \frac{2\|P_s E_s\|^2}{\lambda_{\min}(Q_s)} \|d_m\|^2 \quad (51)$$

From equation (45), hence

$$\dot{V}_\chi \leq -\rho_1 V_\chi + \sigma_1 \|d_m\|^2 \quad (52)$$

where $\rho_1 = \frac{\lambda_{\min}(Q_s)}{2\lambda_{\max}(P_s)}$, $\sigma_1 = \frac{2\|P_s E_s\|^2}{\lambda_{\min}(Q_s)}$.

Solving the differential inequality in equation (52) over the interval $[T_s, t]$ gives

$$V_\chi(t) \leq e^{-\rho_1(t-T_s)} V_\chi(T_s) + \frac{\sigma_1}{\rho_1} \sup_{T_s \leq \tau \leq t} \|d_m(\tau)\|^2 \quad (53)$$

Using equation (45), it follows that

$$\|\chi(t)\| \leq \sqrt{\frac{\lambda_{\max}(P_s)}{\lambda_{\min}(P_s)}} e^{-\frac{\rho_1}{2}(t-T_s)} \|\chi(T_s)\| + \sqrt{\frac{\sigma_1}{\rho_1 \lambda_{\min}(P_s)}} \sup_{T_s \leq \tau \leq t} \|d_m(\tau)\| \quad (54)$$

Therefore, equation (36) holds with $M = \sqrt{\frac{\lambda_{\max}(P_s)}{\lambda_{\min}(P_s)}}$, $\rho = \frac{\rho_1}{2}$, $\gamma = \sqrt{\frac{\sigma_1}{\rho_1 \lambda_{\min}(P_s)}}$.

Equation (54) is an input-to-state stability estimate for the reduced-order sliding dynamics with respect to the mismatched disturbance vector $d_m(t)$. Thus, bounded mismatched disturbances imply bounded reduced-order states. Furthermore, if $\lim_{t \rightarrow \infty} d_m(t) = 0$, then $\lim_{t \rightarrow \infty} \chi(t) = 0$ which implies

$$\lim_{t \rightarrow \infty} z_i(t) = 0, \quad i = 1, 2, 3 \quad (55)$$

Remark 1. The condition $d_m(t) \rightarrow 0$ is required only for exact asymptotic convergence and is not necessary for robustness. For a persistent bounded mismatched disturbance, (36) implies

$$\lim_{t \rightarrow \infty} \sup \|\chi(t)\| \leq \gamma \|d_m\|_\infty \quad (56)$$

showing that the closed-loop sliding dynamics remain ultimately bounded, with a bound proportional to the magnitude of the persistent disturbance.

Theorems 1–3 together show that the proposed controller first enforces fixed-time convergence to the sliding manifold, then guarantees exponential stability of the nominal sliding dynamics, and finally ensures input-to-state stability when bounded mismatched disturbances are present.

5 Simulation Results

To evaluate the effectiveness of the proposed controller, its responses are compared with those of existing benchmark controllers, including feedback linearization control (FBL), continuous proportional-integral sliding mode control (PISMC), adaptive fast terminal sliding mode control (AFTSMC), fractional-order sliding mode control (FOSMC), and multirate output-feedback

discrete-time sliding mode control (MROFDTSMC) [Vernekar and Bandal, 2021; Vo et al., 2024; Shtessel et al., 2014].

The plant parameters are selected according to the magnetic levitation benchmark used in [Vernekar and Bandal, 2021]: coil resistance $R = 28.7(\Omega)$, inductance $L_1 = 0.65(H)$, gravitational acceleration $g_c = 9.81(m/s^2)$, ball mass $m = 11.87 \times 10^{-3}(kg)$, vacuum permeability $\mu_0 = 2.125 \times 10^{-7}(H/m)$, magnetic permeability area $A = 8\pi \times 10^{-4}(m^2)$, coil turns $N = 1024$, and magnetic force constant $Q = 1.4 \times 10^{-4}$.

The desired levitation position was $x_{1d} = 0.01(m)$. The disturbances were selected as exponentially decaying sinusoidal signals

$$d_i(t) = A_i e^{-\lambda t} \sin(\omega_i t), \quad i = 1, 2, 3 \quad (57)$$

where A_i are disturbance amplitudes, $\lambda > 0$ is the decay rate, ω_i are disturbance frequencies.

The proposed controller parameters were selected as $c_0 = 5000$, $c_1 = 1200$, $c_2 = 60$, $a = 300$, $b = 90$, $p = 0.55$, $q = 1.6$, $K = 0.2$, $\phi = 5 \times 10^{-4}$. For the ideal discontinuous reaching law, the theoretical fixed-time upper bound computed from (29) is $T_s \leq 0.0237$ s.

The transient and steady-state performances are evaluated using standard time-domain performance indices.

The integral absolute error is

$$\text{IAE} = \int_0^{T_f} |e_p(t)| dt \quad (58)$$

where $e_p(t) = x_1(t) - x_{1d}$ is the position tracking error.

The integral squared error is

$$\text{ISE} = \int_0^{T_f} e_p^2(t) dt \quad (59)$$

The integral time absolute error is

$$\text{ITAE} = \int_0^{T_f} t |e_p(t)| dt \quad (60)$$

The control energy is

$$J_u = \int_0^{T_f} u^2(t) dt \quad (61)$$

The chattering index is

$$J_{\text{chat}} = \int_0^{T_f} \left| \frac{du(t)}{dt} \right| dt \quad (62)$$

The normalized RMS control input

$$u_{\text{rms}} = \sqrt{\frac{1}{T_f} \int_0^{T_f} u^2(t) dt} = \sqrt{\frac{J_u}{T_f}} \quad (63)$$

Fig. 2 shows the position tracking responses of all compared controllers under the same initial condition and disturbance profile. The feedback linearization controller exhibits the largest transient deviation and the slowest convergence because it does not explicitly compensate for the matched and mismatched uncertainties. The PI-SMC, AFTSMC, FOSMC, and MROFD-TSMC schemes improve the tracking behavior by introducing robust sliding-mode actions; however, their transient responses still contain larger overshoot or slower settling compared with the proposed method. In contrast, the proposed PFxT-ISM drives the ball position rapidly toward the desired levitation height $x_{1d} = 0.01(m)$ with the smallest maximum tracking error. This confirms that the prescribed-time fixed-time reaching law improves both transient convergence and disturbance rejection.

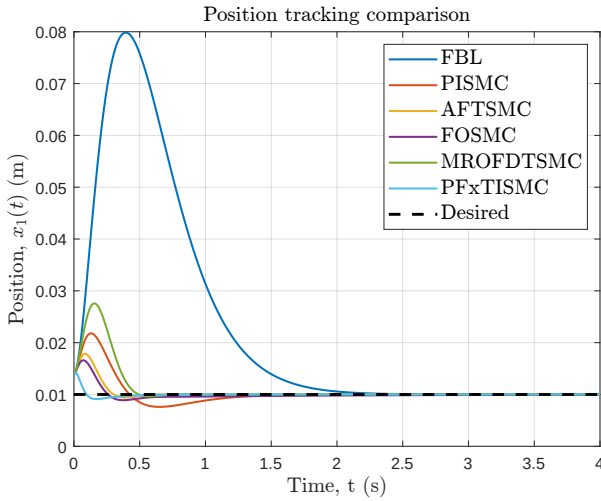


Figure 2. Position tracking $x_1(t)$.

Fig. 3 compares the coil current responses generated by the different controllers. The feedback linearization controller requires a large current peak during the transient stage, which indicates poor robustness and inefficient compensation of nonlinear electromagnetic dynamics. The sliding-mode-based controllers reduce the current excursion significantly. Among them, the proposed PFxT-ISM produces the smoothest and fastest current convergence toward the desired steady-state current. This behavior shows that the proposed controller achieves accurate position regulation without demanding excessive electromagnetic current, which is important for reducing actuator stress and improving practical feasibility.

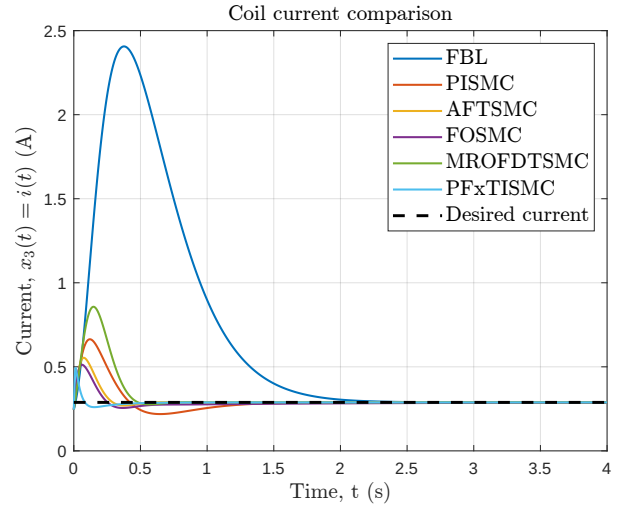


Figure 3. Coil current $x_3(t)$.

Fig. 4 presents the voltage input required by each controller. The feedback linearization method generates the largest voltage peak, reflecting its sensitivity to uncertainty and its inability to suppress transient deviations effectively. The PI-SMC and other benchmark sliding-mode controllers reduce the voltage magnitude but still require noticeable corrective action during the initial transient. The proposed PFxT-ISM maintains a bounded voltage profile while achieving the best tracking performance. Therefore, the proposed method provides a favorable compromise between fast convergence and admissible control effort.

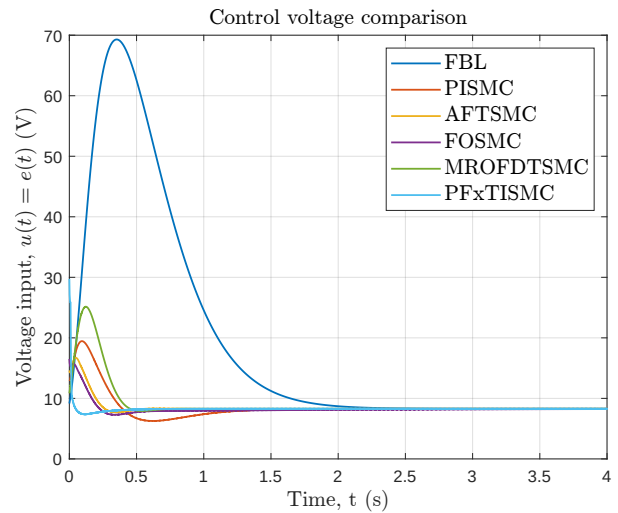


Figure 4. Control voltage $u(t)$.

Fig. 5 illustrates the convergence of the sliding variable for all controllers. The feedback linearization con-

troller has the slowest decay of $s(t)$, while the conventional and benchmark sliding-mode controllers reach the sliding region more rapidly. The proposed PFxT-ISM exhibits the fastest convergence of the sliding variable to zero, which is consistent with the theoretical prescribed-time fixed-time stability result. The rapid decay of $s(t)$ confirms that the proposed reaching law accelerates convergence both far from and near the sliding manifold, thereby improving the transient behavior of the closed-loop system.

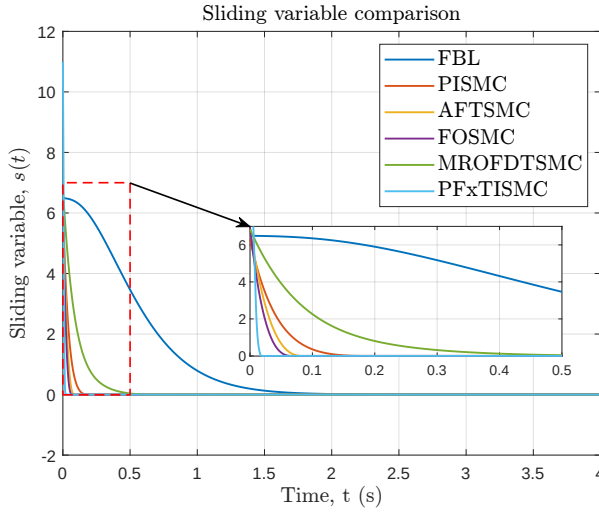


Figure 5. Sliding variable $s(t)$.

Fig. 6 illustrates the convergence of the transformed states z_1 , z_2 , and z_3 under the proposed PFxT-ISM. The three states rapidly approach the origin with small transient oscillations, confirming that the proposed controller stabilizes the feedback-linearized magnetic levitation dynamics. The zoomed response further verifies fast local convergence near the equilibrium, which is consistent with the theoretical fixed-time reaching and sliding-motion stability results.

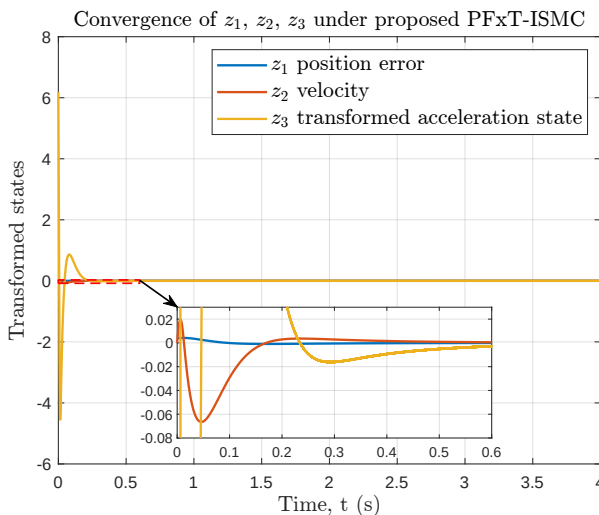


Figure 6. Transformed states z_1, z_2, z_3 under proposed method.

To ensure a normalized comparison, all controllers are evaluated over the same simulation horizon $T_f = 4$ s, and the RMS control input u_{rms} is additionally reported. The quantity J_u is therefore interpreted as a quadratic control-effort index rather than the physical electrical energy consumed by the electromagnetic actuator. Table 1 quantitatively confirms the superiority of the proposed PFxTISM. The feedback linearization controller gives the worst performance, with the largest maximum tracking error $0.069839(m)$, the largest IAE 0.05198 , and the highest control energy 2838.3 . The PI-SMC improves robustness, but its settling time remains relatively long at $1.6093(s)$. The AFTSMC provides fast settling, whereas FOSMC achieves a relatively small maximum error but suffers from the longest settling time. The MROFDTSMC reduces chattering through discrete quasi-sliding motion, but its tracking error remains higher than that of the proposed method. The proposed PFxTISM achieves the smallest maximum tracking error $0.004164(m)$, the lowest IAE 0.00045 , the lowest ISE 8.652×10^{-7} , and the lowest ITAE 7.79×10^{-5} , while also requiring the lowest control energy 278.37 . These results demonstrate that the proposed controller provides the best overall balance among tracking accuracy, transient speed, robustness, and control efficiency.

6 Conclusion

This paper proposed a prescribed-time fixed-time integral sliding mode controller for a nonlinear magnetic levitation system subject to matched and mismatched uncertainties. By combining feedback linearization, an integral sliding manifold, and a fixed-time reaching law, the proposed method provides an explicit upper bound on the reaching time and improves robustness against disturbances. Theoretical analysis established fixed-time reachability of the sliding manifold, exponential stability of the nominal sliding dynamics, and input-to-state stability under bounded mismatched disturbances. Comparative simulations verified that the proposed controller achieves the smallest tracking error, the lowest integral error indices, and the lowest control energy among the tested controllers. These results confirm the effectiveness of the proposed method for robust magnetic levitation control. From a physical viewpoint, the proposed design links nonlinear electromagnetic force-gap dynamics, finite actuator authority, and disturbance rejection within a unified robust stabilization framework for magnetic suspension systems. Future work will focus on experimental validation, actuator saturation, air-gap constraints, and output-feedback implementation.

Table 1. Performance indices

Controller	$T_{\text{set}}(\text{s})$	$\max e_p(t) $	IAE	ISE	ITAE	J_u	$u_{\text{rms}}(\text{V})$
FBL	2.4279	0.069839	0.05198	0.0025104	0.030704	2838.3	26.638
PISMC	1.6093	0.011795	0.00421	2.761e-05	0.001593	318.34	8.921
AFTSMC	0.5507	0.007858	0.00146	8.116e-06	0.000198	299.46	8.652
FOSMC	3.0449	0.006603	0.00199	5.733e-06	0.001390	281.18	8.384
MROFDTSMC	0.7214	0.017569	0.00448	5.703e-05	0.000866	382.55	9.779
PFxTISMC	0.5881	0.004164	0.00045	8.652e-07	7.79e-05	278.37	8.342

References

- Vernekar, P. and Bandal, V. (2021). Robust sliding mode control of a magnetic levitation system: Continuous-time and discrete-time approaches. *arXiv preprint arXiv:2110.12363*.
- Zhu, Q., Wang, S.-M., and Ni, Y.-Q. (2024). A review of levitation control methods for low- and medium-speed maglev systems. *Buildings*, **14**(3), 837.
- Vo, A. T., Truong, T. N., Kang, H.-J., and Le, T. D. (2024). A fixed-time sliding mode control for uncertain magnetic levitation systems with prescribed performance and anti-saturation input. *Engineering Applications of Artificial Intelligence*, **133**, 108373.
- Jastrzebski, M. and Kabzinski, J. (2024). Adaptive control of magnetic levitation system based on fuzzy inversion. *Scientific Reports*, **14**, 24815.
- Han, Y., Yao, X., and Yang, Y. (2024). Disturbance rejection tube model predictive levitation control of maglev trains. *High-Speed Railway*, **2**(1), pp. 57–63.
- Hu, S., Qin, Y., and Yang, J. (2025). Global fast terminal sliding mode control of magnetic levitation system based on improved extended state observer. *International Journal of Control, Automation and Systems*, **23**(5), pp. 1469–1483.
- Zhang, M., Zhang, L., and Shen, L. (2025). Neural network-based adaptive super-twisting sliding mode control for hybrid magnetic levitation system with external disturbance. *Engineering Science and Technology, an International Journal*, **70**, 102177.
- Chen, Y., Yin, Z., Liang, Z., Dong, J., and Li, R. (2025). Fixed-time observer-based sliding mode tracking control for magnetic levitation systems. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, **239**(10), pp. 1819–1831.
- Lan, Q., Yue, P., Liu, N., Sun, J., and Liu, S. (2025). Finite-time position tracking control for magnetic levitation systems with suspension air-gap constraints. *Applied Mathematical Modelling*, **148**, 116258.
- Bhat, S. P. and Bernstein, D. S. (2000). Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, **38**(3), pp. 751–766.
- Polyakov, A. (2012). Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Transactions on Automatic Control*, **57**(8), pp. 2106–2110.
- Shtessel, Y., Edwards, C., Fridman, L., and Levant, A. (2014). *Sliding Mode Control and Observation*. Birkhauser, New York.
- Imangazieva, A. (2024). Robust control of one class of nonlinear systems with a distributed delay. *Cybernetics and Physics*, **13**(1), pp. 42–48.
- Nguyen, B. H., Dang, T. D., Furtat, I., Gushchin, P., Dao, A. Q., and Mai, T. D. (2025). Control of direct current motors with guaranteed speed maintenance in given bounds. *Cybernetics and Physics*, **14**(2), pp. 148–153.
- Sharma, A., Saini, N., and Lather, J. S. (2025). Trajectory control of quadcopter using F-PID controller under disturbances and uncertainties. *Cybernetics and Physics*, **14**(3), pp. 275–284.