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THE NON-STATIONARY SELF-CONSISTENT MODEL OF THE CHARGED SYSTEM

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Abstract
The self-consistent model applicable for the description of a dynamics of the charged systems under the influence of own fields is constructed. Quantum mechanical tasks for one-dimensional and spherically symmetric systems are solved.

Key words
Movement integral, Schrodinger equation.

Introduction
The study of behavior of the charged systems under different conditions is of interest to a row of sections of physics, for example, for the theory of accelerators and physical electronics. The study of dynamics of bunches and bundles of particles in conditions when influence of own forces of a bunch of charges is essential is of special interest. For an analytical study of dynamics fruitful is use of integrals of movement, especially in case of a research of nonstationary systems, see [Efthimiou and Spector, 1994], [Dodonov, Man'ko , and Nikonov, 1992], [Chikhachev, 2006, 2014, 2016]. Papers [Efthimiou and Spector, 1994], [Dodonov, Man'ko, and Nikonov, 1992] contain the description of a row of nonstationary integrals of movement and application of these integrals for receiving exact expressions for propagators — in these articles there is also an extensive list of the quoted operations. In works [Chikhachev, 2006, 2014, 20165] under different conditions the behavior of nonstationary charged systems is studied. In particular, the dynamics in 4-dimensional space (in classical and in quantum mechanical systems) was studied in [Chikhachev, 2006] with using of nonstationary integral of movement ("Meshchersky integral"). It was described for the first time in works [Mestschersky, 1893, 1902]. In [Chikhachev, 2014, 2016] the symmetric systems with use of the same integral of movement were studied nonstationary one-dimensional and spherically, and the concept of "conjugate", integral of the movement allowing to construct model in classical setting is entered. In these operations the electric field was described by one potential — $\Phi(\vec{r}, t)$. In the real operation with use of nonstationary integral of movement ("Meshchersky integral") found the self-consistent self-similar solution for charge flows in a one-dimensional case and in the symmetric task three-dimensional spherically, and more general expression for the electric field described is used two components 4 of potential.

1 Dynamics of One-dimensional Charged System

Generally expression for own electric field of a charge can be written as follows:

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\Phi. \tag{1.1}$$

Here $\vec{E}$ is electric field intensity, $\Phi, \vec{A}$ is potential components 4. In case of one-dimensional system the equation of longitudinal movement along an axis of x can be provided as follows:

$$m\ddot{x} = -\frac{q}{c}\frac{\partial A_x}{\partial t} - q\frac{\partial \Phi}{\partial x}. \tag{1.2}$$

In this equation of A_x is a longitudinal component of vector potential, m, q are the mass and a charge of a particle, c is light velocity, t is time. Owing to dependence on time of components of potential in the equation (1.2) energy isn't the conserved value. Therefore creation of the models allowing, in some sense, to find the conserved value playing an energy role is of interest.

Let's assume that in (1.2) components of potential have an appearance: $\Phi(x,t) = \frac{1}{\xi(t)^2}\varphi(\frac{x}{\xi(t)})$, $A_x(x,t) = \frac{1}{\xi(t)}A(\frac{x}{\xi(t)})$. Instead of x,t we will enter new variables: $x_* = \frac{x}{\xi(t)}, \tau = \int \frac{dt'}{\xi(t')^2}$. Further we will believe $\frac{d\xi}{dt} = \frac{1}{2\tau_0}$, where τ_0 everywhere - a constant with dimensionality of time. Then the left side (1.2) is presented in the form: $m\xi\frac{d^2x_*}{dt} + 2m\frac{dx_*}{dt}\frac{d\xi}{dt} - \frac{mx_*}{4\tau_o^2\xi^3}$, and the right side - $-\frac{1}{\xi^3}\frac{d}{dx_*}(\varphi(x_*) - \frac{1}{2c\tau_0}x_*A(x_*))$. After conversion of derivatives in t to derivatives in τ we will obtain equality:

$$m\frac{d^2}{d\tau^2}x_* = -\frac{d}{dx_*}\left(q\varphi(x_*) - \frac{q}{2c\tau_0}x_*A(x_*) - \frac{mx_*^2}{8\tau_0^2}\right). \tag{1.3}$$

As a result, the movement integral playing an energy role in the studied model system has an appearance:

$$I = \frac{m}{2}\left(\frac{d}{d\tau}x_*\right)^2 + q\varphi(x_*) - \frac{q}{2c\tau_0}x_*A(x_*) - \frac{mx_*^2}{8\tau_0^2}. \tag{1.4}$$

The last three members in the right side of this equation represent effective potential. Let's notice that in addition to integral (1.4) there is also integrated movement integral. Let's consider expression

$$J_I = \int^{x_*}\frac{dy}{\sqrt{\frac{2}{m}(I - q(\varphi(y) - \frac{qy}{2c\tau_0}A(y)) + \frac{y^2}{4\tau_0^2}}} - \tau \tag{1.5}$$

If particles moving in the positive direction of an axis of x_*, that is carried out by equality $\frac{dJ_I}{d\tau} = \frac{\partial J_I}{\partial\tau} + \frac{\partial J_I}{\partial x_*}\dot{x}_* \equiv 0$.

In the studied one-dimensional system of the equation for component $\Phi(x,t)$ and $A_x(x,t)$ can be written down as follows:

$$\begin{aligned}\frac{\partial}{\partial x}\Big(\frac{\partial\Phi}{\partial x} + \frac{1}{c}\frac{\partial A_x}{\partial t}\Big) &= -4\pi qN,\\ \frac{1}{c}\frac{\partial}{\partial t}\Big(\frac{\partial\Phi}{\partial x} + \frac{1}{c}\frac{\partial A_x}{\partial t}\Big) &= \frac{4\pi}{c}J.\end{aligned} \tag{1.6}$$

Here N is density of particles, J is current density.

If $\Phi = \frac{1}{\xi(t)^2}\varphi(\frac{x}{\xi(t)}), A_x = \frac{1}{\xi(t)}A(\frac{x}{\xi(t)})$, that $\frac{\partial\Phi}{\partial x} + \frac{1}{c}\frac{\partial A_x}{\partial t} = \frac{1}{\xi^3}\big(\varphi(x_*) - \frac{x_*A(x_*)}{2\tau_0 c}\big)'$, where $x_* = \frac{x}{\xi}$, and a stroke designates a derivative in x_*. Let's introduce notation : $V(x_*) = \varphi(x_*) - \frac{x_*A(x_*)}{2\tau_0 c}$. Then Maxwell's equation can be written down in a type:

$$\begin{aligned}\frac{1}{\xi^4}V'' &= -4\pi qN,\\ -\frac{1}{2\tau_0 c\xi^5}\big(3V'(x_*) + x_*V''(x_*)\big) &= \frac{4\pi}{c}J.\end{aligned} \tag{1.7}$$

The removed higher than an integral (1.4) can be used as for studying of classical system, and quantum. Further the quantum mechanical system will be considered. In the nonstationary one-dimensional electric field described by two components 4 of potential, the Schrodinger equation has an appearance:

$$\begin{aligned}i\hbar\frac{\partial\Psi(x,t)}{\partial t} &= \Big(\frac{1}{2m}\Big(-i\hbar\frac{\partial}{\partial x} - \frac{q}{with}A_x\Big)^2 + \\ &q\Phi\Big)\Psi = \Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{i\hbar q}{mc}A_x\frac{\partial}{\partial x} + \\ &\frac{i\hbar q}{2mc}\big(\frac{\partial}{\partial x}A_x\big) + \frac{q^2}{2mc^2}A_x^2 + q\Phi\Big)\Psi(x,t).\end{aligned} \tag{1.8}$$

Let's enter new required function: $\Psi = \Psi_1\exp\left\{\frac{i}{\hbar c}\int^x qA(\frac{x'}{\xi(t)})\frac{dx'}{\xi t}\right\}$. Taking into account ratio $\frac{\partial}{\partial t}A(\frac{x}{\xi(t)}) = -\frac{x\dot{\xi}}{\xi^2}A'(\frac{x}{\xi(t)})$ the left side of equation (1.8) can be given by to a look:

$$i\hbar\frac{\partial\Psi}{\partial t} = \Big(i\hbar\frac{\partial\Psi_1}{\partial t} + \frac{x\dot{\xi}}{c\xi^2} + qA\big(\frac{x}{\xi}\big)\Psi_1\Big)\times$$

$$\exp\left\{\frac{i}{\hbar c}\int^x qA\big(\frac{x'}{\xi t}\big)\frac{dx'}{\xi t}\right\}.$$

After conversion of the right side, we will obtain:

$$i\hbar\frac{\partial\Psi_1(x,t)}{\partial t} = \Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + q\Phi - \frac{x\dot{\xi}}{\xi^2}\frac{q}{with}A\big(\frac{x}{\xi}\big)\Big)\Psi_1 \tag{1.9}$$

Let's transform required function by means of equality $\Psi_1 = \frac{1}{\xi^2}\Psi_2(x_*,\tau)\exp\left(\frac{imx_*^2}{4\hbar\tau_0}\right)$ and we will pass to variable $x_* = x/\xi, \tau = \int\frac{dt'}{\xi(t')^2}$. We will obtain the equation:

$$\begin{aligned}i\hbar\frac{\partial\Psi_2(x_*,\tau)}{\partial\tau} &= -\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2(x_*,\tau)}{\partial x_*^2} + \\ &\Big(qV(x_*) + \frac{3i\hbar}{4\tau_0} - \frac{mx_*^2}{8\tau_0^2}\Big)\Psi_2(x_*,\tau).\end{aligned} \tag{1.10}$$

Let's determine, further, density of a charge and a current density. Density of a charge has an appearance:

$$qN = q|\Psi(x,t)|^2 = \frac{q}{\xi^4}|\Psi_2(x_*\tau)|^2,$$

a current density of

$$J = q\frac{\hbar}{2im}\Big(\Psi^*\big(\frac{\partial}{\partial x} + i\frac{q}{\hbar c}A_x\big)\Psi - \Psi^*\big(\frac{\partial}{\partial x} - i\frac{q}{\hbar c}A_x\big)\Psi\Big).$$

Let's put in (1.10) $\Psi_2 = T(\tau)X(x_*)$. Then can be obtained:

$$i\hbar\frac{\dot{T}(\tau)}{T(\tau)} = -\frac{\hbar^2}{2m}\frac{X''(x_*)}{X(x_*)} + qV(x_*) + \frac{3i\hbar}{4\tau_0} - \frac{mx_*^2}{8\tau_0^2}.$$

In this equality the point means a derivative in τ, and a dash is a derivative in x_*. We will use further that $i\hbar\frac{\dot{T}(\tau)}{T(\tau)} = E$, where E is the valid value. Let's designate $qV = U$, then

$$E - \frac{3i\hbar}{4\tau_0} = -\frac{\hbar^2}{2m}\frac{X''(x_*)}{X(x_*)} + U(x_*) - \frac{mx_*^2}{8\tau_0^2}. \quad (1.11)$$

In this case function of $X(x_*)$ is complex. Let's note that in case of such determination of a constant of division putting and right parts of the equations of Maxwell (1.6) equally depend on ξ, that allows to obtain the system of equations depending on one self-similar variable of x_*.

It is convenient to use representation of $X = R(x_*)\exp(i\theta(x_*))$, where R and θ are real-valued functions. For density of a charge expression of $N = \frac{R^2}{\xi^4}$, and for a current density it is possible to obtain $J = q\frac{R^2}{\xi^5}\big(\frac{\hbar\theta'}{m} + \frac{x_*}{2\tau_0 c}\big)$. The equations for R and θ will take a form:

$$-\frac{\hbar^2}{2m}(R'' - R\theta'^2) + \big(U(x_*) - E - \frac{mx_*^2}{8\tau_0^2}\big)R = 0,$$
$$-\frac{\hbar^2}{2m}\big(2R'\theta + R\theta''\big) + \frac{3\hbar R}{4\tau_0} = 0. \quad (1.12)$$

These equations shall be added by ratios (1.7):

$$\frac{d^2}{dx_*^2}U(x_*) = -4\pi q^2R^2, 3U' + x_*U'' = -8\pi\tau_0 q^2R^2\big(\frac{\hbar\theta'}{m} + \frac{x_*}{2\tau_0}\big). \quad (1.13)$$

The equations (1.12) and the ratio following from system (1.13)

$$3U' = -8\pi\tau_0 q^2R^2\frac{\hbar\theta'}{m} \quad (1.14)$$

completely determine functions of R, θ', U.

Let's mark that or psi-function shall be sized - $R(x_*)$ is normalized on multiplier $\frac{1}{l_0^{3/2}}$, where l_0 - the parameter of dimensionality of length, or that from the formal point of view same, the charge of q shall have dimensionality of $[q] = [e]/l_0^{3/2}$, where e - an elementary charge. Let's enter the dimensionless independent variable: $s = x_*/\sqrt{\frac{2\hbar\tau_0}{m}}$. Let's receive the following system for determination of $R, y = \frac{d\theta}{ds}, U$.

$$-\frac{\hbar}{4\tau_0}(R'' - Ry^2) + \big(U - E - s^2\frac{\hbar}{4\tau_0}\big)R = 0,$$
$$2R'y + Ry' - 3R = 0, 3U' = -\frac{8\pi q^2\tau_0}{m}R^2\hbar y. \quad (1.15)$$

Let's put, further,

$$u(s) = \frac{4\tau_0}{\hbar}U, \frac{4\tau_0}{\hbar}E = e_0,$$

$$\sqrt{\frac{32\pi q^2\tau_0^2}{m}}R = x(s).$$

System (1.15) can be reduced to the equations:

$$x'' = x\big(\frac{9u'^2}{x^4} + u - e_0 - s^2\big), u'' = -x^2. \quad (1.16)$$

The system (1.16) decided in case of the initial conditions set in case of $s = 0$. Let's note that average rate - the flux density attitude towards density of a charge, has an appearance:

$$\upsilon = \frac{J}{qN} = \frac{1}{\xi}\sqrt{\frac{\hbar}{2m\tau_0}}\big(s + y\big).$$

the Necessary condition of the nonrelativistic nature of movement of charges is $\sqrt{\frac{\hbar}{2m\tau_0}} << c$, or $\tau_0 >> \frac{\hbar}{2mc^2} = 0.6\cdot 10^{-21}$sec.

Fig. 1 shows the dependence of the function of $x(s)$, characterizing density of a charge from a self-similar variable in case of

$$x(0) = 10, u'(0) = 0, x'(0) = 0,$$

and Fig. 2 shows the same dependence in case of

$$x(0) = 10, u'(0) = 10, x'(0) = 10.$$

Fig. 3 shows dependence of $\upsilon(s) = \hat{\upsilon}(s)\xi\sqrt{\frac{2m\tau_0}{\hbar}}$, is figured by where $\hat{\upsilon} = \frac{J}{qN\xi}$ is average hydrodynamic rate in case of

$$x(0) = 10, u'(0) = 0, x'(0) = 0,$$

and Fig. 4 shows the same dependence in case of

$$x(0) = 10, u'(0) = 10, x'(0) = 10.$$

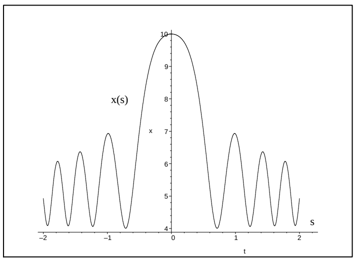

Figure 1. Dependence of $x(s)$ in case of $x(0) = 10, u'(0) = 0, x'(0) = 0$

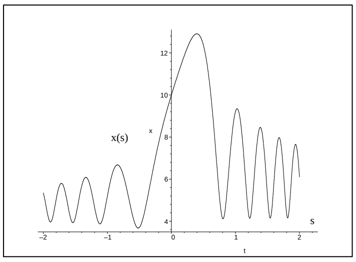

Figure 2. Dependence of $x(s)$ in case of $x(0) = 10, u'(0) = 10, x'$ (0)=10

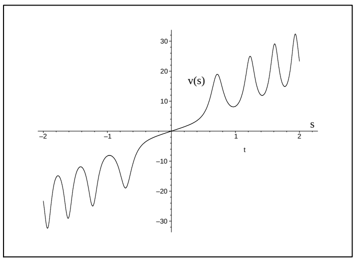

Figure 3. Dependence of $v(s)$ in case of $x(0) = 10, u'(0) = 0, x'$ (0)=0.

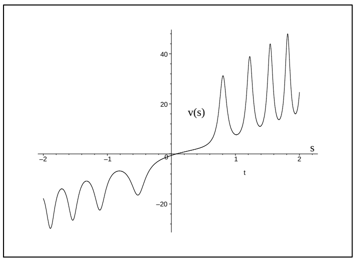

Figure 4. Dependence of $v(s)$ in case of $x(0) = 10, u'(0) = 10, x'(0)$ =10.

2 Dynamics Spherically Symmetric System

In case of spherically symmetric system electric field can be also described by two components 4 of potential - $\Phi(r,t)$ and $A_r(r,t)$ can be also written down in the form of (1.1).

Then the equation of radial movement can be presented in the form:

$$m\ddot{r} = -\frac{q}{c}\frac{\partial A_r}{\partial t} - q\frac{\partial \Phi}{\partial r} + \frac{L}{mr^3}. \qquad (2.1)$$

where r is distance from the center of spherical system, L is a moment square concerning the center. In these equations, Φ is the potential of electric field, A_r is radial a component of vector potential, m, q are the mass and a charge of a particle, L is a particle moment square concerning the center, r is distance from the center, c is the velocity of light, t is time. Owing to dependence on time a potential component in the equation (2.1) energy isn't the remaining size. Therefore creation of the models allowing, in some sense, to find the conversed size playing an energy role is of interest.

Let's put that in (2.1) components of potential have an appearance: $\Phi(r,t) = \frac{1}{\xi(t)^2}\varphi(\frac{r}{\xi(t)}), A_r(r,t) = \frac{1}{\xi(t)}A(\frac{r}{\xi(t)})$. Instead of r, t we will enter new variables: $\rho = \frac{r}{\xi(t)}, \tau = \int \frac{dt'}{\xi(t')^2}$. Further we will believe $\frac{d\xi}{dt} = \frac{1}{2\tau_0\xi}$, where τ_0 everywhere is a constant with dimension of time. Then the right part (2.1) is presented in the form: $m\xi\frac{d^2\rho}{dt^2} + 2m\frac{d\rho}{dt}\frac{d\xi}{dt} - \frac{m\rho}{4\tau_0^2\xi^3}$, and the right part - $-\frac{1}{\xi^3}\frac{d}{\rho}(\varphi(\rho) - \frac{1}{2c\tau_0}\rho A(\rho) + \frac{L}{2m\rho^2})$. After transformation of derivatives in t to derivatives in τ we will obtain equality:

$$m\frac{d^2}{d\tau^2}\rho = -\frac{d}{d\rho}\left(q\varphi(\rho) - \frac{q}{2c\tau_0}\rho A(\rho) + \frac{L}{2m\rho^2} - \frac{m\rho^2}{8\tau_0^2}\right). \qquad (2.2)$$

As a result, the movement integral playing an energy role in the studied model system has an appearance:

$$I = \frac{m}{2}\left(\frac{d}{d\tau}\rho\right)^2 + q\varphi(\rho) - \frac{q}{2c\tau_0}\rho A(\rho) + \frac{L}{2m\rho^2} - \frac{m\rho^2}{8\tau_0^2}. \qquad (2.3)$$

The last 4 composed in this equation represent effective potential. As well as in a one-dimensional case, in addition to integral (2.3) there is an integrated movement integral, determined by expression

$$J_I = \int^{\rho} \frac{dy}{\sqrt{\frac{2}{m}(I - q\varphi(y) - \frac{qy}{2c\tau_0}A(y) - \frac{L}{2my^2} + \frac{my^2}{8\tau_0^2})}} - \tau. \qquad (2.4)$$

If particles move in the positive direction of axis ρ, that is carried out by equality $\frac{dJ_I}{d\tau} = \frac{\partial J_I}{\partial \tau} + \frac{\partial J_I}{\partial \rho}\dot{\rho} \equiv 0$. Let's consider the equations for potential component Φ, A_r :

$$\begin{gathered}\Delta\Phi + \frac{1}{c}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial A_r}{\partial t} = -4\pi q N, \\ \frac{1}{c}\frac{\partial}{\partial t}\frac{\partial \Phi}{\partial r} + \frac{1}{c^2}\frac{\partial^2 A_r}{\partial t^2} = \frac{4\pi}{c}J_r,\end{gathered} \qquad (2.5)$$

here N is density of particles, J_r is density of radial current. Using the given representations a component of potentials, these equations can be given to a look:

$$\frac{1}{\xi^4}\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{d}{d\rho}(\varphi - \frac{\rho}{2c\tau_0}A) = -4\pi q N, \qquad (2.6)$$

$$\frac{3}{2\tau_0 c\xi^5}\big(\varphi-\frac{\rho A}{2\tau_0 c}\big)'+\frac{\rho}{2\tau_0 c\xi^5}\big(\varphi-\frac{\rho A}{2\tau_0 c}\big)''=-\frac{4\pi}{c}J_r. \tag{2.7}$$

The equations (2.6) and (2.7) use ratio $\frac{\partial\Phi}{\partial r}+\frac{1}{c}\frac{\partial A_r}{\partial t}=\frac{1}{\xi^3}(\varphi-\frac{\rho A}{2\tau_0 c})'$. The stroke in (2.7) means a derivative on ρ. At any dependence $\varphi-\frac{\rho A}{2\tau_0 c}$ from ρ is carried out the equation of continuity $\frac{\partial}{\partial t}qn+\frac{1}{r^2}\frac{\partial}{\partial r}r^2 j_r=0$. The condition of system is defined by function of $V(\rho)=\varphi-\frac{\rho A}{2\tau_0 c}$.

In the symmetric electric field non-stationary spherically described by two components 4 of potential, the Schredinger equation has an appearance:

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t}=\Big(\frac{1}{2m}\left(-i\hbar\nabla-\frac{q}{with}\vec{A}\right)^2+ q\Phi\Big)\Psi=\Big(-\frac{\hbar^2}{2m}\nabla^2+\frac{i\hbar q}{mc}\vec{A}\nabla+ \frac{i\hbar q}{2mc}(\nabla\vec{A})+\frac{q^2}{2mc^2}(\vec{A})^2+q\Phi\Big)\Psi(r,t). \tag{2.8}$$

As $\vec{A}=\vec{e}_r A_r$, that $\nabla\vec{A}\equiv A_r\frac{\partial}{\partial r}$, $\nabla\vec{A}\equiv\frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r}$. Let's enter new function required function

$$\Psi=\Psi_1\exp\left\{\frac{i}{\hbar c}\int qA(\frac{r'}{\xi(t)})\frac{dr'}{\xi t}\right\}.$$

Taking into account ratio $\frac{\partial}{\partial t}A(\frac{r}{\xi(t)})=-\frac{r\dot{\xi}}{\xi^2}A'(\frac{r}{\xi(t)})$ the left member of equation (2.8) can be given by to a look:

$$i\hbar\frac{\partial\Psi}{\partial t}=\Big(i\hbar\frac{\partial\Psi_1}{\partial t}+\frac{r\dot{\xi}}{c\xi^2}qA\big(\frac{r}{\xi}\big)\Psi_1\Big)\times$$

$$\exp\left\{\frac{i}{\hbar c}\int qA(\frac{r'}{\xi t}big)\frac{dr'}{\xi t}\right\}.$$

After conversion of the right side, we will obtain:

$$i\hbar\frac{\partial\Psi_1(r,t)}{\partial t}=\Big(-\frac{\hbar^2}{2m}\nabla^2+q\Phi-\frac{r\dot{\xi}}{\xi^2}\frac{q}{c}A\big(\frac{r}{\xi}\big)\Big)\Psi_1 \tag{2.9}$$

Adding expression for electric potential and considering existence of the nonzero orbiting moment, we will obtain:

$$i\hbar\frac{\partial\Psi_1(r,t)}{\partial t}=\Big(-\frac{\hbar^2}{2m}\nabla^2+\frac{q}{\xi^2}\varphi(\frac{x}{\xi})- \frac{r}{2\tau_0\xi^3}\frac{q}{c}A\big(\frac{r}{\xi}\big)+\frac{L}{2mr^2}\Big)\Psi_1 \tag{2.10}$$

Let's pass, further, to variables ρ,τ.

When we will obtain the equation:

$$i\hbar\Big(\frac{\partial\Psi_1(\rho,\tau)}{\partial\tau}-\frac{\dot{\xi}}{\xi}\rho\frac{\partial\Psi_1}{\partial\rho}\Big)=-\frac{\hbar^2}{2m}\Big(\frac{\partial^2\Psi_1(\rho,\tau)}{\partial^2\rho^2} +\frac{2}{\rho}\frac{\partial\Psi_1(\rho,\tau)}{\partial\rho}\Big)+\Big(qV(\rho)+\frac{L}{2m\rho^2}-\frac{m\rho^2}{8\tau_0^2}\Big)\Psi_1(\rho,\tau). \tag{2.11}$$

(the point means a derivative in τ.)

Let's put, further, $\Psi_1=\frac{1}{\xi^2}\Psi_2(\rho,\tau)\exp\left\{\frac{im\rho^2}{4\hbar\tau_0}\right\}$. For Ψ_2 we will obtain the equation:

$$i\hbar\frac{\partial\Psi_2(\rho,\tau)}{\partial\tau}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi_2(\rho,\tau)}{\partial^2\rho^2}+\frac{2}{\rho}\frac{\partial\Psi_2(\rho,\tau)}{\partial\rho}\right) +\left(qV(\rho)+\frac{L}{2m\rho^2}+\frac{i\hbar}{4\tau_0}-\frac{m\rho^2}{8\tau_0^2}\right)\Psi_2(\rho,\tau). \tag{2.12}$$

Let's look for the decision (1.12) in a look: $\Psi_2=T(\tau)R(\rho)$. Can be obtained:

$$i\hbar\frac{\dot{T}}{T}=-\frac{\hbar^2}{2mR(\rho)}\left(\frac{\partial^2R(\rho,\tau)}{\partial\rho^2}+\frac{2}{\rho}\frac{\partial R(\rho,\tau)}{\partial\rho}\right) +\left(qV(\rho)+\frac{L}{2m\rho^2}+\frac{i\hbar}{4\tau_0}-\frac{m\rho^2}{8\tau_0^2}\right). \tag{2.13}$$

Let's put, further, $i\hbar\frac{\dot{T}}{T}=E$, where E is the valid value. In this case function of $R(\rho)$ is complex. As well as in a one-dimensional case, in case of such determination of a constant of division the left and right parts of the equations of Maxwell equally depend on ξ, that allows to obtain the system of equations depending on one self-similar variable ρ, we will obtain:

$$E-\frac{i\hbar}{4\tau_0}=-\frac{\hbar^2}{2mR(\rho)}\left(\frac{\partial^2R(\rho,\tau)}{\partial\rho^2}+ \frac{2}{\rho}\frac{\partial R(\rho,\tau)}{\partial\rho}\right)+\left(qV(\rho)+\frac{L}{2m\rho^2}-\frac{m\rho^2}{8\tau_0^2}\right). \tag{2.14}$$

It is convenient to look for the decision (2.14) in a look: $\rho R=S(\rho)\exp(i\theta(\rho))$, where S,θ - the valid functions. We will obtain system:

$$\begin{cases} E=-\dfrac{\hbar^2}{2m}\left(\dfrac{S''}{S}-\theta'^2\right)+qV(\rho)-\dfrac{m\rho^2}{8\tau_0^2}+\dfrac{L}{2m\rho^2}, \\ -\dfrac{\hbar}{4\tau_0}=-\dfrac{\hbar^2}{2m}\left(\theta''+2\theta'\dfrac{S'}{S}\right). \end{cases} \tag{2.15}$$

This system has to be added with the equation for $V(\rho)=\varphi-\frac{\rho}{2\tau_0 c}A$, i.e. equation (2.6)

$$\frac{1}{\xi^4}\Delta V(\rho)=-4\pi qN,$$

Density of a charge is defined through Ψ. Let's look for further the private decision for wave function, having presented $\Psi_2(\rho,\tau)$ look: $\Psi_2 = \exp iE\tau\frac{S(\rho)}{\rho}\exp\big(i\theta(\rho)\big)$, where E is the valid size, $S(\rho),\theta(\rho)$ is the valid functions. Thus,

$$\Psi = \frac{1}{\xi^2}\exp\big(iE\tau\big)\frac{S(\rho)}{\rho}\exp\big(i\theta(\rho)\big) \\ \times\exp\Big(\frac{iq}{c\hbar}\int A\big(\frac{r'}{\xi}\big)\frac{dr'}{\xi}\Big)\exp\big(\frac{im\rho^2}{4\hbar\tau_0}\big). \tag{2.16}$$

According to (2.16) density of a charge is proportional to the size o $N=\frac{S^2}{xi^4}$. Density of probability of current of J_r has an appearance:

$$J_r = \frac{\hbar}{2im}\Big\{\Psi^*\big(\frac{\partial}{\partial r}+\frac{iq}{\hbar c}A_r\big)\Psi- \\ \Psi\big(\frac{\partial}{\partial r}-\frac{iq}{\hbar c}A_r\big)\Psi^*\Big\} = \Psi\Psi^*\Big\{\frac{\hbar\theta'}{m\xi}+\frac{\rho}{2\xi\tau_0}\Big\}. \tag{2.17}$$

As well as in a one-dimensional case, here or psi-function $\sim l_0^{-1/2}$, shall be normed on multiplier or it is necessary to read $q\sim e/l_0^{-1/2}$.

Let's receive the equation:

$$3V'+\rho V''(\rho) = -8\pi q\tau_0(S^2/\rho^2\Big(\frac{\hbar\theta'}{m}+\frac{\rho}{2\tau_0}\Big).$$

From (2.6) follows

$$V''+\frac{2}{\rho}V' = -4\pi qS^2/\rho^2.$$

Finally, follows: $V' = -8\pi q\tau_0(S^2/\rho^2\big(\frac{\hbar\theta'}{m}\big)$. It is also easy to receive $(\rho^2V')' = -4\pi qS^2$. Let's put $\rho = s\sqrt{\frac{2\hbar\tau_0}{m}}, qV = \frac{\hbar}{4\tau_0}u(s), \nu_0 = \frac{L}{\hbar^2}, \epsilon = \frac{\hbar E}{4\tau_0}$. In the dimensionless variables

$$\frac{S''}{S}-y^2 = u(s)+\frac{\nu_0}{s^2}-\epsilon-s^2, \\ 2S'y+y'S = S, u' = -\frac{16\pi q^2\tau_0}{\hbar}\frac{S^2}{s^2}y = -\kappa_*\frac{S^2}{s^2}y, \tag{2.18}$$

in (2.18) $\frac{d\theta}{ds}=y$, the dash means a derivative in s.

The system (2.18) comes down to the equations:

$$S' = S\Big(u(s)+\frac{\nu_0}{s^2}-\epsilon-s^2+\big(\frac{s^2u'}{\kappa_*S^2}\big)^2\Big), \\ (s^2u')' = -\kappa_*S^2. \tag{2.19}$$

For average radial speed it is possible to obtain:

$$\bar{v}_r = \frac{1}{\xi}\sqrt{\frac{\hbar}{2m\tau_0}}\big(y+s\big).$$

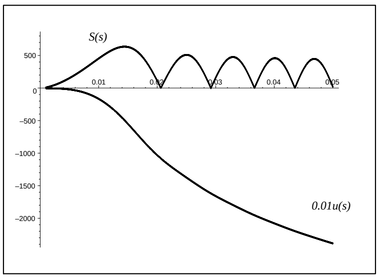

Figure 5. Dependence of $S(s)$ and potential of $u(s)$ from coordinate in case of $S(s_0) = 1, S'(s_0) = 0, u(s_0) = 0, u'(s_0) = 0, s_0 = 10^{-4}$.

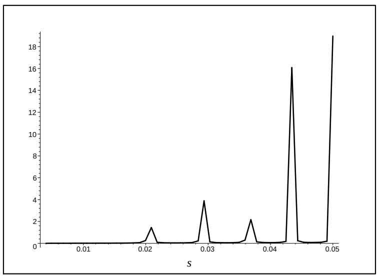

Figure 6. Dependence of $Y(s) = \xi\bar{v}_r\sqrt{\frac{2m\tau_0}{\hbar}}$.

As well as in a one-dimensional case, a condition of the nonrelativistic nature of movement has an appearance: $\tau_0 >> \frac{\hbar}{mc^2}$ is also considered executed in the real operation.

Let's consider the decision (2.19) in case of $l_0 = 1, \nu_0 = 1, \kappa_* = 1$. we will put $S(s_0) = 1, S'(s_0) = 0, u(s_0) = 0, u'(s_0) = 0, s_0 = 10^{-4}$.

In fig. 5 dependences of $S(s), 0.01u(s)$. Dependence of $S(s)$ has oscillatory character whereas $u(s)$ monotonically decreases. Dependence of $Y(s) = y+s = \bar{v}_r(s)\xi\sqrt{\frac{2m\tau_0}{\hbar}}$, provided in fig. 6, has sharp maxima in points where S approaches zero.

Conclusion

In work generalization of integral of the movement of Meshchersky for a case when non-stationary electric field is expressed by means of two a potential component 4 is offered. This generalization is used for a research of behavior of the system interacting with own field. The non-stationary model allowing to reduce a task to self-similar and to the solution of system of the ordinary differential equations is as a result constructed. Private solutions of the received systems are provided.

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