PHYSCON 2009, Catania, Italy, September, 1–September, 4 2009

PRECISE CONTROL OF DYNAMIC NONSTATIONARY OBJECTS ON THE BASE OF MODEL REFERENCE ADAPTIVE APPROACH

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Abstract
This paper deals with the problem of a precise control for dynamic object with a nonstationary mathematical model. A well – known approach on the base of model reference adaptive system theory is used. A new algorithm of adaptation is proposed for such class of systems. An advantage of the algorithm is independence of its operation from intensity and spectral structure of input actions. It makes possible to be sure in dynamic accuracy of such systems.

Key words
Reference model, adaptive system, nonstationary mathematical model, algorithm of adaptation.

1 Introduction

From the origin of model reference adaptive systems (MRAS) research [Parks, 1966; Zemlyakov and Rutkovsky, 1967; Landau, 1969] it is possible to indicate up to hundred monographs and thousands papers on this subject. For such an attention reasons are in a constructability of the task statement and in a beauty of mathematical tools. Really, the reference model permits to know the motion to tend to it. Hence it is possible to apply analogs of the feedback principle what is typical for nature and human activities. Mathematical models (MM) of MRAS are principally multiconnected nonlinear nonstationary ones. The beauty of the MRAS designing is contained for example in the fact that the Lyapunov direct method is used not only for a system stability analysis but for synthesis of MRAS operating algorithms too [Parks, 1966; Zemlyakov and Rutkovsky, 1967; Landau, 1969].

But in spite of intensive theoretical MRAS developments practical applying of such systems is not so impressive. Reasons of such a fact authors attend in a line of essential MRAS deficiencies. Next positions could be referred to such deficiencies as dependence of the MRAS dynamic accuracy from:

1) an intensity and a spectral structure of input actions;

2) level of parametric and coordinate disturbances.

All these facts trend to a small predicted dynamic accuracy of the MRAS that naturally cannot satisfy designers of real systems.

On the other hand modern and perspective dynamic control objects, being operated in conditions of parametric and coordinate disturbance uncertainties, more and more demand dynamic accuracy to be closed to precise one.

Qualitatively under precise control in this work we understand the motion of the system with practically null (or preassigned) displacement with respect to some prescribed desired motion.

In this paper a new algorithm for MRAS operating is proposed. The algorithm:

1) removes the dependence of the MRAS dynamic accuracy from the intensity and the spectral structure of input actions;

2) makes possible to get predicted information about MRAS dynamic accuracy.

And for this algorithm synthesis it is not required an assumption about the quasistationarity of the object functioning regime.

2 Problem statement

Consider an object or a system with MM in the form

$$\frac{d\varphi}{dt}+(A_0+\Delta A(t))\varphi=(B_0+\Delta B(t))g(t)+ \\ +f(t)+L, \tag{1}$$

where t is time; t_0 is a starting point for the system with MM (1); $t \ge t_0$; $\varphi \in R^n$; $g(t) \in R^m$;

$f(t) \in R^n$; $L \in R^n$. Here φ, $g(t)$, $f(t)$ are state, control and disturbance vectors respectively. Matrices $\Delta A(t)$, $\Delta B(t)$ are unknown parametric disturbances. Components of vector L are intended for compensation of coordinate and parametric disturbances. Vectors $\varphi = \varphi(t)$, $g(t)$ are assumed to be available for measurement; matrices A_0 and B_0 are known; ($-A_0$) is a Hurwitz matrix; elements of matrices $\Delta A(t)$, $\Delta B(t)$ and components of vectors $g(t)$, $f(t)$ are continuously differentiable functions.

Let us rewrite the equation (1) in the form

$$\frac{d\varphi}{dt} + A_0\varphi = B_0 g(t) + d(t) + L , \qquad (2)$$

where $d(t) = -\Delta A(t)\varphi + \Delta B(t)g(t) + f(t)$ is the vector-function with continuously differentiable components which are not available for measurement.

It is required to synthesize the algorithms of the vector L components tuning from the condition of the disturbance vector $d(t)$ compensation.

The task will be solved with the condition that components $d_i(t)$ $(i = \overline{1, n})$ values of the vector $d(t) = (d_i(t))$ are not bounded but velocities of their changing are bounded, that is $\left|\frac{d}{dt} d_i(t)\right| \le \mu_{i0}$, $\mu_{i0} = \text{const}$ and numbers μ_{i0} are known and can be quite big.

3 The task solution on the base of model reference adaptive approach

Let us consider MM of desirable system motion in the form

$$\frac{d\varphi_M}{dt} + A_0\varphi_M = B_0 g(t) \qquad (3)$$

where $\varphi_M \in R^n$.

A dynamic link with the MM (3) we accept as the reference model.

We will try to synthesize the algorithm for tuning of L-vector components from the condition that the motion $\varphi = \varphi(t)$ asymptotically converges to the motion $\varphi_M = \varphi_M(t)$ of the system with the MM (3) independently from an intensity and a spectral structure of input actions. In this case it is possible to get predicted information about MRAS dynamic accuracy.

It is interesting to note that traditional adaptation algorithms of MRAS [Parks, 1966; Zemlyakov and Rutkovsky, 1967; Landau, 1969] are not provided asymptotical convergence of the system motion to the reference model motion if the input disturbance vector $f(t)$ is not zero. Here we try to solve the problem independently whether input vectors $g(t)$, $f(t)$ are zero or not.

With notation $\varepsilon = \varphi - \varphi_M$, $y = d(t) + L$, $\frac{d}{dt}L = \psi$, $\frac{d}{dt}d(t) = \mu(t)$, from equations (2) and (3) we get the system

$$\frac{d}{dt}\varepsilon + A_0\varepsilon = y, \qquad \frac{d}{dt}y = \mu(t) + \psi. \qquad (4)$$

Here the term $\psi = \psi(t, \varepsilon)$ is a looking for an adaptation algorithm.

Let us introduce into consideration the vector x that is defined by the equation

$$\tau\frac{d}{dt}x + x = \varepsilon \qquad (5)$$

where $\tau = \text{const} > 0$ is a prescribed small value. At first the system (4) and (5) will be considered with the condition

$$\tau = 0 . \qquad (6)$$

Then the system (4) and (5) could be rewrite in the form

$$\frac{d}{dt}x + A_0 x = y, \quad \frac{d}{dt}y = \mu(t) + \psi . \qquad (7)$$

Let us consider the motion

$$x \equiv 0, \quad y \equiv 0 \qquad (8)$$

of the system (7) and synthesize the algorithm for the vector ψ tuning from the condition that the motion of the system (7) $x = x(t)$, $y = y(t)$ is asymptotically converges to the motion (8). For this goal we take the Lyapunov function in the form [Petrov, Rutkovsky, and Zemlyakov, 1980]

$$V(x, y) = \kappa(x^T P x) + y^T y \qquad (9)$$

where $\kappa = \text{const} > 0$, P is a positive definite matrix to be defined by the Lyapunov equality $-(A_0^T P + PA_0) = Q$, Q is a prescribed negative definite matrix.

The derivative $\frac{dV(x,y)}{dt}$ with respect to the system (7) is defined by an equation

$$\frac{dV(x,y)}{dt} = \kappa(x^T Q x) + 2y^T[\kappa\sigma + \mu(t) + \psi], \qquad (10)$$

where $\sigma = Px$.

Let us synthesize the algorithm for the vector ψ tuning from the condition $\frac{dV(x,y)}{dt} < 0$. For this goal we consider equality

$$\psi = -\kappa\sigma - Ksign(y), \qquad (11)$$

where K is a diagonal matrix $K = diag(k_1, k_2, \dots, k_n)$ and $sign(y)$ is the vector $[sign(y)]^T = [sign(y_1)\ sign(y_2) \dots sign(y_n)]$.

With respect to (11) the equality (10) takes the form

$$\frac{dV(x,y)}{dt} = \kappa(x^T Qx) - 2|y|^T \rho(t), \qquad (12)$$

where $|y|^T = (|y_1|\ |y_2|\ \dots |y_n|)$,
$[\rho(t)]^T = [\rho_1(t)\ \ \rho_2(t)\ \dots\ \rho_n(t)]$,
$\rho_i(t) = k_i - \mu_i(t)sign(y_i) \quad (i = 1, 2, \dots, n)$.

Under condition

$$k_i > \mu_{i0} \qquad (13)$$

it takes place an inequalities $\rho_i(t) > 0$. Then from (9) and (12) we get inequalities

$$V(x, y) > 0, \quad \frac{dV(x,y)}{dt} < 0 \qquad (14)$$

The inequalities (14) garantee asymptotical convergence of the system (7) motion with the adaptation algorithm (11) to the motion (8). These inequalities were synthesized independently of a kind of input actions and coordinate or parametric disturbances entering in the vector $d(t)$ (of course with the condition (13) to be valid). But it is necessary to note that the system (7) with the adaptation algorithm (11) belongs to the class of systems with a discontinuous right part of the MM [Emelyanov, 1967]. In such system it is possible of sliding modes arising.

Let us rewrite the system (7) with the adaptation algorithm (11) in the form

$$\begin{aligned} &\frac{d}{dt}x + A_0 x = y, \\ &\frac{d}{dt}y = \mu(t) - \kappa\sigma - Ksign(y). \end{aligned} \qquad (15)$$

From the system (15) it is evident that sliding modes in a phase space $\{x, y\}$ could arise on one or several of discontinuity hyperplanes $y_i = 0\ (i = 1, 2, ..., n)$. Let it takes place and during of a time interval ΔT on hyperplanes, for example $y_j = 0\ (j = 1, 2, ..., m)$, $m \le n$, the system is moving in sliding modes. Then for this time interval ΔT the equalities $y_j = 0,\ \dot{y}_j = 0 \quad (j = 1, 2, ..., m)$, $m \le n$ take place [Emelyanov, 1967; Utkin, 1992]. During the time interval ΔT components of a vector $z^T = (y_{l+1}, y_{l+2}, ..., y_n)$ can take zero values but a set of such points has a null measure. It means that the representative point in the phase space penetrates these hyperplanes not to delay on them. Then the vector y on this time interval ΔT has m null components which could not break inequalities (14).

Let us introduce the MM of the system (15) in the form

$$\begin{aligned} &\frac{d^2x}{dt^2} + A_0\frac{dx}{dt} + \kappa Px = -R(t)sign(S), \\ &S = \frac{dx}{dt} + A_0 x, \end{aligned} \qquad (16)$$

where $R(t) = diag(\rho_1(t), \dots, \rho_n(t))$ is the diagonal matrix.

We can state that in the section 3 it is proved that the motion of the system with the MM (16) asymptotically converges to the motion

$$x \equiv 0,\ \dot{x} \equiv 0. \qquad (17)$$

4 Analysis of the adaptive system motion

Results of the section 3 were obtained under condition that the vector $sign(y)$ is available to measuring but it was not told how to get such information. Really, if to suppose that the vector y is measured then the necessity in adaptation is eliminated. It follows from the notation $y = d(t) + L$, where the vector of coordinate and parametric disturbances $d(t)$ is not measured. From the MM (4) it is evident that the vector y can be got as the equality $y = \frac{d\varepsilon}{dt} + A_0\varepsilon$ but, according to the problem statement, the vector $\frac{d\varepsilon}{dt}$ is not measured. Just to solve this contradiction the equation (5) was introduced. In this case the vector $\frac{dx}{dt}$ can be measured. Then it is possible to suppose that vectors x and $\frac{dx}{dt}$ will be closed to vectors ε and $\frac{d\varepsilon}{dt}$ respectively if the value τ in (5) is small enough. In fact the equation (5) was introduced for the goal of the vector $\frac{d\varepsilon}{dt}$ estimation.

So for more constructive of the task's solution it is necessary to refuse from the condition (6) and to assume the equality

$$\tau = \tau_0,\ \tau_0 = \text{const} > 0 \tag{18}$$

where τ_0 is a small but constant value.

Under the condition (18) the system of equations (4) and (5) could be rewrite in the form

$$\begin{aligned} &\tau_0 \frac{d^2x}{dt^2} + (E + \tau_0 A_0)\frac{dx}{dt} + A_0 x = y, \\ &\frac{dy}{dt} = \mu(t) + \psi, \end{aligned} \tag{19}$$

where E is a unit matrix.

Now in the MM (19) we leave the adaptation algorithm of the form (11)

$$\psi = -\kappa P x - K sign(S)\ , \tag{20}$$

where $S = \frac{dx}{dt} + A_0 x.$ As vectors x and $\frac{dx}{dt}$ are now available to measuring then the vectors S and $sign(S)$ are available to measuring too. So the adaptation algorithm (20) is realizable in the system with the MM (19).

Let us represent the MM (19) with the adaptation algorithm (20) in the form

$$\begin{aligned} &\tau_0 \frac{d^3x}{dt^3} + (E + \tau_0 A_0)\frac{d^2x}{dt^2} + \\ &+ A_0 \frac{dx}{dt} + \kappa P x = -R(t) Sign(S). \end{aligned} \tag{21}$$

Naturally under the condition $\tau_0 = 0$ the MM (16) and the MM (21) are coincided.

It is possible to suppose that motions of systems with MM (16) and (21) will be closed if the number τ_0 is small enough. Simulation confirms this fact. But some principle questions arise that require analytical solutions. For example, such questions are:

1) It was proved that the motion of the system with the MM (16) asymptotically converges to the motion (17). Is there an interval for values of τ

$$0 < \tau \le \tau_0,\ \ \tau_0 = \text{const} > 0 \tag{22}$$

at which the motion of the system with the MM (21) converges to the motion

$$x \equiv 0,\ \dot{x} \equiv 0,\ \ddot{x} \equiv 0\ ? \tag{23}$$

We can say before that such an interval is not existed for a common case of the matrix $R(t)$ elements changing in the MM (16) that are restricted only by inequalities

$$\begin{aligned} &\rho_i^{\max} \ge \rho_i(t) \ge \rho_i^{\min}, \\ &\rho_i^{\min} = \text{const} > 0 \quad (i = 1, \ldots, n). \end{aligned} \tag{24}$$

2) Let such an interval for possible values τ (22) was not found. Is there in the phase space such a domain that the motion of the system with MM (21) converges to this domain and this domain includes the motion (23)?

We can say before that such domain exists. But if such a domain exists then it could be small enough and the solution of our task at $\tau \ne 0$ could be constructive enough. In this case the system with MM (21) could be quite acceptable for practical application.

3) Let such a convergence domain exists for the system with the MM (21). How we can estimate "dimensions" of such a domain?

In this work we will answer these questions with the help of a simple but enough adequate example.

5 Analysis of the adaptive nonstationary system motion on the base of simple example

Consider a simple example of the MM for a system in the form (16)

$$\frac{dx}{dt} + a_0 x = -\rho(t) sign(x)\ , \tag{25}$$

where x is a scalar coordinate, $a_0 = \text{const} > 0$,

$$\rho^{\max} \ge \rho(t) \ge \rho^{\min}\ ,\ \rho^{\min} = \text{const} > 0\ . \tag{26}$$

It is evident that the motion of the system with the MM (25) converges to the motion

$$x \equiv 0 \tag{27}$$

from any initial point $x(t_0) = x_0$ for $t > t_0$. Nevertheless to illustrate above theory we will show this fact with a method that was used before.

Let us choose for the system with the MM (25) the Lyapunov function in the form

$$V_1(x) \equiv \frac{1}{2}\kappa x^2 \tag{28}$$

where $\kappa = \text{const} > 0$.

A derivative $\frac{dV_1(x)}{dt}$ with respect to the MM (25) could be derived as the expression

$$\frac{dV_1(x)}{dt} = -\kappa(a_0 x^2 + \rho(t)|x|) . \qquad (29)$$

A sliding mode on the line $x = 0$ in the plane $\{x,\, t\}$ does not contradict to the fact that the motion $x(t)$ converges to the motion (27).

Now we consider the system with a MM that is analogous to the MM (21)

$$\tau\frac{d^2x}{dt^2} + \frac{dx}{dt} + a_0 x = -\rho(t)sign(x) . \qquad (30)$$

Let us rewrite the MM (30) in the form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{1}{\tau}(a_0x_1 + x_2 + \rho(t)Sign(x_1)) \end{aligned} \qquad (31)$$

or in a matrix form

$$\dot{x} = Ax + \mu(t,x) \qquad (32)$$

where $x^{\mathrm{T}} = (x_1 \;\, x_2)$, $A = \begin{pmatrix} 0 & 1 \\ -\frac{a_0}{\tau} & -\frac{1}{\tau} \end{pmatrix}$,

$$\mu(t,x) = \begin{pmatrix} 0 \\ -\frac{\rho(t)}{\tau}sign(x_1) \end{pmatrix}.$$

Now we will formulate the set of enough obvious statements.

Statement 1: During the motion of the system with the MM (30) a sliding mode on the line $x_1 = 0$ $(x_2 \neq 0)$ in a phase plane $\{x_1,\, x_2\}$ does not arise.

Statement 2: There are nonstationary functions $\rho(t)$ in the MM (31) that the motion

$$x_1 = 0, \;\; x_2 = 0 \qquad (33)$$

is unstable.

Proof: Consider the Lyapunov function in the form

$$V_2(x) = \kappa(x^T Px) , \qquad (34)$$

where $\kappa = \text{const} > 0$, P is a positive definite matrix to be defined by the Lyapunov equality $-(A_0^T P + PA_0) = Q$, Q is a prescribed negative definite matrix.

A derivative $\frac{dV(x)}{dt}$ according to the system (32) is defined by the equation

$$\frac{dV_2(x)}{dt} = \kappa(x^T Qx) - 2\frac{\rho(t)}{\tau}\sigma_2 sign(x_1) , \qquad (35)$$

where $(Px)^T = (\sigma_1,\, \sigma_2)$, $P = (p_{ij})$ $(i, j = 1,2)$.

Let us take the function $\rho(t)$ in the form

$$\rho(t) = \rho^{\max} sign(x_1) sign(\sigma_2) . \qquad (36)$$

Then the equation (35) takes the form

$$\frac{dV_2(x)}{dt} = \kappa(x^T Qx) + 2\frac{\rho^{\max}}{\tau}|\sigma_2| , \qquad (37)$$

where $\sigma_2 = p_{21}x_1 + p_{22}x_2$, $p_{21} \geq 0$, $p_{22} > 0$.

It is evident that on the phase plane $\{x_1,\, x_2\}$ in a small neighborhood of the point (33) a positive term in the equality (37) exceeds a negative one. So there is some domain including the point (33) that inequalities

$$V_2(x) > 0, \;\; \frac{dV_2(x)}{dt} > 0 \qquad (38)$$

take place for any point of this domain. According to Lyapunov's theorem about an instability, inequalities (38) prove the statement 2

Statement 3: For the system with the MM (32) there is a convergence domain $G(x_1,\, x_2)$.

Proof: Qualitatively, the proof of the Statement 3 follows from the proof of the Statement 2. Really, from the equation (37) it follows that in a small neighborhood of the point (33) there exists some domain including the point (33) that inequalities (38) take places but for points that are displaced in a respectively distant position the inequalities

$$V_2(x) > 0, \;\; \frac{dV_2(x)}{dt} < 0 \qquad (39)$$

take places.

But such a qualitative argument is valid for the concrete function $\rho(t)$ in the form (36) and the concrete Lyapunov function (34). For other nonstationarity functions $\rho(t)$ the motion (33) of the system with the MM (30) could be stable.

Naturally, for a practice applying of the sinthesized adaptive algorithm it is necessary to propose a method for "dimensions" estimation of a convergence domain. In [Zemlyakov and Danilova, 2008] such convergence domain $G(x_1, x_2)$ for the system with MM (32) was obtained in the form of a polygon presented in figure 1. This polygon takes place at $\tau < \frac{a_0}{4}$.

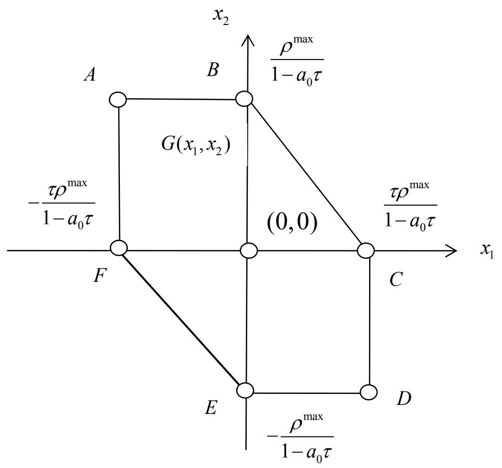

Figure 1. Convergence domain

Acknowledgements

The work reported in this paper is a contribution to Project 09-08-00064, funded by the Russian Foundation for Basic Research and by the Program for Fundamental Research No 15 of Division on the EMMCP of RAS.

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