STEADY-STATE VIBRATION MITIGATION IN A PIECEWISE LINEAR BEAM SYSTEM USING PD CONTROL

Rob H.B. Fey*, Henk Nijmeijer
Department of Mechanical Engineering
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, NL
e-mail: R.H.B.Fey@tue.nl, H.Nijmeijer@tue.nl
fax: +31 (0)40 2461418

*corresponding and presenting author

Ron M.T. Wouters
Department of Technology
YACHT BV
P.O. Box 12610, 1100 AP Amsterdam-ZO, NL
e-mail: ron.wouters@yacht.nl

Abstract
PD-control (control using Proportional and/or Derivative feedback) is applied to a piecewise linear beam system for steady-state vibration amplitude mitigation. Two control objectives are formulated: 1) to reduce the transversal vibration amplitude of the midpoint of the beam at the frequency where the first harmonic resonance occurs, and 2) to achieve this in a larger excitation frequency range. The vibration reduction that is achieved in simulations and validated by experiments is very significant for both objectives. Current results obtained with active PD-control are compared with earlier results obtained using a passive Dynamic Vibration Absorber.

Key words
Vibration mitigation, PD-control, piecewise linear beam system, numerical analysis, experiments

1 Introduction

Currently, demands concerning performance and accuracy of machinery and structures are set at high levels. Due to this the mitigation of vibrations in machinery, structures, and systems has become more important. To achieve this passive [den Hartog, 1985; Hunt, 1979, Korenev and Reznikov, 1993; Mead, 1999], semi-active [Preumont, 2002] and active [Meirovitch, 1990] vibration control can be used.

Piecewise linear systems are frequently met in engineering practice, see for example [Fey and van Liempt, 2002]. Steady-state dynamics of uncontrolled piecewise linear single-dof systems and multi-dof beam systems were studied in [Fey et al., 1996; Shaw and Holmes, 1983; van de Vorst et al., 1996].

In this paper active PD-control will be applied for vibration mitigation of a harmonically excited piecewise linear beam system. In [Bonsel et al., 2004] a linear Dynamic Vibration Absorber (DVA) was used in order to passively reduce the steady-state vibrations in the same piecewise linear beam system. Obviously, application of more advanced (nonlinear) controllers and observers may result in increased vibration reduction but also in increased costs, increased complexity and lower reliability [Doris, 2007].

In the next section first the experimental setup of the piecewise linear beam system will be introduced. The steady-state behavior of the uncontrolled system will be shown in section 3. Subsequently, two control objectives and the PD-controller design approach will be presented in section 4. Section 5 will discuss the numerical model of the system. In section 6 first the separate effect of Proportional feedback and secondly the separate effect of Derivative feedback on the steady-state behavior of the closed loop system will be investigated. Based on the insights obtained, in section 7 two PD-control settings will be determined in order to realize the two control objectives. Experimental and numerical results will be compared. In section 8 a brief comparison between the results obtained in this paper and the results obtained in [Bonsel et al., 2004] using a linear DVA will be carried out. Finally, in section 9 conclusions will be drawn.

2 Experimental setup

Figure 1 shows the schematic representation of the experimental setup of the system. The setup exists of a steel beam (a) supported at each side by a leaf spring (b).

In the middle of beam (a) a one-sided leaf spring (c) is placed flushing to beam (a) and making the system piecewise linear. Spring (c) only makes contact with the main beam for downward deflection of the midpoint of beam (a). The amount of nonlinearity in piecewise linear systems is indicated by the ratio of the one-sided stiffness and the (local) linear stiffness (of beam (a) in this case). Here, this ratio equals 2.7.

The system is transversally excited by a harmonic force generated by an eccentrically rotating mass mechanism, which is attached to the middle of beam (a) and is driven by an electric synchronous motor.

The excitation frequency $f = \omega/(2\pi)$ can be varied between 0 and 60 Hz.

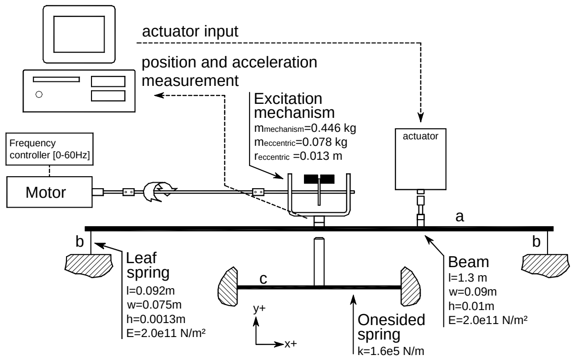

Figure 1: Schematic representation of the experimental setup.

The actuator, which exerts the PD-control force to the system, is placed as near as possible (0.2 m) to the midpoint of the beam. The operation of this actuator is based on the principle that a force is generated when a current flows through a coil, which is placed in a permanent magnetic field. The midpoint transversal displacement and acceleration are measured to determine the control force. These measurement signals are processed by a data acquisition system, which determines an appropriate current amplifier input for the digital PD-controller.

3 Steady-state behavior of the uncontrolled system

In Figure 2 the steady-state response of the uncontrolled system is shown. In this figure the quantity max disp of a periodic solution of the transversal displacement of the beam midpoint $y_{mid}(t)$, defined by:

$$\text{max disp} = \max y_{mid}(t) - \min y_{mid}(t) \qquad (1)$$

is determined for excitation frequencies ranging from 10 to 60 Hz. Note that a value of max disp close to zero does not necessarily mean that the overall vibration level of the beam is close to zero, because the beam may be vibrating in a shape with a node near or in the midpoint of the beam.

Figure 2 shows simulation results and experimental results. Simulation results are based on a numerical model which will be introduced in section 5 and are given for the uncontrolled system *without* actuator as well as the uncontrolled system *with* passive actuator dynamics. In the former case stable periodic solutions are indicated with dashed lines. In the latter case the stable periodic solutions are indicated with solid lines. In both cases unstable periodic solutions are indicated by black dots.

Calculation of (branches of) periodic solutions and their stability and detection of bifurcation points on these branches is based on theory and numerical methods described in e.g. [Fey et al., 1996; Parker and Chua, 1989; Thomsen, 2003].

Experimental results (circles) are only included for the uncontrolled system *with* passive actuator dynamics. A good correspondence can be observed between experimental and simulation results.

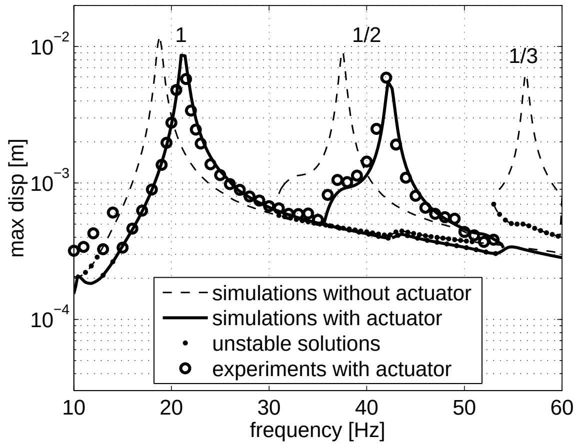

Figure 2: Response of the uncontrolled system.

Figure 2 shows that for both cases (*with* or *without* passive actuator dynamics) a harmonic resonance peak occurs near 20 Hz and a related ½ subharmonic resonance near the double of this frequency. For the case without actuator also a related 1/3 subharmonic resonance is visible near 56 Hz. So, by adding the passive actuator dynamics the global dynamic behavior of the uncontrolled system does not change. Resonance peaks, however, shift to somewhat higher excitation frequencies.

4 Control objectives and PD controller design approach

Two separate control objectives are formulated:

1) Minimize max disp defined by equation (1) at the first harmonic resonance frequency of 21.2 Hz of the uncontrolled piecewise linear beam system (*with* actuator dynamics), see Figure 2.
2) Reduce max disp defined by equation (1) in the frequency range 10-60 Hz. No mathematical optimization criterion will be used to realize this objective. Here, the performance of the PD-control action is evaluated by visual inspection of multiple amplitude-frequency plots such as Figure 2. In this visual inspection mainly will be focused on the success of the suppression of the harmonic *and* subharmonic resonance peaks, which are present in the uncontrolled situation; simultaneously, the appearance of new resonance peaks in the frequency range 10-60 Hz is not permitted. This visual inspection will be carried out for the whole (realizable) design space of the two PD control parameters k_p and k_d, which will be introduced later.

It is emphasized again that vibration reduction of the midpoint of the beam does not guarantee *overall* vibration reduction of the beam, because for the controlled situation this midpoint may behave as a node, while the rest of the beam is still vibrating at high vibration levels. Afterwards, it will be checked if the latter situation, which obviously is undesirable with respect to *overall* vibration reduction, does not occur.

The motivation to use a PD-control force in the mentioned frequency range of 10-60 Hz is based on the observation that in this frequency range only one (nonlinear) normal mode is dominant, see section 5.

The PD-control force is applied 0.2 m from the midpoint of the beam, which is practically the nearest possible position to the beam midpoint. The beam midpoint would have been the most effective actuation position because this is the position where: 1) excitation of the system takes place, 2) the one-sided spring is coupled to the beam, 3) the dominant 2[nd] eigenmode (of the system without one-sided spring) in the frequency range of interest shows a maximum transversal displacement, and 4) the vibration reduction should be realized. A digital PD-controller, which makes use of the transversal displacement measurement and an (indirect) transversal velocity measurement (both at the midpoint of the beam), determines the magnitude of the PD-control force F_c, which should be equal to:

$$F_c = -k_p\ y_{mid} - k_d\ \dot{y}_{mid}\,, \tag{2}$$

where y_{mid} and $\dot{y}_{mid}$ are respectively the midpoint transversal displacement and velocity, and k_p and k_d are the corresponding gains.

To realize each separate control objective two different set-points of the PD-controller will be designed. First, however, the effects of P- and D-action on the system's behavior will be determined separately. With the insight gained later a PD-controller setting will be determined for each separate control objective.

Now, first the dynamic modeling of the system with and without PD-control will be discussed.

5 Dynamic modeling

A 4-dof model is derived for efficient prediction of the dynamics of the system:

$$M\ddot{x} + D\dot{x} + K(y_{mid})x = F \tag{3}$$

with:

$$x = [y_{mid} \quad y_{act} \quad p_2 \quad p_3]^T\,,$$

$$K(y_{mid}) = \begin{Bmatrix} K_l & if & y_{mid} > 0 \\ K_l + K_{os} & if & y_{mid} \le 0 \end{Bmatrix}, \text{ and}$$

$$F = \left[m_e\omega^2 r_e \cos(\omega t) \quad k_m\ G_a u \quad 0 \quad 0\right]^T.$$

M, D and K_l represent respectively the reduced mass, damping and stiffness matrix, derived by dynamic reduction of a linear finite element model including the main beam, the passive actuator dynamics and the periodic excitation mechanism. The Ritz reduction matrix T, relating x to the unreduced dof column q ($q = Tx$), is based on the 2[nd] eigenmode (16.2 Hz, dof p_2) and the 3[rd] eigenmode (54 Hz, dof p_3), see Figure 3, and two residual flexibility modes [Fey et al., 1996] defined for y_{mid} and y_{act} (transversal displacements of the beam midpoint and the actuator position). The 1[st] eigenmode is suppressed by the drive shaft of the excitation mechanism.

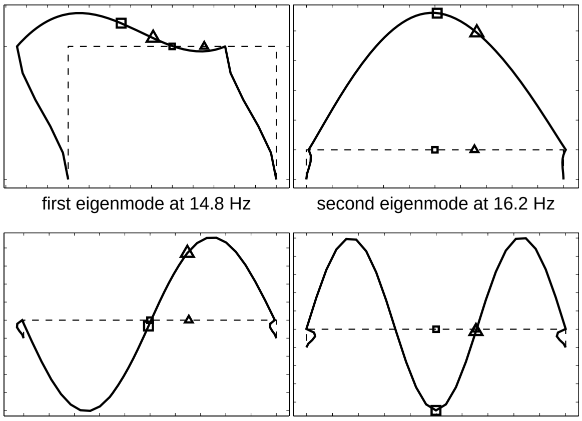

Figure 3: First 4 eigenmodes (black solid lines) of undamped, unreduced linear model. Midpoint position (□), actuator position (Δ).

Note that passive actuator dynamics slightly disturb the (anti-)symmetry of eigenmodes with respect to the beam midpoint. The stiffness of the one-sided spring is represented by matrix K_{os} (containing one non-zero element (1,1) equal to k_{os}). Damping matrix D is based on 2% modal damping, which results in a good match between measured and calculated resonance amplitudes. Elements 1 and 2 of F contain respectively the excitation force and the control force F_c. In the excitation force m_e is the rotating eccentric mass, r_e its eccentricity, and $\omega = 2\pi f$ its angular frequency. In the actuation force k_m is the motor constant, G_a the gain of the current amplifier, and u the voltage, which is chosen so that F_c according to equation (2) is realized.

The velocity signal needed for the controller was determined using analog integration of the measured acceleration signal in combination with an analog low-pass filter to reduce high frequent noise. A model of this filter is also added to the simulation model.

The nonlinear normal mode closely related to eigenmode 2 (with eigenfrequency 16.2 Hz) of the linear beam dominates the response near the first harmonic resonance peak at 21.2 Hz in Figure 2 and in fact in almost the whole frequency range of 10-60 Hz. The first harmonic resonance peak occurs at 21.2 Hz instead of 16.2 Hz due to the one-sided spring [Shaw and Holmes, 1983].

6 Effect of separate P-action and D-action

First the effect of P-action on the response of the piecewise linear system is investigated ($k_d = 0$). Figure 4 shows max disp, see equation (1), in the excitation frequency range 10-60 Hz for several values of k_p. Stable simulation results are validated by experimental results (circles). For larger gains the discrepancies between numerical and experimental results increase somewhat.

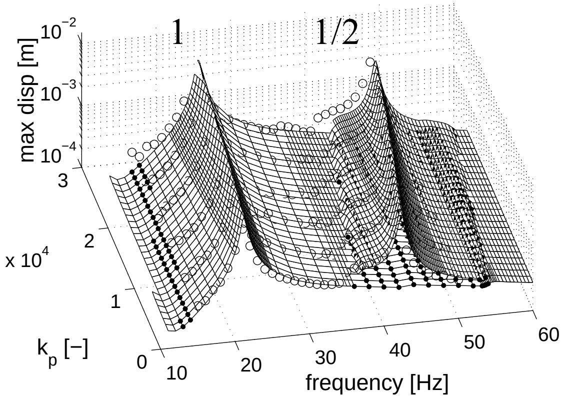

Figure 4: max disp in the range 10-60 Hz for varying Proportional feedback. Surface: simulation results. Circles: experimental results. Black dots: unstable numerical solutions.

Increasing k_p shifts the resonances to higher frequencies because 'stiffness' is added to the system. The ½-subharmonic resonance near 42 Hz shifts approximately twice as much as the harmonic resonance near 21 Hz because it occurs near twice the harmonic resonance frequency due to nonlinear coupling.

In Figure 4 only positive values of k_p are considered. Note that negative values may also be applied and in fact will be used in section 7.

Figure 5 shows the effect of D-action only ($k_p = 0$). Again the experimental and simulation results match (reasonably) well. Derivative feedback significantly suppresses both the harmonic and ½-subharmonic resonances near respectively 21 Hz and 42 Hz.

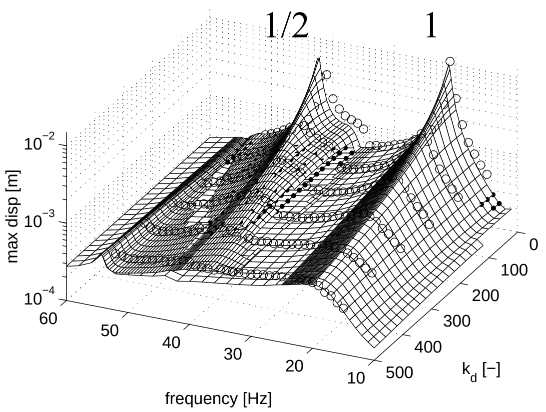

Figure 5: Response of the system for varying Derivative feedback. Surface: simulation results, unstable solutions are indicated by black dots. Circles: experimental results.

7 PD-control

Now the separate effects of P- and D-action on the system behavior are known the effect of combined P- and D-action on the systems behavior can be understood better. Experimentally applicable combinations of k_p and k_d are limited because of: limited power of the amplifier-actuator combination, introduction of unstable behavior, limited accuracy of the measured signals, and limitations of the mechanical design of the actuator, leading to the following constraints:

$$-6\times10^4 \le k_p \le 2.5\times10^4,\ 0 \le k_d \le 600.$$

7.1 Control objective 1

Figure 6 shows a contour plot of max disp (see equation (1)) in [mm] of simulated harmonic solutions for experimentally feasible k_p, k_d combinations at an excitation frequency of 21.2 Hz.

The contours in the grey area refer to unstable harmonic solutions. Here, the stable ½-subharmonic resonance peak coexists, which has relatively high max disp values. Set-point M_2 ($k_p = -4.6\times10^4$, $k_d = 0$) was chosen to fulfill control objective 1, because then neither k_p nor k_d needs to be to set to its limit value, although a slightly higher max disp value results than in set-points M_1 and M_3. This value, however, still is a factor 20 lower compared to the uncontrolled maximum displacement of 10 mm.

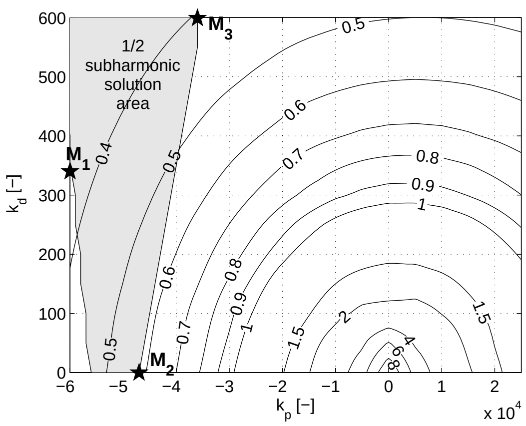

Figure 6: max disp [mm] for Proportional and Derivative feedback combinations at an excitation frequency of 21.2 Hz.

Figure 7 shows the results for the controller settings of setpoint M_2 in a wider frequency range. The controlled response shows that, due to the negative proportional feedback, the first harmonic resonance and ½-subharmonic resonance are shifted to lower frequencies. A local minimum of max disp, see equation (1), of *stable* periodic solutions is now found at 21.2 Hz between the two new resonance frequencies.

It may be noted that application of negative Proportional feedback is quite unusual, since in general it leads to destabilization. Negative P-action in linear systems will move the real parts of the poles in the positive direction of the real axis. However, obviously, also in the latter case negative P-action may shift an anti-resonance to a specific excitation frequency.

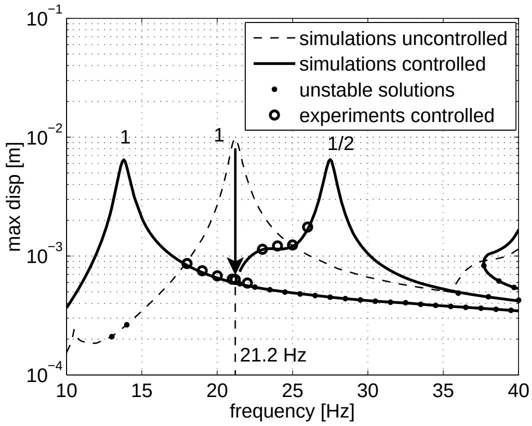

Figure 7: Vibration reduction of the harmonic resonance of the uncontrolled system at 21.2 Hz for set-point M_2. Solid line: stable simulation results, black dots: unstable simulation results. Circles: experimental results. Dashed line: response of uncontrolled system including the passive actuator.

7.2 Control objective 2

When the results of various combinations of P- and D-action are visually compared (not presented here), it appears that actually only D-action is needed to decrease the harmonic and subharmonic resonance peaks in order to maximize overall displacement reduction in the range 10-60 Hz.

Very large P-action could shift all resonances to frequencies above 60 Hz but can not be applied experimentally because amplified measurement noise results in an unstable system.

The circles in Figure 8 show the experimental results for the maximum experimentally applicable D-action (k_p=0, k_d=600) again resulting in a vibration reduction of about a factor 20 (the largest reduction occurs near the harmonic resonance).

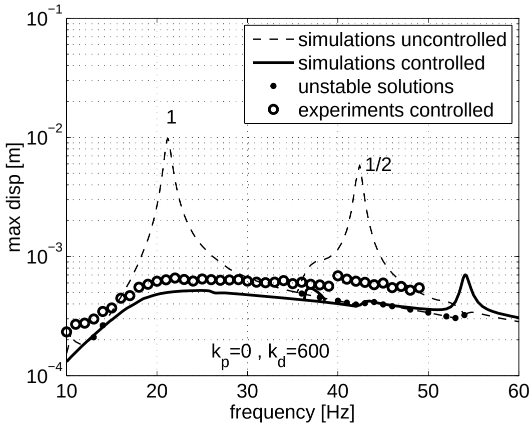

Figure 8: Vibration reduction in the range 10-60 Hz. Solid line: simulation results, circles: experimental results (set-point: k_p=0, k_d=600). Dashed line: uncontrolled system.

The differences between the simulation results and the experimental results in Figure 8 are largely due to the noise in the 'measured' velocity signal (actually the integrated measured transversal acceleration of the beam midpoint) resulting in a limited accuracy of the applied control force. Better correspondence between experimental and simulation results is obtained for lower D-action at the cost of larger max disp values.

8 Comparison with passive control

Passive vibration control of the same system was applied in [Bonsel et al., 2004] by fixing (at the same position) a linear DVA to the beam instead of the PD-controller. The eigenfrequency of this DVA, which is basically a single dof mass-spring-damper system, was tuned to the first harmonic resonance frequency of the piecewise linear system. In [Bonsel et al., 2004] the same two control objectives were formulated as in the current paper. The *undamped* DVA was applied to reduce the vibration amplitude at the first harmonic resonance (control objective 1), whereas the *damped* DVA was used to realize vibration reduction in the frequency range 10-60 Hz (control objective 2).

The amplitudes of periodic solutions obtained by passive vibration reduction are shown for control objective 1 in Figure 9 and for control objective 2 in Figure 10.

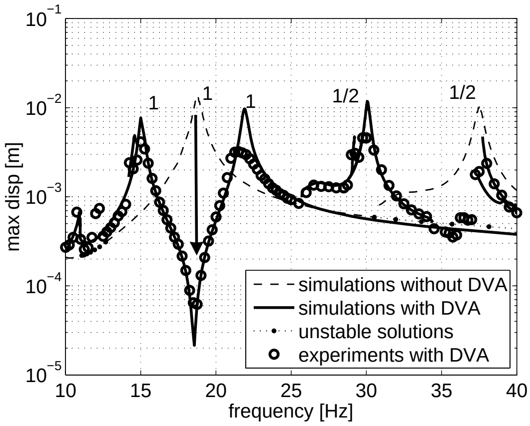

Figure 9: Suppression of the first harmonic resonance peak near 19 Hz using an *undamped* DVA.
Solid lines: simulation results with DVA, circles: experimental results with DVA. Dashed lines: simulation results without DVA.

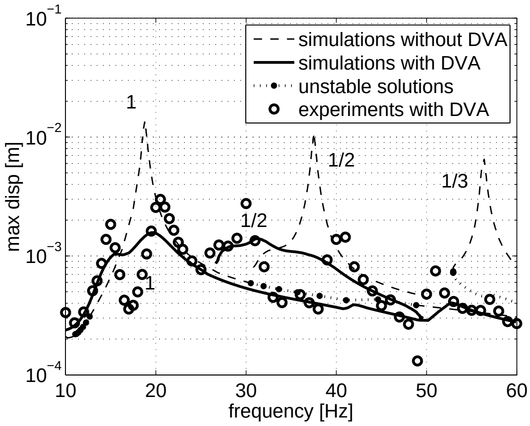

Figure 10: Vibration reduction in the frequency range 10-60 Hz using a *damped* DVA.
Solid lines: simulation results with DVA, circles: experimental results with DVA. Dashed lines: simulation results without DVA.

With respect to control objective 1 it can be seen that the *undamped* DVA (Figure 9) realizes a larger vibration reduction at the harmonic resonance frequency than P-control (Figure 7).

With respect to control objective 2 it is clear that D-control (Figure 8) gives lower max disp values than the *damped* DVA (Figure 10), except for excitation frequencies near 54 Hz. The experimental results of the system with the damped DVA differ substantially from the simulation results due to the fact that the maximum value of the damping coefficient, which could be realized in the experiment, was lower than the optimal numerical value for the DVA damper characteristic. In spite of this the vibration reduction realized with active D-control still is larger than the simulated results with the optimally damped DVA.

9 Conclusions

This paper shows that a linear PD-controller can be effectively used for vibration mitigation of piecewise linear systems for the considered case of flush with a moderate amount of nonlinearity. Simulation results have been validated by experimental results and a (reasonably) good correspondence has been obtained.

First, the effects of P-action and D-action have been investigated separately. Proportional feedback mainly shifts the excitation frequencies where (sub)harmonic resonances occur. Application of D-action results in substantial reduction of the resonance amplitudes in the frequency range of interest (10-60 Hz).

Subsequently, various experimentally feasible combinations of P- and D-action have been investigated to determine which combinations fulfill the two control objectives as good as possible. It appears that pure (negative) P-action gives the best result for the first control objective, whereas pure D-action gives the best result for the second control objective. For the case of flush considered here, the results indicate that the effect of PD-control applied to a piecewise linear system is comparable to a large extent to the effect of PD-control applied to a linear system.

The results obtained with the active PD-controller and the results obtained with the passive DVA have been compared. With respect to the first control objective the undamped DVA results in a lower response amplitude than the active P-controller. With respect to the second control objective, in the frequency range of interest, the active D-controller on average leads to lower response amplitudes than the damped DVA. However, it must be noted that the passive DVA and the active PD controller are in fact two different devices with for instance different mass and different power capacities. This hampers a fair comparison between the performances of the passive and the PD controller (for each of the two control objectives).

As stated before, the vibration reduction realized for the transversal displacement of the midpoint of the beam does not guarantee vibration reduction for transversal displacements of other positions on the beam. However, by visual inspection it was verified that also on other beam positions vibration reduction was achieved, except for excitation frequencies near the second harmonic resonance peak (i.e. 54 Hz), although the response level at this frequency still is low.

Acknowledgements

This work was partially supported by the Dutch-Russian program on interdisciplinary mathematics ‘Dynamics and Control of Hybrid Mechanical Systems’ (NWO grant 047.017.018) and the HYCON Network of Excellence (contract no. FP6-IST-511368).

References

Bonsel, J.H., Fey, R.H.B. and Nijmeijer H. (2004), Application of a dynamic vibration absorber to a piecewise linear system, *Nonlinear Dynamics*, **37**(3), pp. 227-243.

Doris, A. (2007), *Output-feedback Design for Non-smooth Mechanical Systems: Control Synthesis and Experiments.* PhD-thesis, Eindhoven University of Technology.

Fey, R.H.B., van Campen, D.H. and de Kraker, A. (1996), Long term structural dynamics of mechanical systems with local nonlinearities, *ASME Journal of Vibration and Acoustics*, **118**(2), pp. 147-153.

Fey, R.H.B. and Liempt, F.P.H. van (2002), Sine sweep and steady-state response of simplified solar array models with nonlinear elements, in: *Proc. Int. Conf. on Structural Dynamics Modelling*, Funchal, Madeira, Portugal, June 3-5, pp. 201-210.

Hartog, J.P. den (1985), *Mechanical Vibrations, 4th edition*, Dover Publications.

Hunt, J.B. (1979), *Dynamic Vibration Absorbers*, Mechanical Engineering Publications, London.

Korenev, B.G. and Reznikov, L.M. (1993), *Dynamic Vibration Absorbers: Theory and Technical Applications*, John Wiley & Sons, Chichester.

Mead, D.J. (1999), *Passive Vibration Control*, Wiley & Sons, Chichester.

Meirovitch, L. (1990), *Dynamics and Control of Structures*, John Wiley, New York.

Parker, T.S. and Chua, L.O. (1989), *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, Berlin.

Preumont, A. (2002), *Vibration Control of Active Structures- An Introduction, 2nd edition*, Kluwer Academic Publishers, Dordrecht, the Netherlands.

Shaw, S.W. and Holmes, P.J. (1983), A periodically forced piecewise linear oscillator, *Journal of Sound and Vibration*, **90**(1), pp. 129-155.

Thomsen, J.J. (2003), *Vibrations and Stability, Advanced Theory, Analysis, and Tools, 2nd edition*, Springer, Berlin.

Vorst, E.L.B. van de, Assinck, F.H., de Kraker, A., Fey, R.H.B. and van Campen, D.H. (1996), Experimental verification of the steady-state behaviour of a beam system with discontinuous support, *Experimental Mechanics*, **36**(2), pp. 159-165.