AN IDENTIFICATION BY ADAPTING THE FICTITIOUS CONTROLLER TO DATA

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Abstract: In the present paper, we propose a new identification method by adapting the "fictitious controller" to the obtained data in the closed loop experiment. A fictitious controller, which is introduced in this paper, consists of the nominal model with unknown plant parameters and the implemented controller used in the closed loop. One of the key points of the present method is the adaptation of such a controller to the actual experiment data. Moreover, the adaptation can be performed by using an off-line nonlinear optimization. Since the required material in the proposed method is only one-shot experimental data under the normal operation like step responses, this method has a practical advantage in the sense that the costs and time for the identification can be reduced.

Keywords: Tuning, identification, fictitious controller, adaptation

1. INTRODUCTION

As is well known, the issue on system identification is one of the important areas in systems and control (cf. e.g., (Forssel and Ljung, 1999), (Lee *et al.*, 1995), (Verhagen, 1994) and so on). This paper also the relevant topics on system identification, particularly, on parameter identification by using the closed loop experimental data. Here, we propose a new identification method by using the fictitious controller including the plant structure with unknown parameters. One of the key points of the present method from the theoretical points of view is the adaptation of such controllers to the actual experiment data. Moreover, such an adaptation can be performed by using an off-line nonlinear optimization. Since the required material in the proposed method is only one-shot experimental data under the normal operation like step responses, this method has a practical advantage in the sense that the costs and time for the identification can be reduced.

This paper is organized as follows. In Section 2, we prepare the required notations and preparations. In Section 3, we present the main result of this paper. In Section 4, we show an experimental result in order to show the validity of our results. In Section 5, concluding remarks are given.

2. PRELIMINARIES

Let $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and the integers, respectively. Let $\mathbb{R}^n$ denote the set of real vectors of size n and let $\mathbb{R}^{n\times m}$ denote the set of real matrices of size $n \times m$. Let $(\mathbb{R})^{\mathbb{Z}}$ denote the set of discrete time signals. For $w \in (\mathbb{R})^{\mathbb{Z}}$, the value of w at the time $t \in \mathbb{Z}$ is denoted with w_t. For $w \in (\mathbb{R})^{\mathbb{Z}}$ and $a, b \in \mathbb{Z}$ such that $a \leq b$,

$w_{[a,b]}$ denotes the finite time part of w in the time interval $[a,b]$. We regard $w_{[a,b]}$ as an element of $\mathbb{R}^{[b-a+1]}$. Let q denote the shift operator defined by $qw_t := w_{t+1}$ for a time series $w \in (\mathbb{R})^{\mathbb{Z}}$. In the case of $w_{[a,b]}$, we regard $(qw_{[a,b]})_b = 0$.

Consider a single-input single-output, linear, time-invariant, and finite dimensional system in discrete time described by the transfer function $G(q)$. We denote the $i-$th Markov parameter of $G(q)$ with $G_{[i]}$. Let $u_{[0,N]}$ and $y_{[0,N]}$ denote the input and output data, respectively, obtained in the interval $[0,N]$. The output y_t of an operator $G(q)$ with respect to the input $u_{[0,t]}$ is written by the form of $y_t = \sum_{k=0}^{t} G_{[k]} u_{t-k}$. $y_{[0,N]} \in \mathbb{R}^{N+1}$ is regarded as the range of the following Toeplitz matrix operator with respect to $u_{[0,N]}$:

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} G_{[0]} & 0 & \cdots & 0 \\ G_{[1]} & G_{[0]} & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ G_{[N]} & \cdots & G_{[1]} & G_{[0]} \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{pmatrix}, \quad (1)$$

or equivalently,

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} u_0 & 0 & \cdots & 0 \\ u_1 & u_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ u_N & \cdots & u_1 & u_0 \end{pmatrix} \begin{pmatrix} G_{[0]} \\ G_{[1]} \\ \vdots \\ G_{[N]} \end{pmatrix}. \quad (2)$$

We denote the Toeplitz operator of Markov parameters $G_{[i]}$ as

$$\mathcal{T}_{[0,N]}^{G} := \begin{pmatrix} G_{[0]} & 0 & \cdots & 0 \\ G_{[1]} & G_{[0]} & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ G_{[N]} & \cdots & G_{[1]} & G_{[0]} \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}. \quad (3)$$

Similarly, we denote the Toeplitz matrix consisting of truncated time series $w_{[0,N]}$ as

$$\mathcal{T}_{[0,N]}^{w} := \begin{pmatrix} w_0 & 0 & \cdots & 0 \\ w_1 & w_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ w_N & \cdots & w_1 & w_0 \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}. \quad (4)$$

Moreover, we also prepare the following vector expression like a finite time series;

$$[G(q)]_{[0,N]} := \begin{pmatrix} G_{[0]} & G_{[1]} & \cdots & G_{[N]} \end{pmatrix}^{\mathrm{T}} \in \mathbb{R}^{N+1}. \quad (5)$$

By using these notations, it is easy to see that $(G(q)u)_{[0,N]} = \mathcal{T}_{[0,N]}^{G} u_{[0,N]}$. Moreover, Eq.(1) and Eq.(2) are described by

$$y_{[0,N]} = \mathcal{T}_{[0,N]}^{G} u_{[0,N]} = \mathcal{T}_{[0,N]}^{u} [G(q)]_{[0,N]} \quad (6)$$

In Eq.(1), the invertibility of $G(q)$ is equivalent to the nonsingularity of the Toeplitz matrix because of $g_0 \neq 0$. For transfer functions $G(q)$ and $H(q)$, it follows from the well-known commutative property of the product of Toeplitz matrices that $\mathcal{T}_{[0,N]}^{HG} = \mathcal{T}_{[0,N]}^{H} \mathcal{T}_{[0,N]}^{G} = \mathcal{T}_{[0,N]}^{G} \mathcal{T}_{[0,N]}^{H}$ holds.

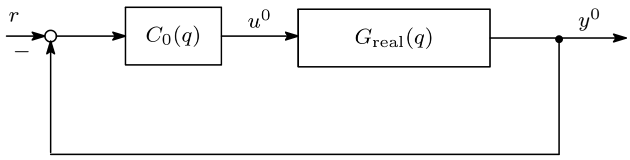

Fig. 1. A closed loop system

3. IDENTIFICATION BASED ON FICTITIOUS CONTROLLER

Consider a conventional feedback control system illustrated in Fig.1. We assume that a plant is a single-input single-output linear time-invariant minimum phase system described by

$$G(\rho, q) = \frac{c^p s^p + c_{p-1} q^{p-1} + \cdots + c_1 q + c_0}{s^d + a_{d-1} q^{d-1} + \cdots + a_1 q + a_0} \quad (7)$$

with the *unknown* parameter vector defined by

$$\rho := [a_0\ a_1\ \cdots\ a_{d-1}\ c_0\ c_1\ \cdots c_p]^{\mathrm{T}} \in \mathbb{R}^{p+d+1}$$

in discrete time. In the following, we use the notation $G(\rho, q)$ as the function of a variable parameter ρ. We also assume that there exists $\rho^* \in \mathbb{R}^{p+d+1}$ such that the dynamics of the real plant is described by $G(\rho^*, q)$, say, $G_{\mathrm{real}}(q) := G(\rho^*, q)$. At the same time, assume that we have the nominal plant

$$G_{\mathrm{nom}}(q) = \frac{c_p^0 q^p + c_{p-1}^0 q^{p-1} + \cdots + c_1^0 q + c_0^0}{a_d^0 q^d + a_{d-1}^0 q^{d-1} + \cdots + a_1^0 q + 1} \quad (8)$$

with the *known* nominal parameters

$$\rho^0 := [a_0^0\ a_1^0\ \cdots\ a_{d-1}^0\ c_0^0\ c_1^0\ \cdots c_p^0]^{\mathrm{T}}.$$

The structure of $G(\rho, q)$, $G_{\mathrm{real}}(q) = G(\rho^*, q)$ and $G_{\mathrm{nom}}(q) = G(\rho^0, q)$ are the same, which corresponds to the situation in which the structure of the dynamics of the plant is known. However, it is natural to consider that ρ^0 and ρ^* are different because of the aging changes under operation, uncertainty on the parameters, and so on. The aim of this paper is to obtain ρ^*, or a parameter ρ which is closer to ρ^* than ρ^0.

We also assume that the controller $C_0(q)$ stabilizes the closed loop and that $C_0^{-1}(q)$ is also proper. It is possible to write the nominal closed loop transfer function from the reference r to the output y described by

$$T_{\mathrm{nom}}(q) = \frac{G_{\mathrm{nom}}(q) C_0(q)}{1 + G_{\mathrm{nom}}(q) C_0(q)}. \quad (9)$$

Under these settings, the first step is to perform the initial experiment and obtain the input and the output data, which are denoted with $u_{[0,N]}^0$ and $y_{[0,N]}^0$, respectively. Next, we introduce the fictitious reference (cf. (Safonov and Tsao, 1997), (Souma *et al.*, 2004)) described by

$$\begin{aligned} \tilde{r}(\rho)_{[0,N]} &= (\tilde{C}(\rho, q)^{-1} u^0)_{[0,N]} + y_{[0,N]}^0 \\ &= \mathcal{T}_{[0,N]}^{(\tilde{C}(\rho)^{-1})} u_{[0,N]}^0 + y_{[0,N]}^0 \end{aligned} \quad (10)$$

with the "*fictitious controller*" $\tilde{C}(\rho, q)$ defined as

$$\tilde{C}(\rho, q) = \frac{G_{\rm nom}(q)}{G(\rho, q)} C_0(q) \tag{11}$$

by using the plant model with an unknown variable parameter ρ. Since $G(\rho, q)$ and $G_{\rm nom}(q)$ have the same structures, the assumption on the invertibility of $C_0(q)$ guarantees that of $\tilde{C}(\rho, q)$. It is easily seen that y^0 is not only the output of the actual closed loop with the controller $C_0(q)$ but also the output of the "fictitious closed loop" $\tilde{T}(\rho, q) := \frac{G(\rho,q)\tilde{C}(\rho,q)}{1+G(\rho,q)\tilde{C}(\rho,q)}$ with the fictitious controller $\tilde{C}(\rho, q)$ with respect to the fictitious reference $\tilde{r}(\rho)$. This is one of the system theoretic interpretations of $\tilde{C}(\rho, q)$ and $\tilde{r}(\rho)$, and the reason why we use the terminology of "fictitious controller". As shown afterwards, $\tilde{C}(\rho, q)$ is only used for the off-line optimization for the identification and is not implemented in the actual closed loop.

Next, we introduce the cost function described by

$$\begin{aligned}\tilde{J}(\rho) &:= \sum_{t=0}^{N} (y_t^0 - (T_{\rm nom}(q)\tilde{r}(\rho))_t)^2 \\ &= \|y^0_{[0,N]} - (T_{\rm nom}(q)\tilde{r}(\rho))_{[0,N]}\|_2^2. \end{aligned} \tag{12}$$

Since $\tilde{J}(\rho)$ is not linear quadratic function with respect to the parameter ρ, the minimization of $\tilde{J}(\rho)$ should be performed by the nonlinear optimization in the recursive way, e.g., Gauss-Newton method. The quantities required in the nonlinear optimization, like Gradient , Hessian and so on, consist of ρ, the initial data and the nominal plant model. Thus, the nonlinear optimization can be perfectly performed off-line by using only one-shot experiment data.

The problem is how the minimization of $\tilde{J}(\rho)$ is related to the identification of $G_{\rm real}(q)$. The next theorem gives an answer to this question, which is core of the proposed method of this paper.

Theorem 3.1. Consider the closed loop system described by Fig. 1 and assume that we have nominal model $G_{\rm nom}(q)$ and the stabilizing controller $C_0(q)$. Let $u^0_{[0,N]}$ and $y^0_{[0,N]}$ be the real experimental data from this closed loop systems. Assume that u_0^0 is nonzero and that

$$S_{\rm nom}(q) := \frac{1}{1 + G_{\rm nom}(q)C_0(q)} = 1 - T_{\rm nom}(q) \tag{13}$$

is invertible. Then, a parameter $\tilde{\rho}$ yields $\tilde{J}(\tilde{\rho}) = 0$ if and only if $\tilde{\rho}$ yields $[G_{\rm real}]_{[0,N]} = [G(\tilde{\rho})]_{[0,N]}$.

[Proof]: Here we give the sketch of the proof. Now it is easy to see that $\tilde{J}(\tilde{\rho}) = 0$ is equivalent to

$$y^0_{[0,N]} - \mathcal{T}^{T_{\rm nom}}_{[0,N]} \tilde{r}(\tilde{\rho})_{[0,N]} = 0. \tag{14}$$

The left-hand side of Eq.(14) can be calculated as follows;

$$\begin{aligned} & y^0_{[0,N]} - \mathcal{T}^{T_{\rm nom}}_{[0,N]} \tilde{r}(\rho)_{[0,N]} \\ &= y^0_{[0,N]} - \mathcal{T}^{T_{\rm nom}}_{[0,N]} \left(\mathcal{T}^{(C(\tilde{\rho})^{-1})}_{[0,N]} u^0_{[0,N]} + y^0_{[0,N]} \right) \\ &= y^0_{[0,N]} - \mathcal{T}^{T_{\rm nom}}_{[0,N]} \left(\mathcal{T}^{((G_{\rm nom}C^0)^{-1})}_{[0,N]} \mathcal{T}^{G(\tilde{\rho})}_{[0,N]} u^0_{[0,N]} + y^0_{[0,N]} \right) \\ &= \mathcal{T}^{S_{\rm nom}}_{[0,N]} y^0_{[0,N]} - \mathcal{T}^{T_{\rm nom}}_{[0,N]} \mathcal{T}^{((G_{\rm nom}C_0)^{-1})}_{[0,N]} \mathcal{T}^{G(\tilde{\rho})}_{[0,N]} u^0_{[0,N]} \\ &= \mathcal{T}^{S_{\rm nom}}_{[0,N]} \left(y^0_{[0,N]} - \mathcal{T}^{G(\tilde{\rho})}_{[0,N]} u^0_{[0,N]} \right) \\ &= \mathcal{T}^{S_{\rm nom}}_{[0,N]} \left(y^0_{[0,N]} - \mathcal{T}^{u^0}_{[0,N]} [G(\tilde{\rho})]_{[0,N]} \right). \end{aligned} \tag{15}$$

At the same time, since y^0 is the real output of $G_{\rm real}$ with respect to the real input u^0,

$$y^0_{[0,N]} = \mathcal{T}^{G_{\rm real}}_{[0,N]} \mathcal{T}^{u^0}_{[0,N]} = \mathcal{T}^{u^0}_{[0,N]} [G_{\rm real}]_{[0,N]} \tag{16}$$

holds. Together with noting that Toeplitz matrix of $S_{\rm nom}(q)$ is nonsingular due to its invertibility, we see that Eq.(14), Eq.(15) and Eq.(16) yield

$$\mathcal{T}^{u^0}_{[0,N]} [G(\tilde{\rho})]_{[0,N]} = \mathcal{T}^{u^0}_{[0,N]} [G_{\rm real}]_{[0,N]}. \tag{17}$$

Due to $u_0^0 \neq 0$, $\mathcal{T}^{u^0}_{[0,N]}$ is nonsingular. Hence $[G(\tilde{\rho})]_{[0,N]} = [G_{\rm real}]_{[0,N]}$ holds.

Conversely, assume that $[G_{\rm real}]_{[0,N]} = [G(\tilde{\rho})]_{[0,N]}$ holds. By using the fact that u^0 and y^0 are the real data and the similar calculation performed in Eq.(15), we see that Eq.(14) holds. (Q.E.D.)

Exactly speaking, the case in which the cost function is to equal to zero is rare. Moreover, the statement of this theorem is related to Markov parameter instead of the real coefficients of the transfer function. From the practical points of view, however, Theorem 3.1 implicitly guarantees that if we sufficiently minimize $\tilde{J}(\rho)$ then we obtain the parameter which is sufficiently close to ρ^* of the real transfer function $G_{\rm real}(q)$.

At this point, the minimization of $\tilde{J}(\rho)$ is performed by using recursive nonlinear optimization like Gauss-Newton method. Thus, it is preferable to provide the properties on the convergence to the minimizer $\tilde{\rho}$. From this points of view, next theorem is more relaxed than Theorem 3.1.

Theorem 3.2. In addition to the assumptions stated in Theorem 3.1, assume that $\tilde{J}(\rho)$ is continuous around $\tilde{\rho} \in \mathbb{R}^{p+d+1}$. Then $\lim_{\rho \to \tilde{\rho}} \sqrt{J(\rho)} = 0$ $\Leftrightarrow$ $\lim_{\rho \to \tilde{\rho}} [G(\rho)]_{[0,N]} = [G_{\rm real}]_{[0,N]}$.

[Proof]: Here we also give the sketch of the proof. Firstly, from the proof of Theorem 3.1, we see that $\lim_{\rho \to \tilde{\rho}} \sqrt{J(\rho)} = 0$ is equivalent to

$$\lim_{\rho\to\tilde\rho}\|y^0_{[0,N]}-\mathcal{T}^{T_{\mathrm{nom}}}_{[0,N]}\tilde r(\rho)_{[0,N]}\|_2$$
$$=\lim_{\rho\to\tilde\rho}\|y^0_{[0,N]}-\mathcal{T}^{u^0}_{[0,N]}[G(\rho)]_{[0,N]}\|_2$$
$$=\lim_{\rho\to\tilde\rho}\|\mathcal{T}^{u^0}_{[0,N]}\left([G_{\mathrm{real}}]_{[0,N]}-[G(\rho)]_{[0,N]}\right)\|_2=0. \quad (18)$$

Secondly, now suppose that $\lim_{\rho\to\tilde\rho}\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2$ does not converge to 0, i.e., for all $\delta>0$ there exists $\exists\epsilon>0$ such that

$$\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2>\epsilon, {}^\forall\rho \text{ s.t.} \|\rho-\tilde\rho\|_2<\delta. (19)$$

Rewrite $\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2$ as

$$\|(\mathcal{T}^{u^0}_{[0,N]})^{-1}\mathcal{T}^{u^0}_{[0,N]}\left([G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\right)\|_2$$
$$\le\lambda\|\mathcal{T}^{u^0}_{[0,N]}\left([G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\right)\|_2 \quad (20)$$

where λ is the maximum singular value of $(\mathcal{T}^{u^0}_{[0,N]})^{-1}$ (which is well-defined from the non-singularity of $\mathcal{T}^{u^0}_{[0,N]}$), and this is positive. From Eq.(19) and Eq.(20), we see that for all $\delta>0$ there exists $\epsilon>0$ such that

$$\|\mathcal{T}^{u^0}_{[0,N]}\left([G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\right)\|_2>\frac{\epsilon}{\lambda}$$
$${}^\forall\rho \text{ s.t. } \|\rho-\tilde\rho\|_2<\delta \quad (21)$$

which contradicts to Eq.(18). Hence, we see that if $\lim_{\rho\to\tilde\rho}\sqrt{J(\rho)}=0$ then $\lim_{\rho\to\tilde\rho}\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2$ also converges to 0. Thirdly, we have to show that $\lim_{\rho\to\tilde\rho}\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2=0$ implies $\lim_{\rho\to\tilde\rho}([G(\rho)]_{[t]}-[G_{\mathrm{real}}]_{[t]})^2=0$ for all $t=0,1,\cdots,N$. Note that $\lim_{\rho\to\tilde\rho}\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2=0$ is equivalent to that for all $\epsilon>0$ there exists $\delta>0$ such that

$$\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2<\epsilon, {}^\forall\rho \text{ s.t.} \|\rho-\tilde\rho\|<\delta. (22)$$

Now, suppose that for all $\delta_1>0$ there exist $\tau\in\mathbb{Z}$ and $\epsilon_1>0$ such that $([G(\rho)]_{[\tau]}-[G_{\mathrm{real}}]_{[\tau]})^2>\epsilon_1$ for all ρ such that $\|\rho-\tilde\rho\|_2<\delta_1$. Due to the arbitrariness of ϵ in Eq.(22), it is possible to substitute ϵ_1 into Eq.(22). Thus, we see that $\epsilon_1>\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2=\sum_{t=0,t\neq\tau}([G(\rho)]_{[t]}-[G_{\mathrm{real}}]_{[t]})^2+([G(\rho)]_{[\tau]}-[G_{\mathrm{real}}]_{[\tau]})^2$ for all ρ such that $\|\rho-\tilde\rho\|<\delta$, which implies $([G(\rho)]_{[\tau]}-[G_{\mathrm{real}}]_{[\tau]})^2<\epsilon_1$. This is the contradiction to $([G(\rho)]_{[\tau]}-[G_{\mathrm{real}}]_{[\tau]})^2>\epsilon_1$. Thus, we see that the convergence of $\lim_{\rho\to\tilde\rho}\|[G(\rho)]_{[0,N]}-[G_{\mathrm{real}}]_{[0,N]}\|_2=0$ implies $\lim_{\rho\to\tilde\rho}([G(\rho)]_{[t]}-[G_{\mathrm{real}}]_{[t]})^2=0$ for all $t=0,1,\cdots N$. Hence $\lim_{\rho\to\tilde\rho}[G(\rho)]_{[0,N]}=[G_{\mathrm{real}}]_{[0,N]}$ holds. The converse direction can be shown in similar way, we omit it here. (Q.E.D.)

Remark 3.1. IFT (cf. (Hjalmarsson *et al.*, 1998)), VRFT (cf. (Campi *et al.*, 2002), (Campi and Savaresi, 2006)) and FRIT(cf. (Souma *et al.*, 2004)) are known as the novel ideas of tuning methods for the controller. The key idea of the present paper is to tune a variable parameter of a fictitious controller which includes the plant parameter by using the data *directly*. Thus, our identification method can be regarded one of the interesting applications of these controller tuning methods to identification. □

Remark 3.2. In (Kaneko *et al.*, 2005), some of the authors provided the identification method based on the tuning of the feedforward controller in the two-degree of freedom of control scheme with only (one-shot) experiment data as one of the applications of the tuning method stated in Remark 3.1. In the real plant in the industries, the augmentation of the controller to the two-degree of freedom control system needs a lot of costs, it is preferable to perform identification in one-degree of freedom of control system from the practical points of view. This is another motivation of this study and one of the advantages over the previous work by the authors in (Kaneko *et al.*, 2005).□

Remark 3.3. The reference (Sala, 2007) discussed *how* conventional closed loop system identification and controller tuning method are unified. On the other hand, as explained in Remark 3.1 and 3.2, the issue of this paper is not an unification of the controller parameter tuning and closed loop system identification but one of the applications of the controller tuning to system identification. Thus, the issue treated in this paper differs from that in the work by Sala (Sala, 2007). □

Remark 3.4. The real measured data u^0 and y^0 include the noise. If it is difficult to neglect the effect of noise, we repeat the experiment with respect to *the same controller* $C_0(q)$ *twice.* This technique is also taken by IFT or VRFT (cf.(Hjalmarsson *et al.*, 1998) and (Campi *et al.*, 2002)). We denote the first experimental data with $\{y_n^{0(1)}:=y^{0(1)}+n_y^{(1)},\ u_n^{0(1)}=u^{0(1)}+n_u^{(1)}\}$ and the second experimental data with $\{y_n^{0(2)}:=y^{0(2)}+n_y^{(2)},\ u_n^{0(2)}=u^{0(2)}+n_u^{(2)}\}$, respectively. Here, $n_y^{(i)}$ and $n_u^{(i)}$ denotes the noise in the i-th experiment on the input and the output, respectively. $y^{0(i)}$ and $u^{0(i)}$ denotes the pure signal we require in this method. The experiment is performed in the closed loop, the correlation between e.g., $n_y^{(1)}$ and $u_n^{0(1)}$ can not be neglected. However, the two experiments are performed in the different time, it is possible to assume that $n_y^{(i)}$ and $n_u^{(i)}$ in the first experiment have no correlation with $n_y^{(j)}$, $n_u^{(j)}$, $y_n^{0(j)}$ and $u_n^{0(j)}$, where $i,j=(1,2)$ or $(2,1)$. Thus, by modifying Eq.(12) as

$$\tilde J_n(\rho)=\left({y_n}^{0(1)}_{[0,N]}-T_{\mathrm{nom}}(q)\tilde r(\rho)^1_{[0,N]}\right)^{\mathrm T}$$
$$\times\left({y_n}^{0(2)}_{[0,N]}-T_{\mathrm{nom}}(q)\tilde r(\rho)^2_{[0,N]}\right) \quad (23)$$

(where, $\tilde r^i(\rho)_{[0,N]}:=C(\rho)^{-1}{u_n}^{0i\prime}_{[0,N]}+{y_n}^{0i\prime}_{[0,N]}$, $i=1,2$), we can approximate the cost function so as to eliminate the effect of the noise. □

Remark 3.5. The characteristics of the obtained parameter depends on $T_{\rm nom}(q)$, because it determines the frequency band in which we can obtain the accurate mathematical model. Moreover, since the initial experimental data reflects on not only the plant dynamics but also the feedback loop property, the initial feedback controller $C_0(q)$ and the reference signal r also play crucial roles in this algorithm. The detailed observations for them are also the issues of the future studies. □

Remark 3.6. In the standard approach, the identifability is clarified as the conditions on the degree of the numerator and the denominator of the controller, PE characteristics of the noise, and the property of the reference signals explicitly. In our approach, the identifability of $G(q)$ depends on whether $\tilde{\rho}$ is the desired local minimum or not, which is related to the issues on numerical computations of non-linear optimization like Gauss Newton method. □

4. EXPERIMENTAL RESULT

In this section, we give an experimental result to show the validity of our approach. The system we address here is described by Fig. 4. The cart

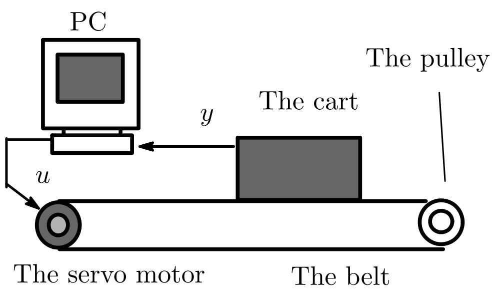

Fig. 2. The cart system

is attached to the belt which is moving by the rolling of the servo motor. The location y (output) from the initial position of the cart is measured by the potentiometer attached in the pulley and is send to the personal computer (PC). And the servo motor is driven by the voltage u (input) from PC. The sampling time of this control system is 0.001[sec].

From the physical points of view, it is known that the transfer function of this plant is described by $y = \beta/(s^2 + \alpha s)u$ in continuous time case. Thus, our aim is to obtain the parameters of the discrete time transfer function as $G(\rho, q) = \frac{c_1 q + c_0}{q^2 + a_1 q + a_0}$ where $\rho := [a_0\ a_1\ c_0\ c_1]^{\rm T} \in \mathbb{R}^4$ is a variable parameter vector. The nominal plant model is described by $G_{\rm nom}(q) = G(\rho^0, q)$ with $\rho^0 := [4.479 \times 10^{-3},\ -1.05,\ 8.898 \times 10^{-3},\ 2.278 \times 10^{-3}]$. We also use a conventional PI controller described by $C(q) = 1 + 0.01\frac{q}{q-1}$. Moreover, we also prepare

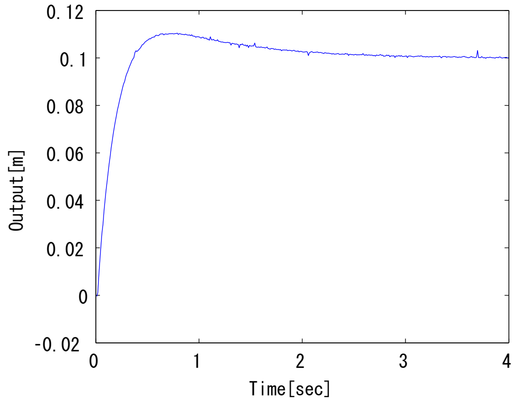

Fig. 3. Output data y^0

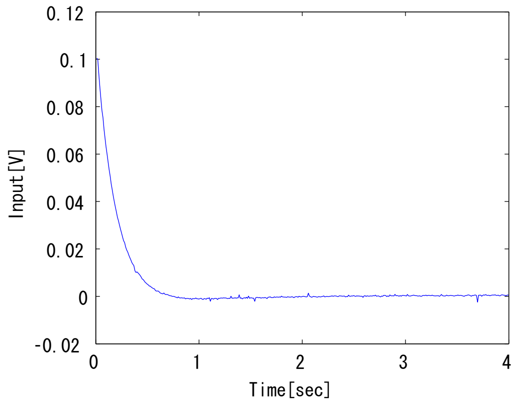

Fig. 4. Input data u^0

the nominal close loop transfer function $T_{\rm nom}(q)$ consisting of $G_{\rm nom}(q)$ and $C_0(q)$ as Eq.(9).

By using $C_0(q)$, we perform the first experiment with the reference signal $r = 0.1$[m]. The output y_0 and the input u_0 are illustrated as Fig.3 and Fig.4, respectively. Next by using the fictitious reference $\tilde{r}(\rho)$ in Eq.(10) and the fictitious controller $\tilde{C}(\rho, q)$ in Eq.(11), we perform the off-line nonlinear optimization so as to minimize $\tilde{J}(\rho)$ in Eq.(12). After the off-line Gauss-Newton optimization with 500 [times] iterations, we obtain $G(\tilde{\rho}, q)$ with $\tilde{\rho} = [0.1121,\ -1.112,\ 1.927 \times 10^{-2},\ 3.9 \times 10^{-2}]$. We illustrate the result of this tuning in Fig.5 where the real line, the chained line, and the dotted line are y_0, $T_{\rm nom}(q)\tilde{r}(\tilde{\rho})$ and $T_{\rm nom}(q)r$, respectively. Since the minimization of $\tilde{J}(\rho)$ can be sufficiently achieved, $T_{\rm nom}(q)\tilde{r}(\tilde{\rho})$ almost coincide with y_0 as shown in this figure. Next, in order to show the validity of the obtained $\tilde{\rho}$, we also perform frequency response experiments. In Fig. 6, we illustrate the output y with respect to $u = \sin(t)$ as the real line. Moreover, we also illustrate the simulation using the obtained $\tilde{\rho}$, and the nominal ρ^0, as the chained line and the dotted line, respectively.

It is clear to see that the output response simulated by using the obtained parameter $\tilde{\rho}$ is much closer to the real response than the response simulated by using the nominal parameter ρ^0.

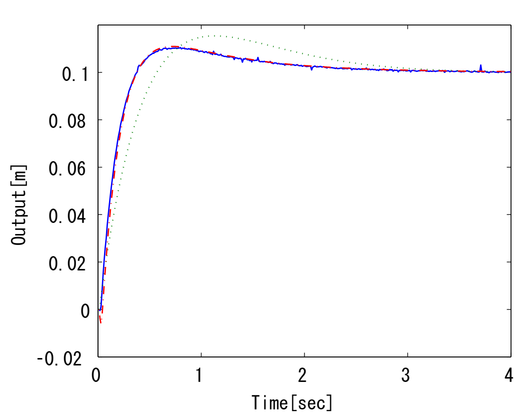

Fig. 5. Identification result (The real line:y_0 The chained line $T_{\rm nom}(q)\tilde{r}(\tilde{\rho})$, The dotted line:$T_{\rm nom}(q)r$)

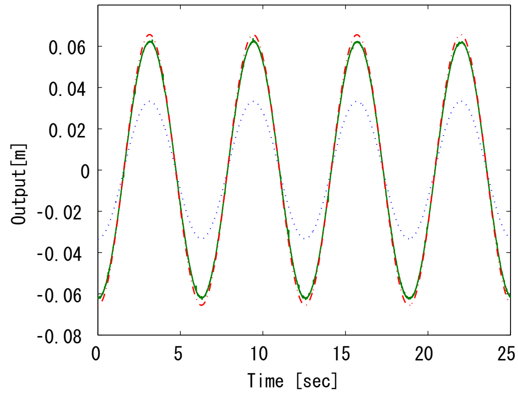

Fig. 6. The validation result w.r.t the frequency response : $1[rad/sec]$ (The real line: the actual response, The chained line: The simulation using $\tilde{\rho}$, The dotted line: The simulation using ρ^0)

Similarly, we illustrate the case in which $u = \sin(100t)$ is applied to this system in Fig.7. In this figure, we also see that the simulation using $\tilde{\rho}$ is much closed to the actual response than the simulation using the nominal parameter ρ^0. Thus, we observe that the obtained parameter reflects the dynamics of this plant sufficiently.

5. CONCLUSION

In this paper, we have proposed a system identification method based on the adaptation of fictitious controllers, which is introduced in this paper, including the structure of a plant model. This method requires only one-shot experiment data, it has a practical advantage in the sense that we can reduce the costs and time for the identification. As future studies, we are studying the issues which have been written as some remarks in Section 3 and the extension to non-minimum phase plants.

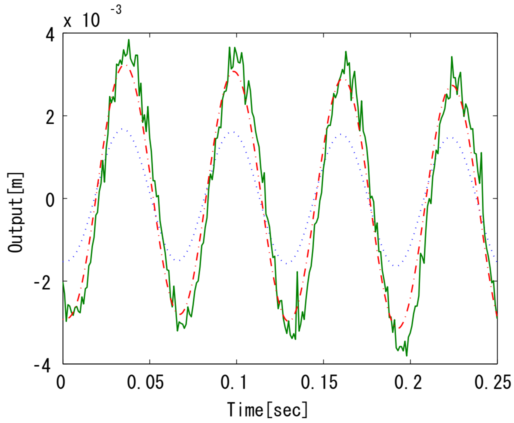

Fig. 7. The validation result w.r.t the frequency response : $100[rad/sec]$ (The real line: the actual response, The chained line: The simulation using $\tilde{\rho}$, The dotted line: The simulation using ρ^0)

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