Uncertain destination dynamics: Multistability, synchronization, and control

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Many systems in nature are characterized by the coexistence of different attractors for a given set of parameters. Examples for such behavior can be found in different fields of science ranging from mechanical systems to chemical systems and ecosystem dynamics. The behavior of multistable systems is rather complicated because of the complexly interwoven basins of attraction for the various coexisting attractors. Which of the attractors is realized depends crucially on the initial condition. Multistable systems are very sensitive to noise leading to a hopping process of the trajectory between different states. One of the system classes where this kind of behavior appears are coupled systems where multistability is often related with the loss of complete synchronization leading to the coexistence of synchronized and nonsynchronized attractors. A particularly interesting dynamics appears in two identical coupled systems when a special coupling is applied. This coupling is designed in such a way that the differential equation for the difference of 2 variables, say for example the first one of each system d(x1-y1)/dt=0 in the long-term limit. In that case this difference will be a constant c which is determined by the initial condition. In the long-term limit the system synchronizes but exhibits in principle infinitely many attractors corresponding to the different synchronization manifolds defined by the parameter c. This phenomenon has been called uncertain destination dynamics. We study two coupled Lorenz systems with a parameter set where each of the system is monostable. This monostability persists when the coupling is turned on and for a particular value of the coupling strength the system becomes multistable and infinitely many attractors emerge all of a sudden. This particular kind of multistability is rather fragile. One can show that a very tiny mismatch in the parameters of the system leads immediately to a disappearance of multistability. If noise is applied to such coupled systems then the trajectory jumps between different attractors as expected. These jumps correspond to a hopping between different synchronization manifolds. We study strategies to control the system in such a way that different attractors can be realized on purpose.