PHYSCON 2017, Florence, Italy, 17–19 July, 2017

MPC BASED COORDINATION FOR THE SUSTAINABLE MANAGEMENT OF PRODUCTION FACTORS IN AGRICULTURE

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Abstract

This article proposes a decision support system based on a Model Predictive Control (MPC) Scheme in order to conciliate sustainable short term economic returns of a set of agricultural production units in a given region and the its long term environmental sustainability. The overall coordination is achieved by a decentralized, adaptive, and hierarchic structure that, on the one end hand, promotes the long term common good by approximating the solution to an infinite horizon optimal control problem, and, on the other hand, provides bounds on the usage of agro-chemical indicators to each one of the production units. This is a very complex challenge and this work is a preliminary effort with emphasis on the problem formulation in a simple context. More precisely, two production units are modeled and an MPC based coordinator to define the production factors for each production unit are considered. In this process, the maximum principle of Pontryagin plays a key role in formulating the dynamics to be taken into account by the coordinator.

Key words

Model Predictive Control, Coordinated Control, Decision Support Systems, Sustainable Agriculture

1 Introduction

Optimal control has been increasingly regarded to provide the kernel for decision support systems for the management and control of production factors in agriculture. In [Lobo Pereira et al., 2013], several pertinent issues are presented on the basis of a selected state-of-the-art. The motivation is huge: there is a clear perception of the difficulty in meeting the future food needs on earth without preserving the essential environmental equilibrium, [Farzin, 2008]. However, data gathering processes, information processing, computational capabilities, and control theory have been growing in sophistication and power at a very fast pace, thus, laying promising perspectives of handling of huge complexity that these problems usually entail.

In this article, we propose a systematic approach to design a coordination control scheme to support the coordination of the management of multiple farms in a given environment that, while seeking to maximize their self short term economic returns, impose constraints on their production factors to ensure a fair contribution of each one of them to the long term environmental sustainability. Of course, this goal is extremely ambitious, and so we will focus on the MPC coordination mechanism and in investigating its properties by restricting ourselves to a simple scenario of two farms under very simple assumptions in terms of the number of variables that intervene in the process. The key goal is to build a basis on which future - more realistic - developments will take place to overcome the achievable performance of the ones currently available for this purpose.

An overview of the literature shows a huge variety of problems whose complexity varies widely. This variety concerns: (i) specification of the scope of the problem, and (ii) the problem specificities once its scope is defined. In the later, the modeling of wealth of dynamics and constraints so that the essentially relevant features are captured and, at the same time, the computational tractability is ensured, and specification of performance functionals and time horizons to be considered are key challenges. In what concerns the former, the extremely vast array of highly intertwined contexts that can be considered - economics, environment, climate, ecology, natural resources (notably, water and soil), spatial (from farm level to a region or country) and time scales (from short horizon of a few production cycles to long horizon returns). In order to deal with the complexity for either concentrated or distributed control and decision problem formulations, researchers have resorted to the specification of architectures enabling the organization of complex systems into multiple interacting monolithic simpler subsystems. However, to the best of our knowledge, there is no approach that integrates in one framework a decentralized structure of independent - possibly conflicting -decision makers - seeking maximal short term returns, and, an optimized and, simultaneously, “fair” global (not necessarily centralized) procedure that establishes production factors quotas for each individual decision-maker so that all cooperate for the common good of a sustainable environment in the medium to the long term.

The structure of this article is as follows. In the next section 2, we will provide a brief overview of the pertinent state-of-the-art. In section 3, the model of a production unit will be discussed in detail. This will be a simplified model in that it includes only the effects in the crop growth of small number of key production factors, notably, fertilizers, pesticides, and biological control. The optimal control problem for each production unit will be formulated and the characterization of its optimal strategy via the Maximum Principle of Pontryagin will be given in section 3. Then, in the ensuing section, 4, we will present the coordinating MPC scheme, including the assumptions under which it is formulated, being the derivation of its dynamics a key issue. This article is closed with some conclusions.

2 Brief State-of-the-art

In [Gorddard et al., 1995; Jones and Cacho, 2000; Manalil et al., 2011; Stiegelmeier et al., 2012], optimal control problems with an increasing degree of sophistication have been considered in order to find the combination of chemical and non-chemical control strategies optimizing the long term economic trade-off between crop yield profits, herbicide costs, and long term adverse effects of weed resistance. In particular, one should point out the effect of weeds resistance to herbicides which, in spite of the modeling difficulties, proved to be an important factor in the problem formulation. Various techniques, such as nonlinear programming, Pontryagin Maximum Principle and dynamic programming, have been used to solve the formulated optimal control problems. The general conclusion of these studies is that a better performance is achieved if a wider range of control methods are available and that, even without explicitly considering environmental factors in the problem, the optimal solutions are environmentally friendlier than the conventional ones. The optimal control of pests has been investigated in a number of articles, notably [Wetzstein et al., 1985; Rafikov and Balthazar, 2005]. Besides the economic issues addressed in the optimal weed control problems, now, issues concerning long term ecosystem equilibrium have been considered. Moreover, optimal strategies seeking long term environmental equilibria have been designed for problems with multiple pest species and by combining the use of fertilizers and pesticides. The Pontryagin Maximum Principle and dynamic programming techniques, [Pontryagin et al., 1962; Clarke et al., 1998; Arutyunov, 2000] have been used to solve these problems. In [Qi, 2010], a systemic approach is adopted in order to formulate optimization and optimal control problems to optimize the economic valuable botanical yield components based on a functional-structural plant growth model in terms of the source-sink dynamics. This model encompasses all pertinent ingredients which encompass both botanical, and ecological yield components, and all other environmental factors. A key challenge here is to ensure the compatibility of this model with the plant model in terms of spatial and temporal scales. In [Huhtala and Laukkanen, 2011], optimal control modeling was used to analyze how public resources should be allocated to small-scale water protection efforts in agriculture or, alternatively, to investments in large-scale waste water treatment plants to control point source loads. In [Risbey et al., 2009], an analysis is performed in the context of the Australian agriculture to show the need of increasingly adaptive policies to take into account the evolution of perceptions of the state of the system and of the intervening processes in the multiple spatial and temporal scales, as well as the increased role of environment changes for which climate variability plays a prominent role. Finally, a much more general context is considered in [Farzin, 2008] where it is argued that the optimal development path in the sense of the maximin criterion of intergenerational justice is too demanding to be practical and too costly for the economically less competitive. This calls for a policy development following an optimal growth approach while encompassing measures to mitigate the intergenerational and intra-generational welfare inequalities.

It is clear from the above that a general abstract control architecture satisfies some of the key identified requirements have been considered, but the articulation of the above discussed conflicting short term and of long term goals is clearly missing. This is the gap that this work intends to mitigate.

3 Production Unit Optimal Management

In this section we discuss three issues corresponding to the three subsections: Production dynamics modeling, optimal control formulation, and the characterization of the optimal control strategy via the maximum principle.

3.1 Dynamics

We consider a model that is simple enough to be tractable and, at the same time, captures sufficiently realistic features of a real life context of the intra-year model of the product growth required to forecast the prospective final production inferred from the current “health” of the plant. Let α be a superindex specifying the production unit. The state of the system $x^\alpha \in \mathbb{R}^4$ is specified by the following variables:

x_1^α - Quantity of primary products, which is considered of dimension 1.

x_2^α - Quantity of collateral products inherent to biological control, which is also considered of dimension 1.
x_3^α - Quantity of weeds which, as mentioned above are considered endemic.
x_4^α - "Distance" of the environment from the sustainability set S, that is $x_4^\alpha = d_S(w)$

where, here, w is the "state" of the environment which, for the purpose at hand, can be given by a measure of the health of the soil in terms of timely natural evolution of all soil ecosystems to a sustainable equilibrium, and S is the set of all values of w for which all soil ecosystems naturally evolve to a sustainable equilibrium within a certain time interval. This is defined to ensure stability and robustness to weather, and some climate changes.

A number of important simplifications are being considered: agrochemical resistance of pests is neglected; the use of the "artificial" variable w; and the health of the state does not affect x_2 and x_3 which are considered extremely well adapted to poor soils.

Before, going into the dynamic of the model, let us dwell on the important point of representing the evolution (i.e., time derivative) of the projected intra-year final primary production x_1, given by the plant health (expected size, leaves color, etc.). The game of the farmer consists in, at each point in time, selecting a strategy yielding maximal profit by the end of the production section (harvest time), by taking into consideration the current health status of the crop, besides other factors such as soil health and forecast of deviations from the normal weather pattern. This requires an intra-year model of the crop status. Without any deviation from the normal external conditions (abnormal weather patterns, unbalanced soil, plagues, etc.) the plants of the crop have different needs of nutrients, water, and sun exposure needs, at different stages of development.

For the sake of simplicity of the illustration but without sacrificing the essential idea, let us consider a model of crop plant with a growth profile with three different piecewise rates: $\gamma_i(\cdot)$, for $i = 1, 2, 3$. That is, we have $\dot{x}_1^\alpha = h_1^\alpha(t)$ where

$$h_1^\alpha(t) = \sum_{i=1}^{3} \gamma_i^1(t)\chi_{[t_{i-1},t_i)}(t)$$

where $t_0 = 0$, $0 < t_1 < t_2 < t_3$, $t_3 = 1$, and $\chi_A(t)$ is the indicator function of A. We may further simplify the dynamics by considering the coefficients γ_i's constant in their subinterval, and typically satisfying $\gamma_1 < \gamma_3 < \gamma_2$. Here and heretofore, the production year is normalized for the interval $[0, 1]$.

The controls available to steer the system $u^\alpha = col(u_1^\alpha, u_2^\alpha, u_3^\alpha)$ is specified by the following variables:

u_1^α - Quantity of fertilizers to improve productivity;
u_2^α - Quantity of agrochemical o fight the growth of weeds;
u_3^α - Size of a selected biological population (biological control) introduced to boost the production of the system.

These controls are constrained as follows: $u_i^\alpha(t) \in [0, \bar{U}_i^\alpha]$ for all $t \in \tau$, $i = 1, 2, 3$, being $\bar{U}_i^\alpha$ the maximum physically allowable value of the control i. In compact form, the dynamics of the production unit α in isolation satisfies the following dynamics within the production cycle $\tau = [0, 1]$:

$$\dot{x}^\alpha = h^\alpha + A(x^\alpha)x^\alpha + B(x^\alpha)u^\alpha,$$

where $u^\alpha = col(u_1^\alpha, u_2^\alpha, u_3^\alpha)$, $x^\alpha = col(x_1^\alpha, x_2^\alpha, x_3^\alpha, x_4^\alpha)$, and $h^\alpha = col(h_1^\alpha, 0, 0, 0)$,

$$A(x^\alpha) = \begin{bmatrix} 0 & -a_{12} & -a_{13} & -a_{14}x_1^\alpha \\ -a_{21} & a_{22} & -a_{23} & 0 \\ -a_{31} & -a_{32} & a_{33} & 0 \\ a_{41} & -a_{42} & 0 & 0 \end{bmatrix}, \text{ and}$$

$$B(x^\alpha) = \begin{bmatrix} b_{11} & -b_{12} & 0 \\ b_{21} & -b_{22} & b_{23} \\ b_{31} & -b_{32}(x_4^\alpha) & 0 \\ b_{41} & b_{42} & 0 \end{bmatrix}.$$

Several remarks are in order: First, note that this model is very close to be linear. This may greatly simplify the analysis. The coefficient above should be time variant in order to reflect seasonal effects. However, if seasonal variations are small or the production in question is not very sensitive to seasonal variations, then we may consider them constant. They also should depend on the state of the environment as it is somewhat unrealistic to consider otherwise. However, the analysis is greatly simplified with this assumption. Finally, note that the variables x_2 and x_3 grow very fast - in fact, often, exponentially fast - w.r.t. the main crop making the consideration of impulsive dynamics reasonable, a framework that we do not exploit here.

3.2 Optimal Production Unit Management Problem

Now, we consider a simplified - but still realistic - version of the unit production intra-year optimal control problem. Let us assume that: (i)The system is time invariant and there is only one production cycle per year. This means that the normalized time interval $[0, 1]$ can be adopted for the problem formulation; (ii) The costs incurred in the crop seeds and biological control are made at the beginning of each period; (iii) The costs of agrochemicals and biologic control are constant; (iv) The financial costs are constant; (iv) There exists upper bounds on the instantaneous values of the controls and (upper or lower) bounds for their integral over the whole production year.

The profit function $J^{\alpha,j}(x_1(0), x, u)$ for the production unit is given by

$$c_1 x_1^\alpha(1) + c_2 x_2^\alpha(1) - c_3 x_1^\alpha(0) - \int_0^1 [c_4 u_1^\alpha(t) + c_5 u_2^\alpha(t) + c_6 u_3^\alpha(t)]\, dt.$$

Here, $x_1^\alpha(0)$ is the amount of biomass associated with the seeds of the main crop. Given the finite fixed area of land available, it may be assumed that there are constants γ_1^α and γ_2^α such that the constraint $\gamma_1^\alpha x_1^\alpha(0) + \gamma_2^\alpha \int_0^1 u_3^\alpha(t)dt \le K^\alpha$ holds. Then, the optimal control to be solved by production unit α can be stated as follows:

$$\begin{aligned}
(P^{\alpha,j})\ \ \text{Min.}\ & -J^{\alpha,j}(x_1^\alpha(0), x^\alpha(\cdot), u^\alpha(\cdot)) \\
\text{s.t.}\ & \dot{x}^\alpha(t) = h^\alpha(t) + A(x^\alpha(t))x^\alpha(t) \\
& \qquad + B(x^\alpha(t))u^\alpha(t) \\
& u^\alpha(t) \in [0, \bar{U}_1^\alpha] \times [0, \bar{U}_2^\alpha] \times [0, \bar{U}_3^\alpha] \\
& \int_0^1 u_i^\alpha(t)dt \le U_i^{\alpha,j}, \quad i = 1,2 \\
& \int_0^1 u_3^\alpha(t)dt \ge U_3^{\alpha,j} \\
& \gamma_1^\alpha x_1^\alpha(0) + \gamma_2^\alpha \int_0^1 u_3^\alpha(t)dt \le K^\alpha \\
& x^\alpha(0) \ge 0
\end{aligned}$$

The quantities $U_i^{\alpha,j}$, $i = 1,2,3$, $j = 1, \dots N$, are defined for each period by the supervisory layer in charge of ensuring the long term environment sustainability. From the above, it is obviously natural to assume that $\bar{U}_i^\alpha > U_i^\alpha$, for $i = 1,2,3$. The supervisory layer should ensure that the data is such that, at the optimum (the optimum value of x is denoted by "hat", i.e., $\hat{x}$), the inequality

$$J^{\alpha,j}(\hat{x}_1^\alpha(0), \hat{x}^\alpha(\cdot), \hat{u}^\alpha(\cdot)) \ge (r_1 + r_2^\alpha) I^{\alpha,j}$$

holds. Here, $I^{\alpha,j}$ is the total investment in the production unit α in the period j, $r_2^{\alpha,j}$ is financial costs rate incurred, and r_1 is the reasonable return on the investment.

We will further "normalize" the problem by considering a new variable $y^\alpha \in \mathbb{R}^3$ defined by $y^\alpha(0) = 0$, and $\dot{y}^\alpha(t) = u^\alpha(t)$, the problem - with the simplification above - becomes fully linear, albeit with box-type control constraints, and endpoint state constraints. That is,

$$\begin{aligned}
(\bar{P}^{\alpha,j})\ \ \text{Min.}\ & \bar{c}^T x^\alpha(1) + \bar{d}^T y(1) + \bar{e}^T x^\alpha(0) \\
\text{s.t.}\ & \dot{x}^\alpha = h^\alpha + A(x^\alpha)x^\alpha + B(x^\alpha)u^\alpha, \\
& \dot{y}^\alpha = u^\alpha,\ y^\alpha(0) = 0,\ x^\alpha(0) \ge 0 \\
& u^\alpha(t) \in [0, \bar{U}_1^\alpha] \times [0, \bar{U}_2^\alpha] \times [0, \bar{U}_3^\alpha] \\
& y_i^\alpha(1) \le U_i^{\alpha,j}, \quad i = 1,2 \\
& y_3^\alpha(1) \ge U_3^{\alpha,j} \\
& \gamma_1^\alpha x_1^\alpha(0) + \gamma_2^\alpha y_3^\alpha(1) \le K^\alpha.
\end{aligned}$$

3.3 Application of the Maximum Principle

In order to pursue with the analysis of the Maximum Principle relations in order to determine the solution to this problem, let us consider a further simplification: The matrices A and B do not depend on x^α, i.e., $A(x^\alpha) = A$ and $B(x^\alpha) = B$. For some practical scenarios, this is not a significant loss of generality. The extra generality afforded by the previous model would imply a much more cumbersome analysis that would make it more difficult to grasp the purpose of the article.

The Maximum Principle states that, for the optimal control process (x, y, u), there exists a multiplier (p, q) that satisfies the adjoint system $-\dot{p}^T = p^T A$, $-\dot{q}^T = 0$, with the transversality conditions

$$\begin{aligned}
&\left(-(p^T(1), q^T(1)), (p^T(0), q^T(0))\right) \\
&\quad \in \nabla J\left((x(1), y(1)), (x(0), y(0))\right) \\
&\qquad + N_C^L\left((x(1), y(1)), (x(0), y(0))\right),
\end{aligned}$$

where u maximizes the Pontryagin funtion $v \to H(x, y, p, q, v) = p^T(Ax + Bv + h) + q^T v$ $[0,1]$ almost everywhere over the set of all feasible controls.

Here, $\nabla J = \left((\bar{c}^T, \bar{d}^T), (\bar{e}^T, 0)\right)$ is the gradient of the cost functional, N_C^L is the limiting Normal cone in the sense of Mordukhovich, C is the joint endpoint state constraints defined by

$$\begin{aligned}
\{((x(1), y(1)), (x(0), y(0))) : x(0) \ge 0, \\
x(1) \in \mathbb{R}^3,\ y(0) = 0, \bar{H}y(1) \le \bar{U}, \\
\bar{\gamma}_1 x(0) + \bar{\gamma}_2 y(1) \le K^\alpha\},
\end{aligned}$$

where $\bar{H} = diag(1,1,-1)$, $\bar{U} = col(U_1, U_2, -U_3)$, $\bar{\gamma}_1 = lin(\gamma_1, 0,0,0)$, and $\bar{\gamma}_2 = lin(0,0,\gamma_2)$.

A few computations and some natural assumptions on the data leads to the conclusion that

$$\begin{aligned}
N_C^L(z) = \partial[\{(\{0\}, \mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^-), \\
(\{0\} \times \mathbb{R}_0^- \times \mathbb{R}_0^- \times \mathbb{R}_0^-, \mathbb{R}^3)\} \\
\cup \{a[(0, (0,0,\gamma_2)), ((\gamma_1, 0,0,0), 0)] : a \ge 0\}].
\end{aligned}$$

Thus, from the boundary conditions of the adjoint variable

$$\begin{aligned}
&\left(-(p^T(1), q^T(1)), (p^T(0), q^T(0))\right) \\
&\quad \in \left((\bar{c}^T, \bar{d}^T), (\bar{e}^T, 0)\right) \\
&\qquad + N_C^L\left((x(1), y(1)), (x(0), y(0))\right),
\end{aligned}$$

we conclude that, there are numbers $\bar{p}_0 \in \mathbb{R}^4$, $\bar{q}_0 \in \mathbb{R}^3$, and $\bar{q}_1 \in \mathbb{R}^3$, with

$$\begin{aligned}
(\bar{q}_1, \bar{p}_0) \in \left(\mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^-, \{0\} \times \mathbb{R}_0^- \times \{0\} \times \mathbb{R}_0^-\right) \\
\bigcup \{\alpha\left((0,0,\gamma_2), (\gamma_1, 0,0,0) : \alpha \ge 0\right\}
\end{aligned}$$

satisfying

$$\begin{aligned}-\bar{p}^T(1) &= \bar{c}^T\\ -\bar{q}^T(1) &= \bar{d}^T + \bar{q}_1\\ \bar{p}^T(0) &= \bar{e}^T + \bar{p}_0\\ \bar{q}^T(0) &= \bar{q}_0.\end{aligned}$$

Notice that the condition on $q(0)$ is, in fact, absent.

Now, the necessary conditions of optimality can be written down as follows:
If $((x^*, y^*), u^*)$ is an optimal control process, then there are numbers $\alpha \geq 0$, $\bar{p}_0 \in \mathbb{R}^4$, and $\bar{q}_1 \in \mathbb{R}^3$ satisfying the transversality conditions (we assume that the problem is normal) and such that u^* maximizes a.e. in $[0, 1]$ the map

$$v \to \langle p^T B + q^T, v\rangle, \quad \forall v \in [0, \bar{U}_1]\times[0, \bar{U}_2]\times[0, \bar{U}_3]$$

where the adjoint variables (p, q) satisfies the adjoint differential equations $-\dot{p}^T = p^T A$, $-\dot{q}^T = 0$, with the boundary conditions $-p(1) = \bar{c}$, $-q(1) = \bar{d} + \bar{q}_1^T$, and $p(0) = \bar{e} + \bar{p}_0^T$. This means that $-q(t) = \bar{d} + \bar{q}_1^T$ for all $t \in [0, 1]$.
Notice that, in the absence of singular arcs, the maximum condition above means that we have, a.e.,

$$\begin{aligned}u_i^*(t) = 0 \quad &\text{if } p^T(t)b_i - (\bar{d} + \bar{q}_1^T)_i < 0\\ u_i^*(t) = \bar{U}_i \quad &\text{if } p^T(t)b_i - (\bar{d} + \bar{q}_1^T)_i > 0.\end{aligned}$$

Here, b_i is the i^{th} column of the matrix B.

4 The optimal coordination control problem

In order to formulate the basic coordination optimal control problem, we consider the variables adopted in the simplified notation. Let $U^k = col(U_1^k, U_2^k, U_3^k)$, be the bounds for the two agrochemicals and to the biological control to be used.

The coordination problem consists in computing the two sequences $\{U^{\alpha,k}\}_{k=1}^N$ and $\{U^{\beta,k}\}_{k=1}^N$ solving the following problem

$$\begin{aligned}(P_c)\ \text{Minimize} \sum_{k=1}^{N}[\pi^{\alpha,k}(x_4^\alpha(k) - x_4^\alpha(k-1))&\\ + \pi^{\beta,k}(x_4^\beta(k) - x_4^\beta(k-1))]& \quad (1)\end{aligned}$$

subject to

$$\begin{aligned}&col(x, y)^\alpha(k+1) = F(col(x, y)^\alpha(k), U^{\alpha,k}) && (2)\\ &col(x, y)^\beta(k+1) = F(col(x, y)^\beta(k), U^{\beta,k}) && (3)\\ &J(col(x, y)^\alpha(k)) \geq s^{\alpha,k} && (4)\\ &J(col(x, y)^\beta(k)) \geq s^{\beta,k} && (5)\\ &x_4(N) = 0 && (6)\end{aligned}$$

where

a) The cost functional (1) to be minimized represents the accumulated "unfair behavior" of the production units in the period. To see this clearly, notice that each parcel represents the deterioration of the environment weighted by the profit relative to its investment given $\pi^{\zeta,k} = \dfrac{J^{\zeta,k} - s^{\zeta,k}}{I^{\zeta,k}}$, $\zeta = \alpha, \beta$.
b) The dynamics (2,3) $col(x, y)^\zeta(k+1) = F(col(x, y)^\zeta(k), U^{\zeta,k})$, $\zeta = \alpha, \beta$, are obtained by solving the optimal control problem $(P^{\zeta,k})$ has a function of the bounds on the controls, $U^{\zeta,k}$ available to the production unit ζ.
c) The constraints (4,5) are imposed in order to ensure the economic sustainability of each production unit.
d) The constraint (6), where $x_4(N)$ is the weighted average $\dfrac{I^{\alpha,N}x_4^\alpha(N) + I^{\beta,N}x_4^\beta(N)}{I^{\alpha,N} + I^{\beta,N}}$ ensures that the desired convergence to environment sustainability is targeted by the resulting coordination strategies.

Item b) requires further considerations. The mapping (2), and (3) are obtained by solving $(\bar{P}^{\alpha,k})$ and $(\bar{P}^{\beta,k})$, respectively, as discussed for the production unit α in the previous section. Since both problems are decoupled and identical, we omit the upper-indices α and β in the discussion leading to the specification of the mapping F that describes the discrete dynamics (2), and (3). Let us consider $((x^*, y^*), u*)$ the optimal control process for $(\bar{P}^{\cdot,k})$. Clearly, we have $x(k+1) = x^*(1)$, $x^*(0) = x(k)$, $y(k+1) = y^*(1)$. Thus, the state is steered from $(\bar{x}(k), y(k))$ to $(x(k+1), y(k+1))$ by solving $(\bar{P}^{\cdot,k})$. Thus,

$$\begin{aligned}x(k+1) &= e^A\bar{x}(k) + \int_0^1 e^{A(1-t)}[Bu^*(t) + h(t)]dt,\\ y(k+1) &= \int_0^1 u^*(t)dt,\end{aligned}$$

where u^* satisfies the maximum condition of the necessary conditions of optimality stated in the previous section, the control constraints $u^*(t) \in \prod_{i=1}^{3}[0, \bar{U}_i]$ as well as the constraints $y_i^*(1) \leq U_i^k$, $i = 1, 2$, $y_3^*(1) \geq U_3^k$, and $\gamma_1 x_1^*(0) + \gamma_2 y_3^*(1) \leq K^\cdot$.
Notice that the map F has an affine structure. By scaling down the u_i^*, i.e. $u_i^* = U_i^k \bar{u}_i^*$ (now, we have that $\bar{u}_i^* \in \left[0, \frac{\bar{U}_i}{U_i^k}\right]$), by noting that

$$Bu^*(t) = \sum_{i=1}^{3} b_i u_i^*(t) = \sum_{i=1}^{3} b_i U_i^k \bar{u}_i^*(t),$$

where b_i is the column i of B, and by denoting

$$\Phi_1 = e^A,$$

$\Phi_2^k = col(\Phi_2^{i,k} : i = 1,2,3)$ where $\Phi_2^{i,k} = \int_0^1 e^{A(1-t)} b_i \bar{u}_i^*(t)dt$, and
$\Phi_3 = \int_0^1 e^{A(1-t)} h(t)dt$,

we obtain, by denoting $U^k = col(U_i^k : i = 1,2,3)$, the discrete dynamics given by

$$x(k+1) = F(\bar{x}(k), U^k) = \Phi_1 x(k) + \Phi_2^k U^k + \Phi_3.$$

Notice that the matrix Φ_1, and the vector Φ_3 are constant, and the matrix Φ_2^k depends on the optimal control obtained in the period k.

A couple of observations are in order which are quite relevant to facilitate the computation of the solution of $(\bar{P}^{\cdot,k})$:

1. The initial state x at each sampling time is the optimum final state of the previous period, i.e., do we have $\bar{x}(k) = x(k)$? If the answer is positive, then each $(\bar{P}^{\cdot,k})$ is a fixed initial state problem and the constraint $\gamma_1 \bar{x}_1(k) + \gamma_1 y_3(k+1) \leq K^{\cdot}$ becomes much simpler.
2. In order to compute the solution to the optimal control problem $(\bar{P}^{\cdot,k})$, it is reasonable to assume $y_i(k+1) = U_i^k$, $i = 1,2,3$, and $\gamma_1 \bar{x}_1(k) + \gamma_2 y_3(k+1) = K^{\cdot}$.

5 Conclusions

This article reports a preliminary research effort targeting the design of a decision support system for the optimal and "fair" management of a number of agricultural productions units seeking to maximize their own short term economic returns while being subject to a coordination control ensuring the medium to long term environment sustainability. This achieved by imposing appropriate production quotas which are computed by running an MPC like scheme. In general, this is a very complex problem and this effort considered the modeling and the decision making optimal control problem for a small scale context in order to prove the concept. Future work will address further refinement of the problem in order to make it mimic some realistic known context, as well as the assessment of the obtained control and coordination strategies.

Acknowledgements

Fernando Lobo Pereira acknowledges the partial support of: FCT R&D Unit SYSTEC - POCI-01-0145-FEDER-006933/SYSTEC funded by ERDF — COMPETE2020 — FCT/MEC — PT2020, Project STRIDE - NORTE-01-0145-FEDER-000033, funded by ERDF — NORTE 2020, and contract no 02.a03.21.0008 of the Ministry of Education and Science of the Russian Federation. Ismael Pena acknowledges the support of FAPESP grant n^o 2013/25135-6.

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