SVD ANALYSIS OF COMPLEX FLOW PATTERNS IN TWO-PHASE FLOWS

Luciano Cantelli
Dipartimento di Ingegneria Elettrica Elettronica e dei Sistemi
Universit degli Studi di Catania
Italy
lcante@diim.unict.it

Arturo Pagano
Dipartimento di Ingegneria Industriale e Meccanica
Universit degli Studi di Catania
Italy
apagano@diim.unict.it

Abstract

The study describes a novel approach for the analysis and classification of air-water two-phase flows in vertical channels, based on the observation of the complex dynamical behavior of experimental flow patterns. An experimental apparatus has been tested by measuring the void fraction time series for a wide range of air and water mass flow rates, corresponding to several kinds of flow patterns. At first, an n-dimensional delayed embedding has been adopted for the representation of the attractors of the experimental time series. This approach has allowed to observe the complex but regular attractor morphology, though affected by noisy features. Therefore, Singular Value Decomposition (SVD) has been applied to the n-dimensional state space in order to determine its eigenvalues and, in particular, the attractor projection onto the eigenvectors state space. This has allowed substantial separation of dominant features of the system dynamics from noise-like dynamics and a satisfactorily unfolded attractor. Moreover, in some cases this new representation has also allowed to observe the existence of a well defined fractal structure. Finally, morphological differences between projected attractors has been observed to posses a high potential as a very effective tool for flow pattern identification and classification.

Key words

Two-phase flows, flow-patterns identification, singular value decomposition.

1 Introduction

Two phase flows are at the basis of several important industrial applications, ranging from oil pipelines, chemical and processing plants to power generation. The type of flow pattern established in the system, i.e. the dynamical distribution of the two phases inside the pipe, and possible transitions from a flow pattern to another indeed represent critical factors for the performances of such systems. This explains the great efforts that have been and are still devoted to flow patterns identification and classification.

As well known, different flow patterns are obtained by varying the mass flow rates of the two phases. In particular, bubbly, slug, plug, churn and annular flows, obtained through a progressive growth of the air flow rate, can be identified as the main flow patterns typically reported in several classifications [Costigan, Whalley, 1997; Mi, Ishii, Tsoukalas, 1998; Keska, Williams, 1999].

While bubbly, slug and plug flows are characterized by a relatively regular but non periodic distribution of air bubbles moving into the water flow, in the churn and annular flows just a thin liquid film surrounding the air flow can be observed, which is highly unstable and characterized by irregular oscillations propagating through it. This observation is important to underline the intrinsically complex dynamical structure of two phase flows, which plays a fundamental role for their classification.

Efficient flow pattern characterisation strongly depends on the technique adopted to measure the void fraction. Several techniques have been proposed for the measure of the void fraction of two-phase flows, ranging from the measurement of the electrical impedance of two phase mixtures [Keska, Williams, 1999; Devia, Fossa, 2003; Lowe, Rezkallah, 1999; Cantelli, Fichera, Guglielmino, Pagano, 2006], to optical techniques based on the measure of the scattering of the interface between the two phases [Keska, Williams, 1999], to the measure of pressure drops in a pipe of specified length [Vial, Camarasa, Poncin, Wild, Midoux, Bouillard, 2000]. Among these techniques, impedance measurements seem to be recognized as the most reliable [Keska, Williams, 1999]; in fact, it is non-intrusive and, most important, less dependent both on internal disturbances and external factors. Two main classes of impedance sensors have been proposed in literature: resistive sensors [Devia, Fossa, 2003; Cantelli, Fichera, Guglielmino, Pagano, 2006] and capacitance sensors [Keska, Williams, 1999; Lowe, Rezkallah, 1999].

The various flow patterns develop as a result of the interaction of several complex transport phenomena. Therefore, flow patterns characterisation is still somewhat confused and controversial; in fact, it depends on the approach to analysis of experimental void fraction time series.

Several studies have been devoted to analyse the dynamical behaviours that characterize two-phase flows in pipes, often on the basis of statistical [Mi, Ishii, Tsoukalas, 1998; Lowe, Rezkallah, 1999] or spectral [Lucas, Walton, 1997; Watson, Hewitt, 1999; Song,Chung, No, 1998] analyses of void fraction-related experimental time series, such as impedance or pressure fluctuations.

Nonlinear techniques have been also adopted for the analysis of pressure fluctuations in horizontal pipes [Drahos , Zahradnik , Puncochar, Fialova , Chen, Bradka, 1991] and in vertical bubble columns [Letzel, Schouten, Krishna, van den Bleek, 1997] or of impedance fluctuations in vertical pipes [Jin, Nie, Ren, Liu, 2003]. Nonetheless, a full explanation of the nonlinear dynamics of two phase flows is still far from being reached. This is due to some relevant unsolved problems which should be addressed when adopting nonlinear time series analyses. In particular, high quality time series are necessary in order to "capture" the complex dynamics of two phase flows. This is possible only if the void fraction time series are sufficiently resolved both in space and time; i.e. the void fraction should be measured over a very thin volume (that can be approximated by a measurement section) with a fast response sensor.

Moreover, an unfolded and noise free attractor in phase space is necessary in order to eliminate false neighbours, i.e. similar phase space representations of dynamically different states [Thompson, Stewart, 1986]. This problem is of primary importance in order to obtain a reliable evaluation of the invariants of the dynamics, such as fractal dimension and Lyapunov exponents, which represent the fundamental parameters to verify the existence of chaos in the system dynamics [Rasband, 1998].

The possibility to achieve high quality time series have been addressed by setting up a resistive void fraction sensor described in [Cantelli, Fichera, Guglielmino, Pagano, 2005].

The aim of this study is to set up an appropriate strategy that allows to obtain an unfolded representation of the attractor in phase space, so that the observation of complex dynamics in two phase flows can be performed on a reliable basis. Moreover, the observation of the morphological differences between the experimental attractors of two-phase flow patterns will be used in order to assess a dynamic-based tool for their classification.

The proposed approach starts from the reconstruction of an n-dimensional representation state space on the basis of Takens' embedding theorem [Takens, 1981], i.e. by means of a set of n delayed versions of the experimental void fraction time series. The analysis of the attractors in this representation space points out the existence of a complex but regular structure in phase space, which constitutes a first hint of deterministic behavior, as already reported in [Cantelli, Fichera, Guglielmino, Pagano, 2005]. Nonetheless, attractors obtained in this way are somewhat noisy as a consequence of the superposition of high order dynamics to the dominant dynamics characterizing the flow pattern. Among the others, the most important high order "noisy" dynamics are those associated to small diameter bubbles dispersed in the liquid slugs and to disturbances on the liquid film enveloping the Taylor bubbles.

In order to address this problem, in the proposed strategy Singular Value Decomposition (SVD) [Broomhead, King, 1986] has been applied to the n-dimensional state space in order to determine its eigenvalues and, in particular, the attractor projection onto the eigenvectors state space. This has allowed substantial separation of the dominant features of the system dynamics from noise-like dynamics. The projections of the n-dimensional attractors in the representations space formed by the three dominant eigenvectors are satisfactorily unfolded.

2 Experimental Apparatus

The experimental apparatus reported in Fig. 1 has been built and tested in order to study the dynamics of two-phase flow in vertical pipes. The two-phase flow test section is a 3 m long vertical pipe of 0.26 m diameter. A resistive probe for void measurements is placed at a distance of more than 100 times the pipe diameter from the mixing section, i.e. over the required entry region for two phase flows, in order to ensure a well established flow regime. The air is supplied to the mixing section by a pressurised tank fed by a compressor. The whole apparatus is also equipped by an electromagnetic flowmeter and three air flow metres. The electromagnetic flow meter is used to measure the water velocity and mass flow rate, whereas the air flow meters are used to set the air flow rate in the range between 10 and 210

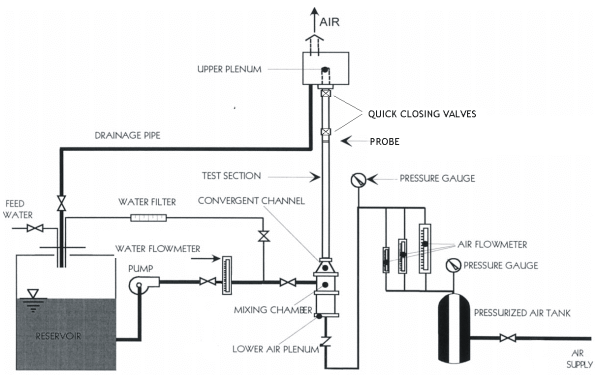

Figure 1. Experimental apparatus

l/min. The water flow rate can be varied in the range 0-150 l/min by means of a series of valves and bypasses placed at the pump outlet. In order to allow the degassing of the working fluid an open tank is placed at the top end of the vertical pipe constituting the test section.

The experimental apparatus is equipped with an impedance probe for void fraction measurements. It is known that the volume fraction of a phase in a two-phase mixture can be determined by measuring the impedance of a mixture if a significant difference in the electrical properties exists between the two phases.

The sensor that has been designed and realised for the present study operates in the resistive range; in fact, a carrier frequency of 20 kHz has been supplied by an external sine wave oscillator to a Wheatstone bridge, by means of an operational amplifier to decouple the input impedance from the load impedance. The instrumentation amplifier assures a high dynamic response and a perfect decoupling of the electronic circuit from the measuring bridge. The amplified output is applied to the electronic rectifier. A cut-off frequency of 200 Hz has been adopted in order to allow adequate removal of the carrier frequency and to avoid aliasing with the sampling frequency. The electronic circuitry has been tested and calibrated before each set of measurements by means of a comparison with precision resistors. The final output has been sent to a data acquisition system at the sampling rate of 1 kHz, in order to record the void fraction time series detected during the experiments.

The probe configuration considered in this study is similar to that proposed in [Costigan, Whalley, 1997; Devia, Fossa, 2003] and has been described in [Cantelli, Fichera, Guglielmino, Pagano, 2005] together with the procedure adopted for its calibration.

3 Phase Space Reconstruction Strategy

In [Cantelli, Fichera, Guglielmino, Pagano, 2005] the experimental time series detected during the experimental tests have been analysed by means of linear tools. The results of these analyses have shown the incapacity of such tools in describing the intrinsic complexity of two phase flows dynamics.

For this reason a different approach has been preliminary adopted in [Cantelli, Fichera, Guglielmino, Pagano, 2006; Cantelli, Fichera, Guglielmino, Pagano, 2005] based on the observation of the morphology of the three-dimensional experimental attractors of various flow patterns in a reconstructed phase space. In order to reconstruct the phase space starting from the experimental scalar observations Takens' method of delays has been adopted [Takens, 1981]. This method is particularly useful as it allows to determine a set of independent variables that can be used to define a representation space.

Considering the generic *d*-dimensional system, Takens' method is based on embedding the original scalar time series into an *m*-dimensional vector. Takens' *Embedding Theorem* [Takens, 1981] ensure that, if $m \geqslant 2d + 1$, the creation of the *m*-dimensional vector results in the reconstruction of a state space containing a smooth manifold for the *d*-dimensional system. In the following *d* will indicate *the system dimension* whereas *m* will indicate *embedding dimension*, which is not the dimension of the system itself but the number of dimensions necessary to correctly embed the attractor in phase space. Takens' method is based on the creation of a *n x w* matrix, where *n* is the length of a window moving through the data and the *w* columns of the matrix represents the number of independent variables used to define the phase space, which must be much greater than the expected minimum embedding dimension of the system in order to ensure that $w > 2d + 1$.

In this way, considering the time series $s(t) = (s_0, s_1, s_2, ..., s_i, ...)$ measuring the variable s of the dynamical system, an independent set of variables is represented according to Takens' method of delay by the matrix having for columns the delayed versions of the experimental time series *s* (each column is delayed of τ time steps from the previous).

According to Takens' theorem the obtained *w*-dimensional system presents the same features of the general dynamics of the above matrix. Once the w-dimensional vector has been constructed, the representation of the attractor is easily obtained by using the columns of matrix *S* as co-ordinates of the phase space. The morphological description performed in [Cantelli, Fichera, Guglielmino, Pagano, 2006] uses just the first three column of the matrix, i.e. the number of variables necessary for a three dimensional pseudo-phase space (the proper phase space is, in fact, *m*-dimensional) where the attractor structure can be analysed. In particular, the attractors obtained for some of the flow patterns have been observed to display a regular fractal structure [Cantelli, Fichera, Guglielmino, Pagano, 2006], which is indeed one of the most important evidences of deterministic chaotic behavior.

In order to expand previous results, in this study a different phase space representation has been adopted, derived from classical delayed embedding based on Takens' theorem and on the application of *Singular Value Decomposition* technique *SVD*, [Broomhead, King, 1986]. The aim of this new representation is to achieve both a reduction of the influence of noise on the time series and, above all, an unfolded representation for which the relevant morphological characteristic of the attractor can be fully exploited. The proposed approach consists in applying *SVD* approach to matrix *S*, i.e. in rotating and translating *S* in order to obtain a new diagonal matrix, equivalent to the original one (i.e. with identical eigenvalues), having the eigenvalues on its diagonal in descending order.

From a mathematical point of view, matrix *S* is factorized into its singular values according to equation:

$$\Lambda = M^T S C \tag{1}$$

where Λ is a diagonal matrix containing the singular values of *S* in decreasing order and *M* and *C* are matrices of the singular vectors associated with Λ . The singular vectors in *M* are those of the square *structure matrix*, $\Phi = XX^T$, whereas the eigenvectors in *C* are those of the square *covariance matrix*, $\Psi = X^T X$. The singular values in Lambda are the square roots of the eigenvalues of either Φ or Ψ (which clearly are the same).

In Λ the high level eigenvalues correspond to the dominant eigenvectors, i.e. those representing the system dynamics, whereas the low level ones correspond to local behaviors or noise components. Therefore, the system can be partitioned into two subsystem: the first deriving from noise free data (i.e. main features and relevant details) and the second from noise, which can be considered superimposed and then eliminated.

Once that the matrices Λ , *M* and *C* have been calculated, an unfolded version of a noise free attractor can be obtained considering the new embedding S_{ev} obtained as:

$$S_{ev} = SC \tag{2}$$

In particular, only the first *m* dominant singular vectors of S_{ev} will be necessary to describe the system dynamics. It is worth noting that the three-dimensional attractor plotted in the pseudo-phase space spanned by the first three eigenvectors will appear different from the original attractor in the delayed phase space. Nonetheless, it is morphologically equivalent to the original attractor as it has been obtained from it through a rotation and a translation of the coordinate system. In other words, it represents an alternative representation of the same dynamics, that can be preferred for it is unfolded and noise-free.

4 Results

The approach described so far has been used in order to obtain a denoised representation of the dynamics and, in particular, an unfolded attractor. The *SVD* technique has been applied to the delayed embedding *S* of the experimental void fraction time series. The delayed embedding has been created considering a delay $\tau = 1$ and $w = 50$ in order to be sure that the number *w* of variables considered is sufficiently greater than *m*, i.e. than the (unknown) dimension of the system. The length *n* of the observation window has been set at 10000 data samples in order to be wide enough to obtain a well defined attractor in phase space, i.e. an attractor whose morphology remains unchanged if further data samples are added. In order to observe the performances of the proposed strategy, in Fig. 2 and 3 are reported the attractors of the same operating condition in two different embedding. In particular, the phase space adopted in Fig. 2 is the basic delayed embedding, whereas in Fig. 3 is reported the projection on the phase space spanned by the first three eigenvectors of the improved embedding obtained through application of *SVD*.

From the analysis of the plot it is possible to observe the attractor in Fig. 2, i.e. in the delayed phase space, is affected by noise and is not sufficiently unfolded. On the other hand, the reduction of noise and the satisfactory unfolding is immediately apparent for the attractor in the representation space defined by the first three eigenvectors obtained through SVD approach.

The two attractors reported in Fig. 2 and 3 appear different but are the expression of the same dynamical behavior. This means that they are morphologically equivalent, i.e. they are characterized by the *same invariants of the dynamics*, such as fractal dimension and Lyapunov exponents [Thompson, Stewart, 1986]. As well known, the algorithms for the calculation of these invariant characteristics are strongly affected by noise in the time series. Therefore, the approach herein pro-

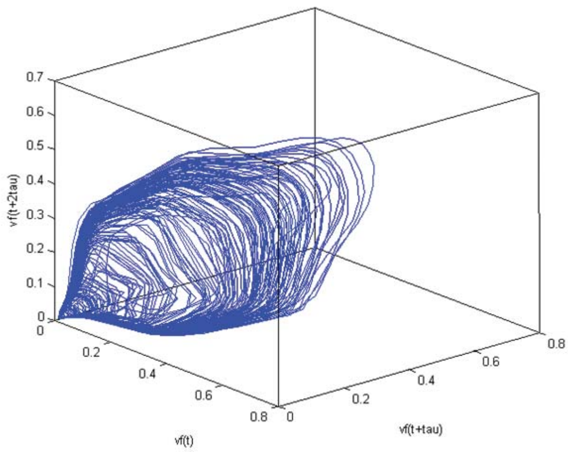

Figure 2. Attractor of the experimental void fraction in a delayed phase space (τ = 3 samples): air mass flow rate 10 litres/min - water mass flow rate 84.2 litres/min.

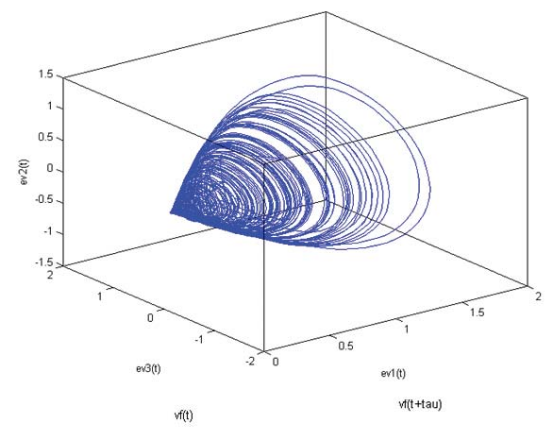

Figure 3. Attractor of the experimental void fraction in the phase space spanned by the three dominant singular vectors of the n-D embedding; operating condition is the same as in Fig.1.

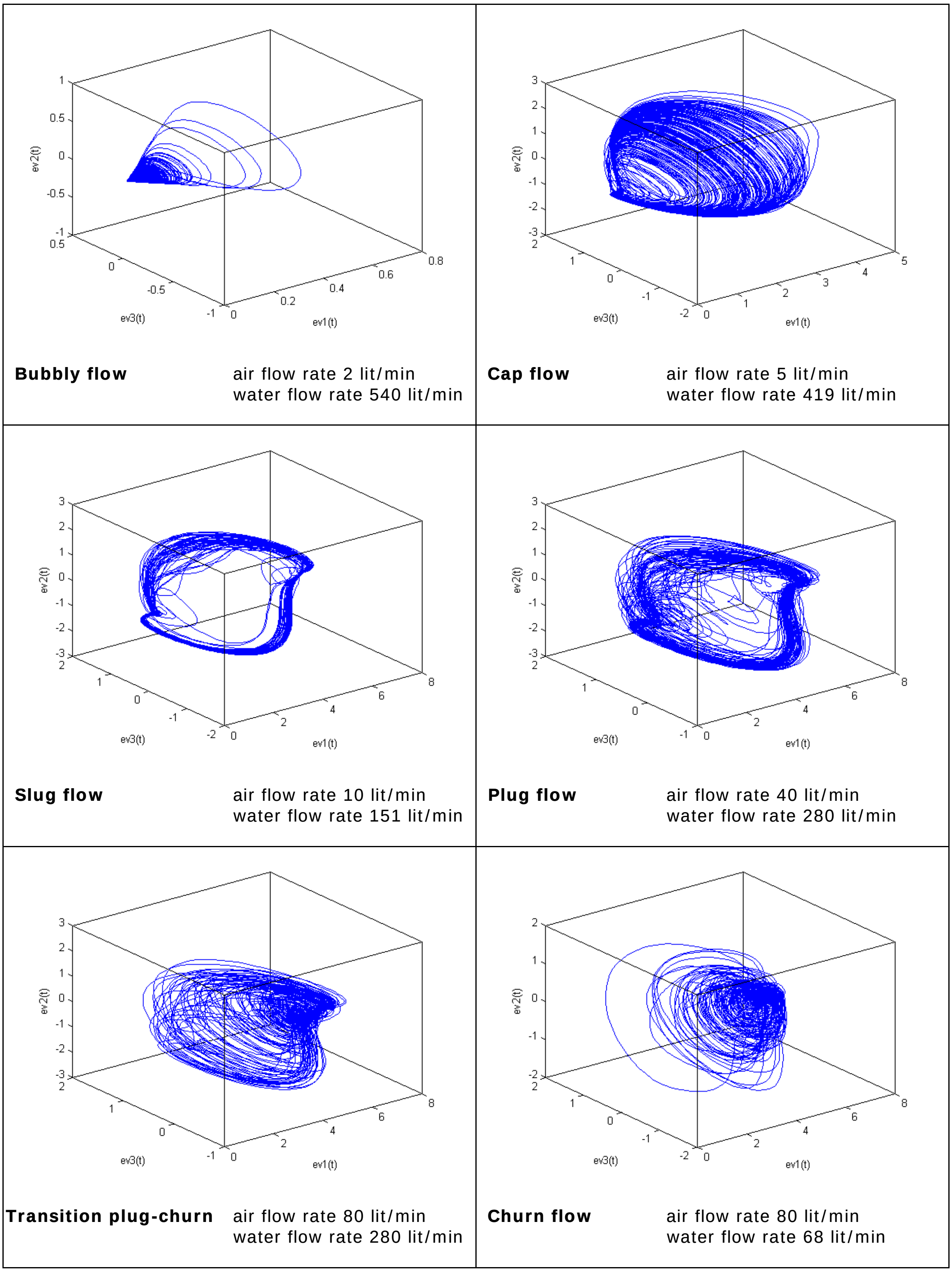

Figure 4. Attractors of the void fraction experimental time series in the phase space spanned by the three dominant singular vectors for various flow patterns.

posed will be used in future studies in order to obtain an unfolded and noise-free embedding which will allow a more reliable determination of the invariant characteristics of the dynamics, such as fractal dimension and Lyapunov exponents, with the aim of achieving a deeper knowledge of nonlinear dynamics in two-phase flows. Fig. 4 reports the attractors obtained with the embedding strategy proposed in the present study for the experimental time series corresponding to different flow patterns. As a first observation, all of the attractors are characterized by a well defined structure in phase space, which is sufficiently unfolded and less compromised by the presence of noise. This observation holds also for those attractors that are characterized by a more complex structure, in particular those corresponding to the most unstable flows, such as churn flow or the transition flow between plug and churn flow. A detailed analysis of these plots is out of the scope of the present study, but is worth observing that the representation in the phase space spanned by the principal vectors is very effective in underlining the differences between the various flow patterns. Each type of flow pattern is, in fact, characterized by a specific morphology, sufficiently different from that of other flow patterns. In particular, different regions of phase space are occupied by the attractors of the various flow patterns; notice, for example, how the attracting region for the case of bubbly flow is opposite to that for the case of churn flow (with respect to the first eigenvector co-ordinate). Moreover, each attractor "fills" the phase space in a different way; for example, the attractor of cap flow entirely fills the 3-D phase space as opposite to that of slug flow, which is almost two-dimensional and distributes around a sort of limit cycle. In a similar way, the attractor of the plug flow still distributes around a sort of limit cycle but fills the phase space more than that of the slug flow. This difference is very important as the degree of filling of the phase space is strictly related to the fractal behavior of the attractor [Thompson, Stewart, 1986; Rasband, 1998], with the fractal dimension corresponding, in practice, to a *measure* of this filling. It is worth noting that the last considerations all refer to the general class of slug flow, class to which both cap and plug flow belong; nonetheless, relevant differences can be observed between attractors.

Previous observations draw a clear picture of how a refined phase space analysis, i.e. performed on the basis of SVD projection, possesses a high potential as a very effective tool for flow pattern identification. This potential will be further exploited in future study through the analysis of Poincar maps and evaluation of the main invariant characteristics of the dynamics, such as fractal dimension and Lyapunov exponents.

5 Conclusions

A novel approach has been proposed for the analysis and identification of two-phase flow patterns, based on the assessment of an appropriate phase space for the representation of the complex attractors that characterizes this kind of flows. Proper definition of the phase space is fundamental for obtaining an unfolded attractor which is the first step for the evaluation of the invariant characteristics of the dynamics, such as fractal dimension and Lyapunov exponents.

The proposed strategy is based on the determination of the principal vectors of a classical delayed embedding and on the projection of the attractor on the state space spanned by these vectors. This allows to separate the dominant features of the dynamics, corresponding to a subset of the main principal vectors, from noise in the time series, which corresponds to higher order principal vectors. In the present study the attention have been focused on obtaining an unfolded and noise-free representation of the attractor in the pseudo-phase space spanned by the first three principal vectors. The potential of this approach for flow pattern identification and classification have been reported.

On the basis of the proposed strategy, further analyses will be developed in order to verify the potential of Poincar maps and of invariant characteristics of the dynamics for a full exploitation of the complex behavior of two phase flows and as a dynamics-based classification tool.

References

Broomhead, D.S., King, G.P. (1986) *Extracting Qualitative Dynamics from Experimental Data*. Physica D, Vol. 20, 217-236.

Cantelli, L., Fichera, A., Guglielmino, I.D., Pagano, A. (2006) *Non-Linear Dynamics of Air-Water Mixtures in Vertical Pipes: Experimental Trends*. Int. J. Bifurc. nd Chaos, Vol. 16-9.

Cantelli, L., Fichera, A., Guglielmino, I.D., Pagano, A. (2005) *Preliminary Analyses of the Experimental Dynamics of an Air-Water Two-Phase Flow in a Vertical Pipe*. XXIII Congr. UIT 2005, Parma, June 20-22.

Costigan, G., Whalley, P.B. (1997) *Slug flow regime identification from dynamic void fraction measurement in vertical air-water flows*. Int J Multiphase Flow, Vol. 23, pp. 263 282.

Devia, F., Fossa, M. (2003) *Design and optimisation of impedante probes for void fraction measurements*. Flow Measurem. Instrum., Vol. 14, pp. 139-149.

Drahos, J., Zahradnik, J., Puncochar, M., Fialova, M., Chen, K., Bradka, F. (1991) *Effect of operating conditions of the characteristics of pressure fluctuations in a bubble column*. Chemical Engineering Processing, Vol. 29, 107-125.

Jin, N.D., Nie, X.B., Ren, Y.Y., Liu, X.B. (2003) *Characterization of oil/water two-phase flow patterns based on nonlinear time series analysis*. Flow Meas. Instr., Vol. 14, 169-175.

Keska, J.K., Williams, B.E. (1999) *Experimental comparison of flow pattern detection techniques for air-water mixture flow*. Exper. Thermal and Fluid Science, Vol. 19, pp. 1-12.

Letzel, H., Schouten, J., Krishna, R., van den Bleek, C.M. (1997) *Characterization of regimes and regime transitions in bubble columns by chaos analysis of pressure signals*. Chemical Engineering Science, Vol. 52, pp. 4447-4459.

Lowe, D., Rezkallah, K.S. (1999) *A capacitance sensor for the characterization of microgravity two-phase liquid-gas flows*. Meas. Sci. Tech., Vol. 10, pp. 965-

975.
Lucas, G.P., Walton, I.C. (1997) *Flow rate measurement by kinematic wave detection in vertically upward, bubbly two-phase flows*. Flow Meas. Instr.,Vol. 8(3-4),pp.133-143.
Mi, Y., Ishii, M., Tsoukalas, L. H. (1998) Vertical two-phase flow identification using advanced instrumentation and neural networks, Nucl. Eng. Design, Vol.184, 409-420, 1998.
Rasband, S.N. (1990) *Chaotic Dynamics of Nonlinear Systems*. J. Wiley e Sons Editions, New York, 1990.
Song, C.H., Chung, M.K., No, H.C. (1998) *Measurements of void fraction by an improved multi-channel conductance void meter*. Nucl. Eng. Design, Vol. 184, 269-285.
Takens, F., (1981) *Lecture Notes in Mathematics, Dynamical System and Turbulence*. D.A. Rand e L.S. Young Editions, Springer, New York.
Thompson, J.M.T., Stewart, H.B., (1986) *Nonlinear Dynamics and Chaos*. J. Wiley e Sons Editions, New York.
Vial, C., Camarasa, E., Poncin, S., Wild, G., Midoux, N., Bouillard, J. (2000) *Study of hydrodynamic behaviour in bubble columns and external loop airlift reactors through analysis of pressure fluctuations*. Chemical Engineering Science, Vol. 55, pp. 2957-2973.
Watson, M.J., Hewitt, G.F. (1999) *Pressure effects on the slug to churn transition*. Int. J. Multiph. Flow, Vol. 25, 12251241.