NON-LINEAR PI AND PID REGULATORS IN MECHANICAL SYSTEM CONTROL

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Abstract
In the paper the problem of program positions stabilization for holonomic mechanical systems through integral and proportional-differential regulators is reduced to the study of the stability problem of the integro-differential equations of Volterra type. The paper presents the development of the Lyapunov functional method to solution of this problem. This allows to justify an expansion of the class of PI and PID regulators in solving the problem of program positions stabilization for controlled holonomic mechanical systems.

Key words
mechanical system, stabilization problem, PI and PID regulators, Volterra equation.

1 Introduction

The complexity of the stabilization problem of program motions of robot manipulators consists in the necessity of investigating a nonlinear controlled system with unknown parameters of the system and acting forces and also with incomplete measurement of the phase variables.

Numerous works in this direction are devoted to the use of PI controllers [Berghuis and Nijmeijer, 1993], [Berghuis and Nijmeijer, 1993], [Burkov, 1995], [Burkov, 1998], [Nunes et al., 2008], [Burkov, 2009], [Canudas de Wit and Fixot, 1991], [Siciliano and Villani, 1996], [Andreev and Peregudova, 2017] and PID [Arimoto and Miyazaki, 1984], [Anan'evskii and Kolmanovskii, 1989], [Kelly, 1995], [Ortega et al., 1995], [Cervantes and Alvarez-Raminez, 2001], [Loria et al., 2000], [Santibanez et al., 2010] controllers. Using simple examples it can be shown that adding integral components to the controller does not always help stabilize the program motions of robot manipulators. When using standard PI or PID controllers, the control law depends not only on the current system states, but also on the previous ones. The arising process is in the general case described by integro-differential equations of Volterra type. On this base the methods of constructing PI and PID regulators in the problem of mechanical systems control have been obtained.

The development of the Lyapunov functional method in the stability study of the non-autonomous equations [Andreev, 2009] makes it possible to obtain new effective results on the construction of controllers with integral components.

There are two contributions of this paper. In the first part of the paper new theorems of LaSalle and Krasovskii type on the limit behavior of the solutions, on asymptotic stability and the instability of the zero solution are proved for Volterra type equations. The second part of the report outlines the results of solving the program positions stabilization problem for a holonomic mechanical system based on the construction of new types of nonlinear integral and integro-differential regulators.

2 The quasi-invariance principle for an integro-differential equation of Volterra type

Consider a nonlinear integro-differential equation of Volterra type

$$\dot{x} = f\left(t, x\left(t\right)\right) + \int\limits_0^t g\left(s - t, x\left(t\right), x\left(s\right)\right) ds \qquad (1)$$

where $x \in R^n$, R^n is n-dimensional linear real space with the norm $\|x\|$; f, g are the functions defined and continuous, respectively, in the domains $R^+ \times D$ ($D \subset R^n$) and $R^- \times D \times D$, $R^- = (-\infty, 0]$, the function f satisfies the Lipschitz condition:

$$\|f(t, x^{(2)}) - f(t, x^{(1)})\| \leqslant L_1 \|x^{(2)} - x^{(1)}\| \qquad (2)$$

for each compact set $K_1 \subset D$, $L_1 = L_1(K_1) = const > 0$ the function g satisfies the following conditions: for each compact set $K_2 \subset D \times D$ the following inequalities are hold

$$\|g(t,x,y)\| \leqslant g_1(\tau, K_2) \quad \forall (t,x,y) \in R^- \times K_2$$
$$\int\limits_{-\infty}^{0} g_1(\tau, K_2) d\tau < +\infty \tag{3}$$

$$\begin{aligned}&\|g(\tau, x^{(2)}, y^{(2)}) - g(\tau, x^{(1)}, y^{(1)})\| \leqslant \\ &\leqslant L_{21}\|x^{(2)} - x^{(1)}\| + L_{22}\|y^{(2)} - y^{(1)}\|\end{aligned} \tag{4}$$

where $L_{2j} = L_{2j}(K_2)$.

Under these conditions, for each initial point $x_0 \in D$ there exists a unique solution $x = x(t, x_0)$ ($x(0, x_0) = x_0$) of the equation (1) defined on the interval $[0, \alpha)$ while $x(t, x_0) \to \partial D$ for $t \to \alpha - 0$.

Consider a family of shifts $\{f^\tau(t,x) = f(t+\tau, x), \quad \tau \in R^+\}$ of the function $f = f(t,x)$. Using the precompactness property of this family, we find the set of limit functions [Artstein, 1977]

$$f^*(t,x) = \frac{d}{dt} \lim_{\tau_k \to +\infty} \int\limits_0^t f(\tau_k + s, x) ds \tag{5}$$

In this case, each function $f^*(t,x)$ can be extended to $t \in R^-$. Thus, the domain of its definition can be the region $R \times D$ for almost all $t \in R$ [Artstein, 1977].

For the equation (1) we define the family of limit integro-differential equations

$$\dot{x}(t) = f^*(t, x(t)) + \int\limits_{-\infty}^{t} g(s-t, x(t), x(s)) ds \tag{6}$$

Let $x = x(t, x_0)$ be some solution (1) defined and bounded by some compact set $K \subset D$ for all $t \geqslant 0$. We define in a classical way a positive limit point $p \in D$ and the corresponding positive limit set ω^+

$$p = \lim_{t_k \to +\infty} x(t_k, x_0)$$
$$\omega^+ = \{p \in D : x(t_k, x_0) \to p, \; t_k \to +\infty\}$$

With respect to the family of equations (6) the following property of the set ω^+ holds.

Theorem 2.1. Let $x = x(t, x_0)$ be some solution of (1) defined and bounded by some compact set $K \subset D$ for all $t \geqslant 0$. Then, for each limit point $p \in \omega^+$ there exists a solution $x = x(t)$ ($t \in R$) of some equation (6) such that $x(0) = p$, $\{x(t), t \in R\} \subset \omega^+$.

Suppose that for equation (1) one can find a Lyapunov functional candidate as

$$V(t, x_t) = V_1(t, x(t)) + \int\limits_0^t V_2(s-t, x(t), x(s)) ds \tag{7}$$

where ($x_t = x(s)$, $0 \leqslant s \leqslant t$), V_1 and V_2 are some nonnegative scalar functions which are defined and continuous in the domains $R^+ \times D$ and $R^- \times D \times D$.

Assume the existence of the upper right-hand derivative of the functional (7) along the trajectories of the system (1)

$$\dot{V}^+(t, x_t) = \lim_{h \to 0+} \frac{V(t, x_{t+h}) - V(t, x_t)}{h} \tag{8}$$

such that the following estimate holds

$$\dot{V}^+(t, x_t) \leqslant -W(t, x_t)$$
$$W(t, x_t) = W_1(t, x(t)) + \int\limits_0^t W_2(s-t, x(t), x(s)) ds \tag{9}$$

where $W_1(t, x)$ and $W_2(\tau, x, y)$ are some non-negative functions defined and continuous in the domains $R^+ \times D$ and $R^- \times D \times D$. These functions satisfy in these domains conditions as (2),(3) and (4).

Hence, in particular, for a continuous function $x : R \to K$ ($K \subset D$ is compact set) there exists an integral

$$\int\limits_{-\infty}^{t} W_2(s-t, x(t), x(s)) \, ds \tag{10}$$

Consider a family of shifts $\{W_1^\tau(t,x) = W_1(\tau + t, x), \quad \tau \in R^+\}$. We introduce the limit functions of W_1 as folllows

$$W_1^*(t,x) = \frac{d}{dt} \lim_{\tau_k \to +\infty} \int\limits_0^t W_1(\tau_k + s, x) ds \tag{11}$$

defined on the domain $R \times D$ for almost all $t \in R$.

Theorem 2.2. Suppose that for the equation (1) one can find a functional $V = V(t, x_t)$ bounded for all continuous function $x : R^+ \to D$ whose upper right-hand derivative satisfies inequality (9). Then for each bounded by some compact set $K \subset D$ solution $x = x(t)$ of equation (1) the set ω^+ consists of the solutions of the equation (1) which are satisfied the following equalities

$$W_1(t, x(t)) = 0, \; W_2(s-t, x(t), x(s)) = 0, \; t \geqslant s \tag{12}$$

Proof. For the solution $x = x(t, x_0)$ due to the condition (9) the function $V(t, x_t(x_0))$ ($x_t(x_0) = x(s, x_0)_t$ $0 \leqslant s \leqslant t$) is monotonically decreasing. Therefore, the following holds

$$\lim_{t \to +\infty} V(t, x_t) = V^* \geqslant 0 \tag{13}$$

From the inequality (7) also for all $T>0$ one can find

$$\begin{array}{l} V(t+T,x_{t+T})-V(t-T,x_{t-T})\leqslant \\ \leqslant -\int\limits_{t-T}^{t+T} W_1(\tau,x(\tau))d\tau- \\ -\int\limits_{t-T}^{t+T}(\int\limits_{0}^{\tau} W_2(s-\tau,x(\tau),x(s))ds)d\tau \end{array} \tag{14}$$

Let ω^+ be positive limit set and $p\in\omega^+$ be positive limit point defined by the sequence $t_k\to+\infty, x(t_k,x_0)\to p$. As in the proof of Theorem 2.1 one can find the solution $x=\phi(t)$ of the equation (6) which passes through the point p, $\phi(0)=p$. Thereafter, for the sequences $t_m^*\to+\infty$ $T_m^*\to+\infty$ constructed in the theorem 2 we have

$$\begin{array}{l} V\left(t,x_t^{(m)}\right)\Big|_{t=t_m^*+T_m^*} - V\left(t,x_t^{(m)}\right)\Big|_{t=t_m^*-T_m^*}\leqslant \\ \leqslant -\int\limits_{-T_m^*}^{T_m^*} W_1\left(\tau,x^{(m)}(\tau)\right)d\tau- \\ -\int\limits_{-T_m^*}^{T_m^*}\left(\int\limits_{-t_m^*}^{\tau} W_2\left(s-\tau,x^{(m)}(\tau),x^{(m)}(s)\right)ds\right)d\tau \end{array}$$

Hence, passing to the limit for $m\to+\infty$ and taking into account (13) we obtain

$$\begin{array}{l} W_1(\tau,\phi(\tau))=0 \\ \int\limits_{-\infty}^{\tau} W_2(s-\tau,\phi(\tau),\phi(s))\,ds=0 \end{array} \tag{15}$$

for all $t\in R$ and correspondingly $W_2(s-\tau,\phi(\tau),\phi(s))=0,\ \tau\geqslant s$. Theorem is proved.

Theorem 2.2 is a theorem of the invariance principle for equation (1).

Suppose that in equation (1) the following holds $f(0)=0$, $g(\tau,0,0)=0$ and therefore equation (1) has a zero solution $x(t,0)=0$.

From Theorem 2.2 it is easy to derive the following sufficient conditions for the asymptotic stability and instability of a solution in which we denote by $a:R^+\to R^+$ a function of Hahn type.

Theorem 2.3. Suppose that for the equation (1) one can find a functional (7) with a function $V_1(t,x)\geqslant a(\|x\|)$ whose upper right-hand derivative satisfies the inequality (9). In this case, there are no solutions $x=\phi(t)$ of the equation (6) satisfying the following equalities

$$\begin{array}{l} W_1(t,\phi(t))=0 \\ W_2(s-t,\phi(t),\phi(s))=0,\quad s\leqslant t \end{array} \tag{16}$$

for all $t\in R$ besides the zero solution $\phi(t)=0$. Then, the zero solution $x=0$ of the equation (1) is asymptotically stable.

Theorem 2.4. Suppose that for the equation (1) one can find a functional (7) with a function $V_1(t,x)$ that takes negative values in any sufficiently small neighborhood of $x=0$ and with an upper right-hand derivative satisfying the inequality (9). In this case, there are no solutions $x=\phi(t)$ of the equation that satisfy relations

$$\begin{array}{l} V_1(t,\phi(t))<0,\quad W_1(t,\phi(t))=0 \\ W_2(s-t,\phi(t),\phi(s))=0 \end{array} \tag{17}$$

Then, the zero solution $x=0$ of equation (1) is unstable.

The proofs of Theorems 2.3 and 2.4 are derived directly from Theorem 2.2.

3 The problem of program position stabilization for the holonomic mechanical system on the base of PI-regulator

Consider a controlled mechanical system with n degrees of freedom described by Lagrange equations as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot q}\right)-\frac{\partial T}{\partial q}=-\frac{\partial\Pi}{\partial q}+Q(q,\dot q)+U \tag{18}$$

where q is the vector of generalized coordinates, $T=\dot q' A(q)\dot q/2$ is the kinetic energy of the system with inertial matrix $A(q)$, $\dot q=dq/dt$, $Q(q,\dot q)$ is the vector of generalized dissipative and gyroscopic forces, $Q(q_i,0)=0$, $Q'\dot q\leqslant 0$, $\Pi=\Pi(t,q)$ is the potential energy, U is the generalized control force, $()'$ is the transpose operation. Suppose that included in (18) functions are defined and continuous for all $q\in R^n$, restrictions on the control U are not imposed.

We can represent equations (18) resolved with respect to $\ddot q$ in the form

$$\left.\begin{array}{l} \frac{dq}{dt}=\dot q \\ \frac{d\dot q}{dt}=A^{-1}(q)\left(C(q,\dot q)\dot q+Q(q,\dot q)-\frac{\partial\Pi}{\partial q}+U\right) \end{array}\right\} \tag{19}$$

The coefficients of the matrix $C=(c_{jk})$ of inertial forces are defined by the following equality

$$\begin{array}{l} c_{jk}=\frac{1}{2}\sum\limits_{i=1}^{n}\left(\frac{\partial a_{ik}}{\partial q_i}-\frac{\partial a_{kj}}{\partial q_i}-\frac{\partial a_{ij}}{\partial q_k}\right)\dot q_j \\ j,k=1,...n \end{array} \tag{20}$$

We consider the stabilization problem of program position of equilibrium

$$\dot q=0,\quad q=q_0=const \tag{21}$$

We show that this problem is solved by means of an integral regulator such as

$$U = -\frac{\partial \Pi_u(t,q)}{\partial q} - \\ -\left(\frac{\partial f}{\partial q}\right)' \int\limits_0^t P(\nu - t)\left(f(q(t)) - f(q(\nu))\right) d\nu \quad (22)$$

where $\Pi_u \in R \times R^n \to R$ is some continuously differentiable function, $P : R \to R^{n\times n}$ is some nonnegative matrix function with a derivative $\partial P(s)/\partial s$ as

$$x' \frac{\partial P(s)}{\partial s} x \geqslant \alpha(s)\|x\|^2 \quad (23)$$

where $\alpha(s) > 0,\ f : R \to R^n$ is some differentiable function which has finite number of the prototypes $f(c)$ in any bounded domain $\{q \in R^m : \|q\| \leqslant \mu = const\}$. In other words, it has finite number of solutions of the equation $f(q) = c$.

Let us make the change of variables such as $x = q - q^0,\ y = q'$. Then, the equations (18) with the controller (22) can be written as

$$\left.\begin{array}{l} \frac{dx(t)}{dt} = y(t) \\ \frac{dy(t)}{dt} = A_1^{-1}(x(t))(C_1(x(t),y(t))y(t) + \\ +Q_1(x(t),y(t)) - \\ -\frac{\partial \Pi(t,x(t))}{\partial t}\left(\frac{\partial f_1(x(t))}{\partial x}\right)' \int\limits_0^t P(\nu - t)(f(x(t)) - \\ -f(x(\nu)))d\nu) \end{array}\right\} \quad (24)$$

where the subscript "1" denotes the functions which are obtained from the corresponding functions included in (21) and (22) as a result of the aforementioned change of variables.

We will use the Lyapunov functional candidate as

$$V = \tfrac{1}{2}y'(t)A_1(x(t))y(t) + \Pi_1(t,x(t)) + \\ +\tfrac{1}{2}\int\limits_0^t (f(x(t)) - f(x(\nu)))'P(\nu - t)(f(x(t)) - \\ -f(x(\nu)))d\nu$$

where $\Pi_1(t,x) = \Pi(t,x) + \Pi_u(t,x)$.

For the time derivative of this functional due to the equation (24) we obtain

$$\dot{V} = \\ = -\frac{1}{2}\int\limits_0^t (f_1(x(t)) - f_1(x(\nu)))' \frac{\partial P(\nu - t)}{\partial \nu}(f_1(x(t)) - \\ -f_1(x(\nu)))d\nu \leqslant 0 \quad (25)$$

Accordingly to Theorems 2.2 – 2.4 one can simply obtain the following results.

Theorem 3.1. Let the controller (22) be such that the function $\Pi_1(t,x)$ is definitely positive and $\partial \Pi_1/\partial t \leqslant 0$. Let also in some region $\{0 < \|x\| < \Delta\}$ the following inequality hold

$$\frac{\partial \Pi_1}{\partial x} > 0$$

Then, this controller ensures the stabilization of the program position (21). Moreover, each motion of the system (24) approaches arbitrarily close to the set

$$\{\dot{x}(t) = 0, \quad \frac{\partial \Pi(t,x(t))}{\partial x} = 0\}$$

as $t \to +\infty$.

Theorem 3.2. Let the controller (22) be such that the function $\Pi_1(t,x)$ takes any sufficiently small negative values in the neighborhood $x = 0$,

$$\partial \Pi_1/\partial t \leqslant 0$$

and the following inequality holds

$$\left\|\frac{\partial \Pi_1}{\partial x}\right\| > 0 \quad (26)$$

in the domain $\{0 < \|x\| < \Delta, \quad \Pi_1(x) < 0\}$. Then, the controller (22) is destabilizing.

Theorems 3.1 and 3.2 represent the basis for constructing integral regulators both linear (proportional-integral) and non-linear ones in the problem on stabilization of program positions of equilibrium of holonomic mechanical systems in a nonlinear formulation.

Theorems 2.1 and 2.2 can also be applied to the solution of the problem of stabilizing the program position (21) by use nonlinear integro-differential regulators such as

$$U = -F_U(t,q(t),\dot{q}(t)) - \int\limits_0^t P(\nu - t)\frac{\partial \Pi_U(q(\nu))}{\partial q} d\nu$$

$$U = -\frac{\partial \Pi_U(t,q(t))}{\partial q} - \int\limits_0^t P(\nu - t)\dot{q}(\nu)d\nu$$

where F_U is the control component of the dissipative force type satisfying conditions (2), $\dot{q}'F_U(t,q,\dot{q}) \geq \alpha_0\|\dot{q}\|^2$, $\alpha_0 > 0$, $P : R^- \to R^{n\times n}$ is the gain matrix function, $P' = P$, $F_U \in C^2(R^n \to R)$, for which the conditions (3) and (4) take place.

4 Conclusion

The paper presents the development of the Lyapunov functional method in the study of the stability of an integro-differential equation of Volterra type. On this basis new results have been obtained to stabilize the program positions of robot manipulators with integral feedback. The control of the type of nonlinear integral regulator allows to solve the stabilization problem without measuring the velocities. The structure of the regulators constructed is such that it does not require exact values of inertial parameters of the system, it allows to take into account the action of uncontrolled forces. The results his paper develop the works [Andreev, Blagodatnov, and Kilmetova, 2013], [Andreev and Rakov, 2015], [Andreev and Peregudova, 2015], [Andreev and Peregudova, 2017].

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