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CONTROL OF MECHANICAL OSCILLATIONS FOR MAGNETOSTRICTIVE ACTUATOR

Pavel A. Shavrin
Department of Industrial Automation
Togliatti State University
Russia
shavrin@avtograd.ru

Sergey A. Kochetkov
Department of Industrial Automation
Togliatti State University
Russia
ser711@yandex.ru

Sergey A. Kiselyov
Department of Industrial Automation
Togliatti State University
Russia
kiselyov@avtograd.ru

Abstract

The problem for control of mechanical oscillations of ultrasonic range for nonlinear magnetostrictive actuator is considered. Some features of the control algorithm are demonstrated on an example of the control system design for magnetostrictive vibrator. The solution of the problem is obtained by creating the sliding motion in such a manner that a stable limit cycle appears in the state space of the mechanical subsystem. Furthermore, the parameters of that cycle can be arbitrary modified with respect to the desired behavior of the closed loop system. Thus, the amplitude, frequency and shape of the mechanical oscillations can be adjusted on the prescribed manner.

Key words

Magnetostrictive actuator, mechanical oscillations, sliding mode control, amplitude modulation.

1 Introduction

As it is well known the magnetostrictive vibrator is a source of the mechanical oscillations of ultrasonic range. This property defines its wide application in the numerous technical and technology areas. At present there are a lot of ways and methods to control the magnetostrictive transformers including various circuits and control systems of power converters [Syrkin, 1972], [Donskoi et al., 1982]. However, in spite of the numerous investigations of magnetostrictive effect and its application only such a question like a steady mode model can be regarded as finally finished. Here, the theoretical basis of that model is in the various harmonic linearization and harmonic balance methods, because the magnetostrictor presents generally speaking a nonlinear system [Syrkin et al., 1970].On the other hand these techniques are valid just in analysis and calculation for the steady mode of the nonlinear system. The motivation to use a steady mode model is magnified by an essential property of the magnetostrictive transformer to be a high quality resonant system. It follows that practically all the most of control algorithms are based on the different automatic frequency control (AFC) techniques realizing some extremal principles (maximum of power consumed, maximum of acoustic intensity if it is available for measurement, minimum of phase angle between current and voltage, minimum of deviation from the resonant frequency of mechanical subsystem and so on) [Donskoi et al., 1982]. At the same time the absence of the adequate mathematical model describing the plant in the state space does not permit to realize all the magnetostrictor's features. In particular, the questions of adjusting the parameters of mechanical oscillations (amplitude, frequency, shape) on determined time law stay still open. By the way, the above mentioned AFC algorithms are useless in principle for solution of such a task to say nothing of the transient modes at start up which can only be estimated after manufacturing and experimental testing a control system hardware. As to various power converters which have one and more additional reactive units then in a viewpoint of control algorithm these units just increase the order of system. Meanwhile, it is always desirable to reduce the order of the plant equations especially in the case of nonlinear system. In this point of view additional reactive units are accessible just as a result of control strategy applying the state space extension, of course, if it is not a question of the power converter ability to work.

The proposed paper presents an effort to design a control system for the high quality nonlinear magnetostrictive actuator provided with amplitude modulation of ultrasonic oscillations and invariant to the variations of mechanical load. With this aim in mind an exemplary control system design is conducted and some features of the algorithm inaccessible to existing systems are demonstrated. In the first step a linearized magnenostrictor is examined. A solution of the control problem is reached by the selection of a nonlinear surface in the state space and creating the sliding motions on that surface [Utkin, 1992]. So a stable limit cycle appears in the subspace of mechanical variables. The obtained algorithm is considered as a basic solution. A full order state observer is used for the estimation of unmeasured variables. So it is a question of sliding mode control at the deviation between model and plant [Bondarev et al., 1985]. The computer simulations illustrate the main theoretic results and features of the control system.

2 Linearized Model and Basic Solution of Control Problem

It is well known that the most effective generation of the ultrasonic oscillations is observed when polarizing the magnetic system of the magnetostrictive vibrator with the constant current [Syrkin, 1972]. So considering the magnetostrictor in the vicinity of the equilibrium point defined by a polarization and applying some harmonic linearization technique you can get a linearized model. If, furthermore, this model can be supported with some experimental results, then it can be examined as correct model. Let us choose a half-bridge voltage inverter as a power converter for magnetostrictive transformer. In this case the differential equations of the linearized magnetostrictive vibrator in the terms of equivalent electric circuit have a form

$$\begin{cases} L\dfrac{di}{dt} = -Ri - u + e; \\ C_m\dfrac{du}{dt} = i - \dfrac{u}{R_m} - i_m; \\ L_m\dfrac{di_m}{dt} = u. \end{cases} \tag{1}$$

Here L, R are the electric parameters; i is electric current; u, i_m are the mechanical speed and shift respectively; C_m, L_m, R_m denote the equivalent mechanical mass, softness and load; e is output voltage of power inverter considered as a control input. The control input e is discontinuous and accepts the values $e \in \{-E; E\}$.

Let us state the control problem as a problem of creating the stable limit cycle in the state space of mechanical subsystem via an appropriate selection of control input. Let all the parameters and variables be known or accessible for measurement. According to the standard procedure of discontinuous control system design [Utkin, 1992], let us examine the mechanical subsystem described by the last two equations of (1). The current i is admitted as a "fictitious" control. Rewriting this system in the new variables $x_1 = \sqrt{C_m}u$; $x_2 = \sqrt{L_m}i_m$, we get

$$\begin{cases} \dot{x}_1 = -\alpha x_1 - \omega x_2 + bi; \\ \dot{x}_2 = \omega x_1, \end{cases} \tag{2}$$

where

$$\alpha = \frac{1}{R_m C_m}; \quad \omega = \frac{1}{\sqrt{L_m C_m}}; \quad b = \frac{1}{\sqrt{C_m}}.$$

Let

$$bi = Kx_1 - x_2^3; \quad K > \alpha. \tag{3}$$

Then denoting $K - \alpha = a$ we can obtain the closed loop system

$$\begin{cases} \dot{x}_1 = (a - x_1^2)x_1 - \omega x_2; \\ \dot{x}_2 = \omega x_1, \quad a > 0. \end{cases} \tag{4}$$

As you can see these equations are similar to the Van der Pole system and present a particular case of Lienard equation [Kamke, 1959]. As it is well known this system can generate the stable oscillations. Making a replacement

$$x_1 = X\cos\theta, \quad x_2 = X\sin\theta$$

and applying then an integral averaging, we receive a system

$$\begin{cases} \dot{X} = \frac{3}{8}\left(\frac{4}{3}a - X^2\right)X; \\ \dot{\theta} = \omega, \end{cases} \tag{5}$$

which has a general solution of the form

$$X = \sqrt{\frac{4aCe^{at}}{3(1 + Ce^{at})}}; \quad \theta = \omega t + \theta_0,$$

where C is an arbitrary constant and $C \in (-\infty, -1) \cup (0, \infty)$. It is easy to notice if $a > 0$ then $\lim_{t\to\infty} X = \sqrt{\frac{4a}{3}}$. In other words the oscillation amplitude is directed to the prescribed value determined by the coefficient K. The oscillation frequency is equal to the eigenfrequency of mechanical subsystem. Thus, a stable limit cycle really appears in the mechanical system of magnetostrictive vibrator. In the terms of equivalent electric circuit it corresponds to a current resonance in the contour $L_m - C_m$.

Let us pay an attention that expression (3) forms some nonlinear surface in the phase space of initial system (1). Now you can set a problem of creating the sliding mode on that surface [Utkin, 1992]. Selecting a switching function in the form

$$s = bi - Kx_1 + x_1^3,$$

and differentiating it on the system trajectories we obtain

$$\dot{s} = -b\frac{R}{L}i - \frac{1}{L}x_1 + \frac{b}{L}e + (3x_1^2 - K)(-\alpha x_1 - \omega x_2 + bi).$$

As you can see if

$$e = -E\,\mathrm{sign} s$$

then

$$s\dot{s} < 0$$

at enough large value of E. So a sliding motion arises in the surface $s = 0$. In this case the closed loop system has the desired features and the design goal is reached in such a way. This control algorithm can be considered as a basic solution of the problem.

Now let us examine the control problem for the linearized magnetostrictive transformer when incomplete information of the system variables is available. More exactly, it is assumed that system parameters (L, R, C_m, L_m, R_m) are known and the electric current i is measurable. The solution of the problem in these conditions is well known. That is an application of the full order or reduced order state observer. For example in the given situation a full order state observer has a form

$$\begin{cases} L\dfrac{d\hat{\imath}}{dt} = -R\hat{\imath} - \hat{u} + e + k_1\bar{\imath}; \\ C_m\dfrac{d\hat{u}}{dt} = \hat{\imath} - \dfrac{\hat{u}}{R_m} - \hat{\imath}_m - k_2\bar{\imath}; \\ L_m\dfrac{d\hat{\imath}_m}{dt} = \hat{u}; \\ \bar{\imath} = i - \hat{\imath}, \end{cases} \tag{6}$$

where symbol "ˆ" denotes an estimate of the variable. It is obvious that convergence of the estimates can be always managed by an appropriate selection of the coefficients k_1, k_2. In particular if $k_1 > 0$, $k_2 > 0$ then trivial solution of the corresponding homogeneous differential system is asymptotically stable.

The computer simulations illustrate ability to work and some features of the proposed algorithm. The following parameters of equivalent electric circuit are used for simulation:

$$L = 0.36\,mH,\ R = 2\,\Omega,\ L_m = 10.8\,\mu H,$$
$$C_m = 4.823\,\mu F,\ R_m = 150\,\Omega.$$

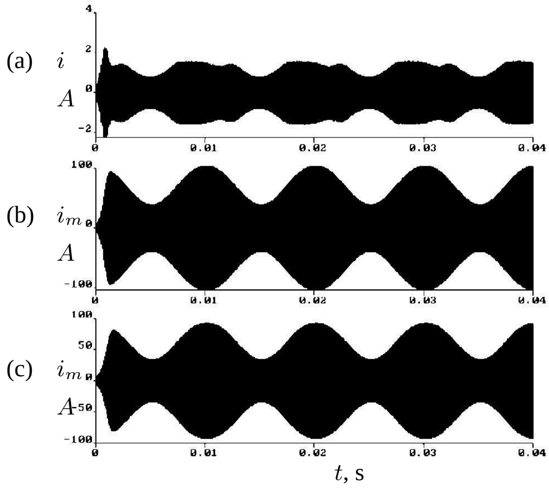

Figure 1. Simulation results for the closed loop system when modulated by a low frequency ((a) is an electric current i; (b) is an equivalent of the mechanical shift i_m; (c) is the same when doubled mechanical load).

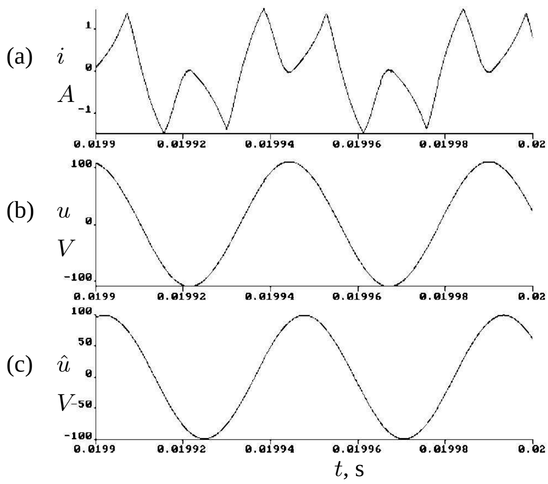

Figure 2. Simulation results for the control system with asymptotic state observer.

The voltage of a DC supply is $300\,V$. So the control parameter $E = 150\,V$. The simulation results for the closed loop system when modulated by a low frequency according to the expression $75 + 25\cos 628t$ are depicted in the Fig.1. The conditions of the test include a variation of the equivalent mechanical load in double the above denoted parameter R_m.
Figure 2 shows a simulation of the control system with asymptotic state observer at the conditions of the previous test. The state observer demonstrates a good convergence to the true variables. But it is only possible at an exact correlation between parameters of observer and the plant. However, in practice this situation does not take place as a rule. The mechanical parameters determined by a geometry and the features of material are practically constant and can be tuned. At the same time the electrical parameters especially

the inductance L can vary in several times of the magnitude since the magnetostrictive vibrator presents a nonlinear system as it was mentioned before. Nevertheless a sliding mode observer is useful in these conditions.

3 Sliding Mode Control of Nonlinear Magnetostrictive Vibrator

Let us consider a nonlinear model of the magnetostrictive transformer taking into account a nonlinearity of electric inductance L. In this case the polarized vibrator is described by the following system in terms of equivalent electric circuit [Syrkin, 1972], [Syrkin et al., 1970]

$$\begin{cases} \dfrac{d\psi}{dt} = -Ri - u + e; \\ C_m \dfrac{du}{dt} = i - \dfrac{u}{R_m} - i_m; \\ L_m \dfrac{di_m}{dt} = u. \end{cases} \tag{7}$$

where $\psi = \psi(i)$ is a nonlinear and monotonic function. Now we would like a closed loop system to be similar to the linearized one examined in the previous section. As before it is assumed that current i is measured and parameters C_m, L_m, R_m describing mechanical properties of the magnetostrictive vibrator are known.

Making note that

$$\frac{d\psi}{dt} = L\frac{di}{dt}, \quad \text{where} \quad L = L(i)$$

and denoting $\frac{1}{R_m} = g$ we can rewrite system (7) in the form of (1) as

$$\begin{cases} L \quad \dfrac{di}{dt} = -Ri - u + e; \\ C_m \dfrac{du}{dt} = i - gu - i_m; \\ L_m \dfrac{di_m}{dt} = u. \end{cases} \tag{8}$$

We also suppose that $\inf_i L = L_0$, where $L_0 > 0$ is well defined value.

Let us propose an observer

$$\begin{cases} \hat{L} \quad \dfrac{d\hat{\imath}}{dt} = -R\hat{\imath} - \hat{u} + e + v; \\ C_m \dfrac{d\hat{u}}{dt} = \hat{\imath} - g\hat{u} - \hat{\imath}_m - Mz; \\ L_m \dfrac{d\hat{\imath}_m}{dt} = \hat{u}; \\ \mu \dfrac{dz}{dt} = v - z; \\ v = V\text{sign}\bar{\imath}; \\ \bar{\imath} = i - \hat{\imath} \end{cases} \tag{9}$$

and control algorithm

$$\begin{aligned} &e = -E\text{sign}s, \quad s = \hat{\imath} - Mz - f(\hat{u}), \\ &f(\hat{u}) = \left(g + a\left(1 - \tfrac{\hat{u}^2}{{U_z}^2}\right)\right)\hat{u}, \end{aligned} \tag{10}$$

where symbols $\hat{}$ and $\bar{}$ denote an estimate and an error of the variable respectively.

Let $\hat{L} < L_0$ then $\inf_i \bar{L} > 0$ where $\bar{L} = L - \hat{L}$. Subtracting the first equation of (9) from the first one of (8) we obtain

$$\hat{L} \quad \frac{d\bar{\imath}}{dt} = -R\bar{\imath} - \bar{u} - v - \bar{L}\frac{di}{dt}. \tag{11}$$

Thus, we can set a hierarchy sliding mode control problem [Utkin, 1992]. Here the sliding surfaces are

$$s_1 = \bar{\imath} = 0 \quad s_2 = \hat{\imath} - Mz - f = 0 \tag{12}$$

and controls are v and e. According to the hierarchy control procedure [Utkin, 1992] let the sliding mode appear on the surface $s_1 = 0$ first and then on the surface $s_2 = 0$. Considering (11) and calculating an equivalent control from the equation $\dot{s}_1 = 0$ we get

$$v_{eq} = -\bar{u} - \bar{L}\frac{di}{dt}. \tag{13}$$

Making notice that in sliding mode

$$\hat{\imath} = i \quad \Rightarrow \quad \frac{d\hat{\imath}}{dt} = \frac{di}{dt}$$

and taking (13) into account we can define an equivalent control e_{eq} from the equation $\dot{s}_2 = 0$ as

$$e_{eq} = Ri + u - \frac{ML}{\mu + M\bar{L}}(\bar{u} + z) + \frac{L\mu}{\mu + M\bar{L}}\frac{df}{dt}. \tag{14}$$

Substitution of e_{eq} into the (13) in consideration of (8) gives

$$v_{eq} = -\bar{u} + \frac{M\bar{L}}{\mu + M\bar{L}}(\bar{u} + z) - \frac{\mu\bar{L}}{\mu + M\bar{L}}\frac{df}{dt}. \quad (15)$$

Then

$$\mu\frac{dz}{dt} = -\frac{\mu}{\mu + M\bar{L}}(\bar{u} + z) - \frac{\mu\bar{L}}{\mu + M\bar{L}}\frac{df}{dt}. \quad (16)$$

Thus denoting $\mu + M\bar{L} = \tau$ we get

$$\tau\frac{dz}{dt} = -(\bar{u} + z) - \bar{L}\frac{df}{dt}. \quad (17)$$

In sliding mode the error system takes a form

$$\begin{cases} C_m\dfrac{d\bar{u}}{dt} = -g\bar{u} - \bar{\imath}_m + Mz; \\ L_m\dfrac{d\bar{\imath}_m}{dt} = \bar{u}; \\ \tau\frac{dz}{dt} = -\bar{u} - z - \bar{L}\frac{df}{dt}. \end{cases} \quad (18)$$

Considering the homogeneous system

$$\begin{cases} C_m\dfrac{d\bar{u}}{dt} = -g\bar{u} - \bar{\imath}_m + Mz; \\ L_m\dfrac{d\bar{\imath}_m}{dt} = \bar{u}; \\ \tau\frac{dz}{dt} = -\bar{u} - z. \end{cases} \quad (19)$$

we can find that trivial solution is asymtotically stable when $M > 0$. It follows that perturbed system (18) is also stable. At the same time according to the equation $s_2 = 0$, $\hat{\imath} - Mz = f(\hat{u})$ is obtained. Then

$$\begin{cases} C_m\dfrac{d\hat{u}}{dt} = -g\hat{u} + f(\hat{u}) - \hat{\imath}_m; \\ L_m\dfrac{d\hat{\imath}_m}{dt} = \hat{u}, \end{cases} \quad (20)$$

and finally a closed loop system can be written as

$$\begin{cases} C_m\dfrac{d\hat{u}}{dt} = b\left(1 - \dfrac{\hat{u}^2}{{U_z}^2}\right)\hat{u} - \hat{\imath}_m; \\ L_m\dfrac{d\hat{\imath}_m}{dt} = \hat{u}, \end{cases} \quad (21)$$

Simulation results illustrate some features of the nonlinear magnetostrictive vibrator supplied with the proposed algorithm. Figure 3 shows the low frequency

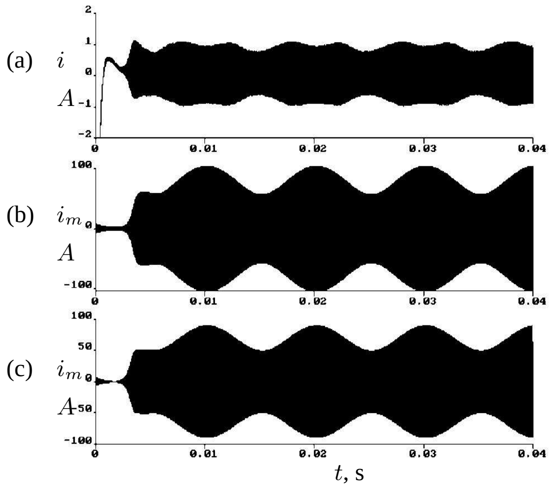

Figure 3. Simulation of the closed loop system for nonlinear magnetostrictive vibrator when modulated by a low frequency ((a) is an electric current i; (b) is an equivalent of the mechanical shift i_m; (c) is the same when doubled mechanical load).

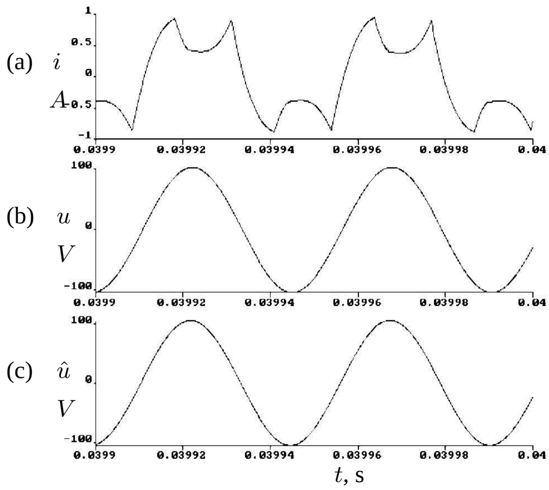

Figure 4. Simulation of the control system with sliding mode observer.

modulation at the test conditions of the linearized model. The ferrite nonlinearity is approximated by an expression

$$i = \frac{1}{L_f}(\psi + n\psi^3),$$

where $L_f = 3.4\,mH$, $n = 0.02$. The next Figure 4 depicts equivalent electric values of the mechanical speed u and its estimate $\hat{u}$.

4 Conclusion

In this paper a sliding mode approach is applied to control of mechanical oscillations of a magnetostrictive vibrator. The solution of the control problem is reached by selection of a nonlinear sliding surface so that a stable limit cycle appears in the state space of mechanical variables. Application of an asymptotic state observer for estimation of unmeasured variables does not ruin sliding motion and makes an effective sliding mode control possible. But it is unacceptable in nonlinear situation. As it was shown by theoretic analysis and computer simulation, a good remedy is sliding mode observer which permits not only to save the main properties of basic algorithm but to increase the robustness as well. Thereby, a deterministic controller for magnetostrictive vibrator is presented which allows to adjust the mechanical oscillations in arbitrary manner.

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