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GUIDANCE AND CONTROL OF A LAND-SURVEY SPACECRAFT WITH A SMOOTH CONJUGATION OF BOUNDARY CONDITIONS

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Abstract

New statement of the optimization problem by the spacecraft (SC) rotation maneuver with the general boundary conditions is considered. Methods for exact numeric and approximate analytic solution of the stated problem, and also some results on synthesis of the SC guidance laws with a smooth conjugation of boundary conditions are presented. Methods and results on the SC robust pulse-width and digital attitude control are also represented.

Key words

spacecraft, guidance, control

1 INTRODUCTION

The dynamic requirements to the attitude control systems (ACSs) for remote sensing spacecraft (SC) are:

- guidance the telescope's line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- stabilization of an image motion at the onboard optical telescope focal plane.

Moreover, these requirements are expressed by rapid angular manoeuvering and spatial compensative motion with a variable vector of angular rate. Increased requirements to such information satellites have motivated intensive development of the gyro moment clusters (GMCs) based on excessive number of gyrodines (GDs) — single-gimbal control moment gyros. Mathematical aspects of the SC nonlinear gyromoment control were represented in a number of research works (Junkins and Turner, 1986; Hoelscher and Vadali, 1994) et al., including authors' papers (Somov *et al.*, 1999; Somov *et al.*, 2005; Somov *et al.*, 2007). The paper suggests new results on guidance, pulse-width and digital robust attitude control of the agile land-survey spacecraft.

2 MATHEMATICAL MODELS

We introduce the inertial reference frame (IRF) $\mathbf{I}_\oplus$ $(\mathrm{O}_\oplus \mathrm{X}_\mathrm{e}^\mathrm{I}\mathrm{Y}_\mathrm{e}^\mathrm{I}\mathrm{Z}_\mathrm{e}^\mathrm{I})$, the geodesic Greenwich reference frame (GRF) $\mathbf{E}_\mathrm{e}$ $(\mathrm{O}_\oplus\mathrm{X}^\mathrm{e}\mathrm{Y}^\mathrm{e}\mathrm{Z}^\mathrm{e})$ which is rotated with respect to the IRF by angular rate vector $\boldsymbol{\omega}_\oplus \equiv \boldsymbol{\omega}_\mathrm{e}$ and the geodesic horizon reference frame (HRF) $\mathbf{E}_\mathrm{e}^\mathrm{h}$ $(C\,\mathrm{X}_c^\mathrm{h}\mathrm{Y}_c^\mathrm{h}\mathrm{Z}_c^\mathrm{h})$ with origin in a point C and ellipsoidal geodesic coordinates altitude H_c, latitude B_c and longitude L_c. There are standard defined the body reference frame (BRF) $\mathbf{B}$ $(\mathrm{O}xyz)$ with origin in the SC mass center O, the orbit reference frame (ORF) $\mathbf{O}$ $(\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o})$, the optical telescope (sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}}$ $(\mathrm{O}x^\mathrm{s}y^\mathrm{s}z^\mathrm{s})$ and the image field reference frame (FRF) $\boldsymbol{\mathcal{F}}$ $(\mathrm{O}_i x^i y^i z^i)$ with origin in center O_i of the telescope focal plane $y^i\mathrm{O}_i z^i$. The BRF attitude with respect to the IRF is defined by quaternion $\boldsymbol{\Lambda}_\mathrm{I}^b \equiv \boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \lambda_3\}$, and with respect to the ORF – by vector-column $\boldsymbol{\phi} = \{\phi_i, i = 1,2,3 \equiv 1 \div 3\}$ of Euler-Krylov angles ϕ_i. Let us vectors $\boldsymbol{\omega}(t)$, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are standard denotations of the SC body vector angular rate, the SC mass center's position and progressive velocity with respect to the IRF. Further the symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\,\cdot\,\}$, $[\,\cdot\,]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^\mathrm{t}$ for matrixes are conventional denotations. The GMC's angular momentum (AM) vector $\boldsymbol{\mathcal{H}}$ have the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = \mathrm{h}_g \sum \mathbf{h}_p(\beta_p)$, there h_g is constant own AM value for each GD #p, $p = 1 \div m$ with the GD's AM unit $\mathbf{h}_p(\beta_p)$ and vector-column $\boldsymbol{\beta} = \{\beta_p\}$. Within precession theory of the control moment gyros, for a fixed position of the SC flexible solar array panels (SAPs) with some simplifying assumptions and for $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model is appeared as follows

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2;\ \mathbf{A}^o\,\{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}\} = \{\mathbf{F}^\omega, \mathbf{F}^q\}, \qquad (1)$$

$$\mathbf{F}^\omega = \mathbf{M}^\mathrm{g} - \boldsymbol{\omega} \times \mathbf{G} + \mathbf{M}^\mathrm{e} + \mathbf{M}_d^o(t, \boldsymbol{\Lambda}, \boldsymbol{\omega}) + \mathbf{Q}^o(\boldsymbol{\omega}, \dot{\mathbf{q}}, \mathbf{q});$$

$$\mathbf{F}^q = \{-((\delta^q/\pi)\Omega_j^q \dot{q}_j + (\Omega_j^q)^2 q_j) + \mathrm{Q}_j^q(\boldsymbol{\omega}, \dot{q}_j, q_j)\};$$

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q \\ \mathbf{D}_q^{\mathrm{t}} & \mathbf{I} \end{bmatrix}; \; \begin{array}{l} \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q \dot{\mathbf{q}}; \\ \mathbf{G}^o = \mathbf{J}\,\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}); \end{array}$$

where $\boldsymbol{\omega} = \{\omega_i\}$, $\mathbf{q} = \{q_j, j = 1 \div n^q\}$; vector $\mathbf{M}^{\mathrm{e}}$ presents the orientation engine unit (OEU) torques, vector $\mathbf{M}_d^o(\cdot)$ is an external torque disturbance, and $\mathbf{Q}^o(\cdot), \mathrm{Q}_j^q(\cdot)$ are nonlinear continuous functions.

The OEU is based on six thermal-catalytic jet engines (JEs) with a pulse-width modulation (PWM) of the JE thrust. For the PWM of normalized command τ_r by the thrust inclusion $P^n(t, \tau_r^d) \in \{0, 1\}$, $r \in \mathbb{N}_0 \equiv [0, 1, 2, ...)$ by each JE, namely

$$P^n(t, \tau_r^d) = \begin{cases} 1 & t \in [t_r, t_r + \tau_r^d); \\ 0 & t \in [t_k + \tau_r^d, t_{r+1}), \end{cases} \quad (2)$$

ratio

and the modulation characteristic is described by the

$$\tau_r^d = \begin{cases} 0 & \tau_r < \tau_{\mathrm{m}}; \\ \tau_r & \tau_{\mathrm{m}} \leq \tau_r < \tau^{\mathrm{m}}; \\ \tau^{\mathrm{m}} & \tau^{\mathrm{m}} \leq \tau_r < T_u^{\mathrm{e}}; \\ T_u^{\mathrm{e}} & \tau_r > T_u^{\mathrm{e}}. \end{cases} \quad (3)$$

Taking into account a transport delay T_{zu}^d dynamic processes on the normalized thrust $P_d^n(t)$ for each JE are presented by the differential equation $T^{\mathrm{d}} \dot{P}_d^n + P_d^n = P^n(t - T_{zu}^d, \tau_r^d)$ with the initial condition $P_d^n(t_0) = 0$, where a time constant T^d accepts two values T_+^d or T_-^d according to the ratio:

$$if\, P^n = 1\; then\; T^d = T_+^d\; else\; T^d = T_-^d.$$

For everyone j-th JE $D_j, j = 1 \div 6$ there is compared the vector $\mathbf{P}_j(t) = P^{\mathrm{m}} P_d^n(t)\ \mathbf{p}_j$ of the current jet thrust with fixed unit $\mathbf{p}_j$ beginning in a point $\mathrm{O}_j^{\mathrm{d}}$, where P^{m} is the current maximal thrust value, identical for all JEs. The point $\mathrm{O}_j^{\mathrm{d}}$ arrangement is defined by a radius-vector $\boldsymbol{\rho}_j$. The OEU control torques concerning axes $\mathrm{O}x$, $\mathrm{O}y$ and $\mathrm{O}z$ are created by JEs' pairs. Logic of the command τ_{jr} formation for inclusion everyone j-th JE takes into account a sign of a command signal v_{ir} on channel $i = x, y, z$ and is described by such algorithm: $\tau_{ir} = |\mathrm{v}_{ir}|$; $\mathrm{s}_{ir} = \mathrm{sign}\ \mathrm{v}_{ir}$, $i = x, y, z$, and then, for example for $i = x$:

$$if\, \mathrm{s}_{xr} > 0\, then\, (\tau_{1r} = \tau_{xr} \& \tau_{2r} = 0)$$
$$else\, (\tau_{1r} = 0 \& \tau_{2r} = \tau_{xr}).$$

Formed by the OEU the control torque vector $\mathbf{M}^{\mathrm{e}}$ is calculated by formula $\mathbf{M}^{\mathrm{e}} \equiv \mathbf{M} = \sum \boldsymbol{\rho}_j^d \times \mathbf{P}_j$.

At matrix $\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \partial \mathbf{h}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}$ the GMC torque vector $\mathbf{M}^{\mathbf{g}}(\boldsymbol{\beta}, \dot{\boldsymbol{\beta}})$ is presented as follows:

$$\mathbf{M}^{\mathbf{g}} = -\dot{\boldsymbol{\mathcal{H}}} = -h_g \mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta})\, \mathbf{u}^{\mathrm{g}}; \quad \dot{\boldsymbol{\beta}} = \mathbf{u}^{\mathrm{g}}, \quad (4)$$

$\mathbf{u}^{\mathrm{g}} = \{\mathrm{u}_p^{\mathrm{g}}\}$, $\mathrm{u}_p^{\mathrm{g}}(t) = a^{\mathrm{g}}\, \mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u_{pk}^g, d^g), \bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}}), T_u]$ with constants a^{g}, d^g, $\bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}}$ and a control period $T_u = t_{k+1} - t_k$, $k \in \mathbb{N}_0$; discrete functions $u_{pk}^g \equiv u_p^g(t_k)$ are outputs of digital nonlinear control law (CL), and functions $\mathrm{Sat}(x, a)$ and $\mathrm{Qntr}(x, a)$ are general-usage ones, while the holder model with the period T_u is such: $y(t) = \mathrm{Zh}[x_k, T_u] = x_k\ \forall t \in [t_k, t_{k+1})$.

At given the SC body angular programmed motion $\boldsymbol{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$, $\boldsymbol{\varepsilon}^p(t) = \dot{\boldsymbol{\omega}}^p(t)$ with respect to the IRF $\mathbf{I}_{\oplus}$ during time interval $t \in \mathrm{T} \equiv [t_{\mathrm{i}}, t_{\mathrm{f}}] \subset \mathrm{T}_{t_0}$, $t_{\mathrm{f}} \equiv t_{\mathrm{i}} + T$, and for forming the vector of corresponding continuous control torque $\mathbf{M}^{\mathbf{g}}(\boldsymbol{\beta}(t), \dot{\boldsymbol{\beta}}(t))$ (4), the vector-columns $\dot{\boldsymbol{\beta}} = \{\dot{\beta}_p\}$ and $\ddot{\boldsymbol{\beta}} = \{\ddot{\beta}_p\}$ must be component-wise module restricted:

$$|\dot{\beta}_p(t)| \leq \bar{\mathrm{u}}_{\mathrm{g}} < \bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}}, \;\; |\ddot{\beta}_p(t)| \leq \bar{\mathrm{v}}_{\mathrm{g}},\ \forall t \in \mathrm{T}, \quad (5)$$

where values $\bar{\mathrm{u}}_{\mathrm{g}}$ and $\bar{\mathrm{v}}_{\mathrm{g}}$ are constant.

Collinear pair of two GDs was named as *Scissored Pair Ensemble* (*SPE*) into well-known original work *J.W. Crenshaw* (1973). Redundant multiply scheme, based on six gyrodines in the form of three collinear GD's pairs, was named as *3-SPE*. Fig. 1 presents a simplest arrangement of this scheme into a canonical orthogonal gyroscopic basis $\mathrm{O}x_{\mathrm{c}}^{\mathrm{g}} y_{\mathrm{c}}^{\mathrm{g}} z_{\mathrm{c}}^{\mathrm{g}}$. By a slope of the GD pairs suspension axes in this basis it is possible to change essentially a form of the AM variation domain $\mathbf{S}$ at any direction. Based on four gyrodines the minimal redundant scheme *2-SPE* is easily obtained from the *3-SPE* scheme – without third pair (GD #5 and GD #6). In park state of above schemes one can have a vector of summary normed GMC's AM $\mathbf{h}(\boldsymbol{\beta}) \equiv \sum \mathbf{h}_p(\beta_p) = \mathbf{0}$.

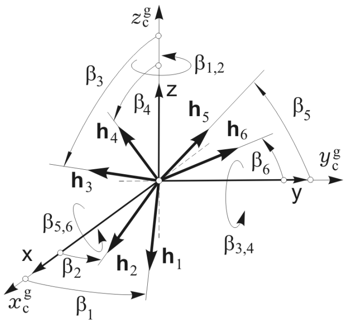

Figure 1. The scheme *3-SPE*

3 THE PROBLEM STATEMENT

After separating a SC from buster and disclosing the SAPs at any time moment $t = t_0$ the angular rate vector accepts a value $\boldsymbol{\omega}(t_0) \in \mathbf{S}_\omega$ from the bounded convex domain $\mathbf{S}_\omega$. Let a constant command angular rate vector $\boldsymbol{\omega}^{\mathrm{c}} = \{\omega_i^{\mathrm{c}}\}$ is given. First problem consists in synthesis of the OEU pulse-width control so that components ω_i^{c} should be reached with given accuracy $|\omega_i(t) - \omega_i^{\mathrm{c}}| \leq \delta_\omega\ \forall t \geq t_0 + T^{\mathrm{r}}$ for some acceptable duration T^{r} of damping mode.

At initial the SC damping, guidance on the Sun and on the Earth, signals of a block of angular rate sensors (ARSs) and the GPS/GLONASS navigation signals with period $T_q^{\mathrm{e}} = 1\,s$ are applied for forming the OEU pulse-width control with period $T_u^{\mathrm{e}} = 4\,s$. The model of the ARS block for measuring the SC body rate vector represents by set of three same channels for measurement $\omega_i(t), i = x, y, z$, moreover model of each its channel takes into account own dynamical properties, a noise and systematic errors, a time sampling, quantization and limit levels. Because of the SC small measuring base at the SC attitude determination by the GPS/GLONASS navigation signals, the accuracy is poor, $\approx 0^\circ.5$. That accuracy is enough for initial the SC guidance on the Sun and on the Earth and for next the SC attitude stabilization into the ORF by the OEU during the gyrodine rotors' spinup and initial preparing the SC attitude determination by strapdown inertial system (SIS) with astronomical correction.

At the SC gyromoment attitude control, applied onboard SIS is based on inertial gyro unit corrected by the fine fixed-head star trackers. Contemporary filtering & alignment calibration algorithms give finally a fine discrete estimating the SC angular coordinates by the quaternion $\mathbf{\Lambda}_s^{\rm m} = \mathbf{\Lambda}_s \circ \mathbf{\Lambda}_s^{\rm n}$, $s \in \mathbb{N}_0$, where $\mathbf{\Lambda}_s \equiv \mathbf{\Lambda}(t_s)$ and $\mathbf{\Lambda}_s^{\rm n}$ is a "noise-drift" digital quaternion, and a measurement period $T_q = t_{s+1} - t_s \leq T_u$ is multiply with respect to a control period T_u.

For initial the SC guidance simultaneously on the Sun and on the Earth and also for the SC's telescope guidance on given part of the Earth surface by next scanning in designated direction, the SC's spatial rotation maneuver (SRM) is needed. Into the IRF the SC's SRM is described by kinematic relations

$$\dot{\mathbf{\Lambda}}(t) = \tfrac{1}{2}\mathbf{\Lambda} \circ \boldsymbol{\omega}(t);\ \dot{\boldsymbol{\omega}}(t) = \boldsymbol{\varepsilon}(t);\ \dot{\boldsymbol{\varepsilon}}(t) = \mathbf{v}, \qquad (6)$$

where $\dot{\boldsymbol{\varepsilon}}(t) \equiv \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$, during given time interval T_p, e.g. $\forall t \in \mathrm{T}_p \equiv [t_{\rm i}^p, t_{\rm f}^p]$, $t_{\rm f}^p \equiv t_{\rm i}^p + T_p$. The optimization problem consists in determination of time functions $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ for the boundary conditions on left ($t = t_{\rm i}^p$) and right ($t = t_{\rm f}^p$) trajectory ends

$$\mathbf{\Lambda}(t_{\rm i}^p) = \mathbf{\Lambda}_{\rm i};\ \boldsymbol{\omega}(t_{\rm i}^p) = \boldsymbol{\omega}_{\rm i};\ \boldsymbol{\varepsilon}(t_{\rm i}^p) = \boldsymbol{\varepsilon}_{\rm i}; \qquad (7)$$

$$\mathbf{\Lambda}(t_{\rm f}^p) = \mathbf{\Lambda}_{\rm f};\ \boldsymbol{\omega}(t_{\rm f}^p) = \boldsymbol{\omega}_{\rm f};\ \boldsymbol{\varepsilon}(t_{\rm f}^p) = \boldsymbol{\varepsilon}_{\rm f} \qquad (8)$$

with optimization of the integral quadratic index

$$\mathrm{I}_2 = \tfrac{1}{2}\int_{t_{\rm i}^p}^{t_{\rm f}^p} \langle \mathbf{v}(\tau), \mathbf{v}(\tau) \rangle\, d\tau \Rightarrow \min. \qquad (9)$$

Optimizing this functional is topologically equivalently to optimizing the most practice important functional

$$\mathrm{I}_1 = \bar{\mathrm{v}} \equiv \tfrac{1}{T_p}\int_{t_{\rm i}^p}^{t_{\rm f}^p} |\mathbf{v}(\tau)|\, d\tau \Rightarrow \min, \qquad (10)$$

which have the clear physical sense: the mean value $\bar{\mathrm{v}}$ of the "control" module $\mathrm{v}(t) \equiv |\mathbf{v}(t)|$ – a module by derivative of the BRF acceleration vector during process of the SC rotation maneuver with respect to inertial reference frame $\mathbf{I}_\oplus$.

Principle problem gets up on the SC angular guidance at a spatial course motion (SCM) when a space opto-electronic observation is executed at given time interval, namely $t \in \mathrm{T}_n \equiv [t_{\rm i}^n, t_{\rm f}^n]$. This problem consists in determination of quaternion $\mathbf{\Lambda}(t)$ by the SC BRF $\mathbf{B}$ attitude with respect to the IRF $\mathbf{I}_\oplus$, angular rate vector $\boldsymbol{\omega}(t)$, vectors of angular acceleration $\boldsymbol{\varepsilon}(t)$ and its derivative $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ in the form of explicit functions, proceed from principle requirement: optical image of the Earth given part must to move by desired way at focal plane $y^i\mathrm{O}_i z^i$ of the telescope.

Onboard algorithms are needed for the SC guidance at a SRM taking into account the restrictions (5) to vectors $\dot{\boldsymbol{\beta}}(t)$ and $\ddot{\boldsymbol{\beta}}(t)$. Here for given time interval T_p a problem consists in determination the explicit time functions $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t)$ for the boundary conditions (7), (8) and also for given condition

$$\dot{\boldsymbol{\varepsilon}}(t_{\rm f}^p) = \dot{\boldsymbol{\varepsilon}}_{\rm f} \equiv \boldsymbol{\varepsilon}_{\rm f}^* + \boldsymbol{\omega}_{\rm f} \times \boldsymbol{\varepsilon}_{\rm f}, \qquad (11)$$

which presents requirements to a *smooth conjugation* of a SRM with guidance at next the SC's SCM.

At a land-survey SC lifetime up to 5 years its structure inertial and flexible characteristics are slowly changed in wide boundaries, the SAPs from time to time are rotated with respect to the SC body on angle γ and the communication antennas are pointing for information service. Therefore inertial matrix $\mathbf{A}^o$ and partial frequencies Ω_j^q of the SC structure are not complete certain. Problems consist in synthesis of the SC gyromoment guidance laws at its both the SCM and the SRM, and also in designing the GMC's robust digital control law $\mathbf{u}_k^g = \{u_{pk}^g\}$ on the quaternion values $\mathbf{\Lambda}_s^{\rm m}$ when the SC structure characteristics are uncertain and its damping is very weak, decrement $\delta_j^q \approx 5 \cdot 10^{-3}$ in (1).

4 Optimization of a Spatial Rotation Maneuver

Of course, optimizing one-axis motion is elementary problem which have analytic solution by Pontryagin's maximum principle. In result, the SC optimal on index (9) motion with respect to any k axis is presented by the analytic function $\varphi_k(t)$ in a class of the five degree polynomials (splines) (Somov, 2008*b*).

Developed analytical approach is based on necessary and sufficient condition for solvability of Darboux problem. At general case the solution is presented as result of composition by three ($k = 1 \div 3$) simultaneously derived elementary rotations of embedded bases $\mathbf{E}_k$ about units $\mathbf{e}_k$ of Euler axes, which positions are defined from the boundary conditions (7) and (8) for initial spatial problem. For all 3 elementary rotations with respect to units $\mathbf{e}_k$ the boundary conditions are analytically assigned. Into the IRF $\mathbf{I}_\oplus$ the quaternion $\mathbf{\Lambda}(t)$ is defined by the production

$$\mathbf{\Lambda}(t) = \mathbf{\Lambda}_{\rm i} \circ \mathbf{\Lambda}_1(t) \circ \mathbf{\Lambda}_2(t) \circ \mathbf{\Lambda}_3(t), \qquad (12)$$

where $\mathbf{\Lambda}_k(t) = (C(\varphi_k(t)/2), S(\varphi_k(t)/2)\mathbf{e}_k)$, $C(\alpha) \equiv \cos\alpha$, $S(\alpha) \equiv \sin\alpha$, and functions $\varphi_k(t)$ present the elementary rotation angles in analytical form.

Let the quaternion $\mathbf{\Lambda}^* \equiv (\lambda_0^*, \boldsymbol{\lambda}^*) = \tilde{\mathbf{\Lambda}}_{\rm i} \circ \mathbf{\Lambda}_{\rm f} \neq \mathbf{1}$ have the Euler axis unit $\mathbf{e}_3 = \boldsymbol{\lambda}^*/S(\varphi^*/2)$ by 3-rd elementary rotation where angle $\varphi^* = 2\arccos(\lambda_0^*)$. For elementary rotations there are applied next the boundary quaternion values:

$$\mathbf{\Lambda}_1(t_{\rm i}^p) = \mathbf{\Lambda}_1(t_{\rm f}^p) = \mathbf{\Lambda}_2(t_{\rm i}^p) = \mathbf{\Lambda}_2(t_{\rm f}^p) = \mathbf{1};$$
$$\mathbf{\Lambda}_3(t_{\rm i}^p) = \mathbf{1};\ \mathbf{\Lambda}_3(t_{\rm f}^p) = (C(\varphi_3^f/2), \mathbf{e}_3 S(\varphi_3^f/2)), \quad (13)$$

where $\varphi_3^f = \varphi^*$ and $\mathbf{1}$ is a single quaternion. Unit $\mathbf{e}_1$ of 1-st elementary rotation's on Euler's axis is selected by simple algorithm (Somov, 2008*b*), then unit $\mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1$ is defined. All vectors $\boldsymbol{\omega}_k(t) = \dot{\varphi}_k(t)\mathbf{e}_k$, $\boldsymbol{\varepsilon}_k(t) = \ddot{\varphi}_k(t)\mathbf{e}_k$ and $\dot{\boldsymbol{\varepsilon}}_k(t) = \dddot{\varphi}_k(t)\mathbf{e}_k$ have analytic presentations (Somov, 2008*b*). Suggested approach have large advantages with respect to rotations by standard Euler-Krylov angels (Somov, 2007).

For nonlinear problem (6) – (9) Hamilton function is

$$H = -\tfrac{1}{2}\langle \mathbf{v}, \mathbf{v}\rangle + \tfrac{1}{2}\langle \mathrm{vect}(\tilde{\mathbf{\Lambda}} \circ \mathbf{\Psi}, \boldsymbol{\omega}\rangle + \langle \boldsymbol{\mu}, \boldsymbol{\varepsilon}\rangle + \langle \boldsymbol{\nu}, \mathbf{v}\rangle$$

with associated quaternion $\mathbf{\Psi}(t) = \mathbf{C}_\varphi \circ \mathbf{\Lambda}(t)$, where $\mathbf{C}_\varphi = (\mathrm{c}_{\varphi 0}, \mathbf{c}_\varphi)$ is the normed quaternion (Branetz and Shmyglevsky, 1973) with a vector part $\mathbf{c}_\varphi = \{\mathrm{c}_{\varphi k}\}$. At the notation $\mathbf{p} = \mathrm{vect}(\tilde{\mathbf{\Lambda}} \circ \mathbf{\Psi}) = \tilde{\mathbf{\Lambda}} \circ \mathbf{c}_\varphi \circ \mathbf{\Lambda}$ the associated differential system have the form

$$\mathbf{p}(t) = \tilde{\mathbf{\Lambda}}(t) \circ \mathbf{c}_\varphi \circ \mathbf{\Lambda}(t); \dot{\boldsymbol{\mu}} = -\tfrac{1}{2}\mathbf{p}(t); \dot{\boldsymbol{\nu}} = -\boldsymbol{\mu}. \quad (14)$$

The optimality condition $\partial H/\partial \mathbf{v} = -\mathbf{v} + \boldsymbol{\nu} = \mathbf{0}$ give the structure of optimal "control"

$$\mathbf{v}(t) = \mathbf{c}_\varepsilon - \mathbf{c}_\omega(t - t_{\rm i}^p) + \tfrac{1}{2}\int_{t_{\rm i}^p}^{t} \left(\int_{t_{\rm i}^p}^{\tau} \mathbf{p}(s)ds\right) d\tau,$$

where vectors $\mathbf{c}_\varphi$, $\mathbf{c}_\omega = \{\mathrm{c}_{\omega k}\}$ and $\mathbf{c}_\varepsilon = \{\mathrm{c}_{\varepsilon k}\}$ must be numerically defined taking into account the boundary conditions (7) and (8). Standard Newton iteration method was applied for numerical obtaining the "control" $\mathbf{v}(t)$ which is a strict optimal on index (9) for the nonlinear optimization problem. Moreover analytical solution of the "start" problem (initial point) was applied in the form of approximate optimal motion (12) and (13) with the constant vectors $\mathbf{c}_\varphi$, $\mathbf{c}_\omega$ and $\mathbf{c}_\varepsilon$. Values of these constant vectors are numerically corrected by an iteration procedure using a combine numerical integration of direct (6) and associated (14) differential systems which are linearizated at neighbourhood of numerical solution on previous iteration. At such initial point the Newton's iteration process have a rapid convergence: usually there is needed only 2 – 3 iterations for obtaining a numerical solution with fine accuracy. Difference between approximate optimal spatial motion (analytic solution of "start" problem) and strict optimal spatial motion is very light — up to 5 % by functional I_1 (10) for the SC practical rotational maneuvers (Somov, 2007).

Fast onboard algorithms for the SC gyromoment guidance at a SRM with restrictions to $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t)$, corresponding restrictions to $\mathbf{h}(\boldsymbol{\beta}(t)), \dot{\boldsymbol{\beta}}(t)$ and $\ddot{\boldsymbol{\beta}}(t)$ in a class of the SC angular motions, were elaborated. Developed analytical approach to the problem is based on approximate optimal motion (12) with boundary conditions (7), (8) and (11). Here functions $\varphi_k(t)$ are selected in a class of splines by five and six degree, moreover a module of a angular rate $\dot{\varphi}_3(t)$ in a position transfer ($k = 3$) may be limited. The technique is based on the integral's properties for the AM of the mechanical system "SC+GMC" and allows to evaluate vectors $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ in analytical form for a preassigned SC motion $\mathbf{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$, $\dot{\boldsymbol{\varepsilon}}(t)\, \forall t \in \mathrm{T}_p$.

Into orthogonal canonical basis $\mathrm{O}x_{\rm c}^{\rm g} y_{\rm c}^{\rm g} z_{\rm c}^{\rm g}$, see Fig. 1, the GD's AM units have next projections:

$$x_1 = C_1;\ x_2 = C_2;\ y_1 = S_1;\ y_2 = S_2;$$
$$x_3 = S_3;\ x_4 = S_4;\ z_3 = C_3;\ z_4 = C_4;$$
$$y_5 = C_5;\ y_6 = C_6;\ z_5 = S_5;\ z_6 = S_6,$$

where $S_p \equiv \sin\beta_p$ and $C_p \equiv \cos\beta_p$. Then vector-column $\mathbf{h}(\boldsymbol{\beta}) \equiv \{\mathsf{x}, \mathsf{y}, \mathsf{z}\} = \{\Sigma x_p, \Sigma y_p, \Sigma z_p\}$ and matrix $\mathbf{A}_{\rm h}(\boldsymbol{\beta}) = \partial\mathbf{h}/\partial\boldsymbol{\beta}$ have the form

$$\mathbf{A}_{\rm h}(\boldsymbol{\beta}) = \begin{bmatrix} -y_1 & -y_2 & z_3 & z_4 & 0 & 0 \\ x_1 & x_2 & 0 & 0 & -z_5 & -z_6 \\ 0 & 0 & -x_3 & -x_4 & y_5 & y_6 \end{bmatrix}.$$

For *3-SPE* scheme singular state is appeared when the matrix Gramme $\mathbf{G}(\boldsymbol{\beta}) = \mathbf{A}_h(\boldsymbol{\beta})\mathbf{A}_h^{\rm t}(\boldsymbol{\beta})$ loses its full rang, e.g. when $\mathsf{G} \equiv \det\mathbf{G}(\boldsymbol{\beta}) = 0$. At introducing the denotations

$$x_{12} = x_1 + x_2;\ \ x_{34} = x_3 + x_4;\ \ y_{12} = y_1 + y_2;$$
$$y_{56} = y_5 + y_6;\ \ z_{34} = z_3 + z_4;\ \ z_{56} = z_5 + z_6;$$
$$\tilde{x}_{12} = x_{12}/\sqrt{4 - y_{12}^2}\,;\ \tilde{x}_{34} = x_{34}/\sqrt{4 - z_{34}^2};$$
$$\tilde{y}_{12} = y_{12}/\sqrt{4 - x_{12}^2}\,;\ \ \tilde{y}_{56} = y_{56}/\sqrt{4 - z_{56}^2};$$
$$\tilde{z}_{34} = z_{34}/\sqrt{4 - x_{34}^2}\,;\ \ \tilde{z}_{56} = z_{56}/\sqrt{4 - y_{56}^2}$$

components of the GMC explicit vector tuning law

$$\mathbf{f}_\rho(\boldsymbol{\beta}) \equiv \{f_{\rho 1}(\boldsymbol{\beta}), f_{\rho 2}(\boldsymbol{\beta}), f_{\rho 3}(\boldsymbol{\beta})\} = \mathbf{0} \quad (15)$$

are applied for $\rho = \mathrm{const}, 0 < \rho < 1$, in the form

$$f_{\rho 1}(\boldsymbol{\beta}) \equiv \tilde{x}_{12} - \tilde{x}_{34} + \rho\,(\tilde{x}_{12}\,\tilde{x}_{34} - 1);$$
$$f_{\rho 2}(\boldsymbol{\beta}) \equiv \tilde{y}_{56} - \tilde{y}_{12} + \rho\,(\tilde{y}_{56}\,\tilde{y}_{12} - 1);$$
$$f_{\rho 3}(\boldsymbol{\beta}) \equiv \tilde{z}_{34} - \tilde{z}_{56} + \rho\,(\tilde{z}_{34}\,\tilde{z}_{56} - 1).$$

Analytical proof have been elaborated that vector tuning law (15) ensures absent of singular states by this GMC scheme for all values of the GMC AM vector $\mathbf{h}(t) \in \mathbf{S} \setminus \partial\mathbf{S}$, e.g. inside all its variation domain. For the representation

$$x_{12} = (\mathsf{x} + \Delta_x)/2;\quad x_{34} = (\mathsf{x} - \Delta_x)/2;$$
$$y_{56} = (\mathsf{y} + \Delta_y)/2;\quad y_{12} = (\mathsf{y} - \Delta_y)/2;$$
$$z_{34} = (\mathsf{z} + \Delta_z)/2;\quad z_{56} = (\mathsf{z} - \Delta_z)/2$$

and the denotation $\boldsymbol{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ one can obtain the nonlinear vector equation $\boldsymbol{\Delta}(t) = \boldsymbol{\Phi}(\mathbf{h}(t), \boldsymbol{\Delta}(t))$. At a known vector $\mathbf{h}(t)$ this equation have single solution $\boldsymbol{\Delta}(t)$, which is readily computed by method of a simple iteration. Further the units $\mathbf{h}_p(\beta_p(t))$ and vector-columns $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ are calculated by explicit analytical relations $\forall t \in \mathrm{T}_p$. For the *2-SPE* scheme such evaluation is carried out by the explicit analytical formulas only.

5 Guidance at a course motion

Analytic matching solution have been obtained for problem of the SC angular guidance at the SCM $\forall t \in \mathrm{T}_n$. The solution is based on a vector composition of all elemental motions in the GRF $\mathbf{E}_\mathrm{e}$ using next reference frames: the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, the SRF $\boldsymbol{\mathcal{S}}$ and the FRF $\boldsymbol{\mathcal{F}}$. Vectors $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are presented in the GRF $\mathbf{E}_\mathrm{e}$:

$$\mathbf{r}^\mathrm{e} = \mathbf{T}_\mathrm{I}^\mathrm{e}\,\mathbf{r}; \quad \mathbf{v}^\mathrm{e} = \mathbf{T}_\mathrm{I}^\mathrm{e}\,(\mathbf{v} - [\omega_\oplus \mathbf{i}_3 \times]\mathbf{r}_\mathrm{o}),$$

where matrix $\mathbf{T}_\mathrm{I}^\mathrm{e} = [\rho_\mathrm{e}(t)]_3$ and angle $\rho_\mathrm{e}(t) = \rho_\mathrm{e}^\mathrm{i} + \omega_\oplus(t - t_\mathrm{i})$. Vectors $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ and $\mathbf{v}_\mathrm{e}^\mathrm{s}$ are defined as

$$\boldsymbol{\omega}_\mathrm{e}^\mathrm{s} = \{\omega_{\mathrm{e}i}^\mathrm{s}\} = \mathbf{T}_b^\mathrm{s}(\boldsymbol{\omega} - \tilde{\boldsymbol{\Lambda}} \circ \omega_\oplus \mathbf{i}_3 \circ \boldsymbol{\Lambda}); \;\; \mathbf{v}_\mathrm{e}^\mathrm{s} = \tilde{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} \circ \mathbf{v}_\mathrm{o}^\mathrm{e} \circ \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s},$$

where $\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_\mathrm{I}^b; \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{I} \circ \boldsymbol{\Lambda}_\mathrm{I}^b \circ \boldsymbol{\Lambda}_b^\mathrm{s}$ and $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} \circ \boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$, and constant matrix $\mathbf{T}_b^\mathrm{s}$ represents the telescope fixation on the SC body. For any observed point C the oblique range D is analytically calculated as $\mathrm{D} = |\mathbf{r}_c^\mathrm{e} - \mathbf{r}^\mathrm{e}|$. If matrix $\mathbf{C}_\mathrm{h}^\mathrm{s} \equiv \tilde{\mathbf{C}} = \|\tilde{c}_{ij}\|$ defines the SRF $\boldsymbol{\mathcal{S}}$ attitude with respect to the HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, then for any point $\mathrm{M}(\tilde{y}^i, \tilde{z}^i)$ at the telescope focal plane $y^i\mathrm{O}_i z^i$ the components $\tilde{V}_y^i(\tilde{y}^i, \tilde{z}^i) = \dot{\tilde{y}}^i$ and $\tilde{V}_z^i(\tilde{y}^i, \tilde{z}^i) = \dot{\tilde{z}}^i$ of normed vector by an image motion velocity are appeared as follows:

$$\begin{bmatrix} \dot{\tilde{y}}^i \\ \dot{\tilde{z}}^i \end{bmatrix} = \begin{bmatrix} \tilde{y}^i & 1 & 0 \\ \tilde{z}^i & 0 & 1 \end{bmatrix} \begin{bmatrix} q^i \tilde{\mathrm{v}}_{\mathrm{e}1}^\mathrm{s} - \tilde{y}^i\, \omega_{\mathrm{e}3}^\mathrm{s} + \tilde{z}^i\, \omega_{\mathrm{e}2}^\mathrm{s} \\ q^i \tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s} - \quad \omega_{\mathrm{e}3}^\mathrm{s} - \tilde{z}^i\, \omega_{\mathrm{e}1}^\mathrm{s} \\ q^i \tilde{\mathrm{v}}_{\mathrm{e}3}^\mathrm{s} + \quad \omega_{\mathrm{e}2}^\mathrm{s} + \tilde{y}^i\, \omega_{\mathrm{e}1}^\mathrm{s} \end{bmatrix}. \quad (16)$$

Here normed focal coordinates $\tilde{y}^i = y^i/f_e$ and $\tilde{z}^i = z^i/f_e$, where f_e is the telescope equivalent focal distance; function $q^i \equiv 1 - (\tilde{c}_{21}\tilde{y}^i + \tilde{c}_{31}\tilde{z}^i)/\tilde{c}_{11}$, and vector of normed SC's mass center velocity have the components $\tilde{\mathrm{v}}_{\mathrm{e}i}^\mathrm{s} = \mathrm{v}_{\mathrm{e}i}^\mathrm{s}/\mathrm{D}, i = 1 \div 3$. By (16) for given image velocity $\tilde{\mathrm{W}}_y^s = \mathrm{const}$ and conditions $\tilde{V}_y^i(0,0) = \tilde{\mathrm{W}}_y^i = -\tilde{\mathrm{W}}_y^s$; $\tilde{V}_z^i(0,0) = 0$; $\partial \tilde{V}_y^i(0,0)/\partial \tilde{z}^i = 0$ calculation of vector $\boldsymbol{\omega}_\mathrm{e}^\mathrm{s}$ is carried out by the relations

$$\omega_{\mathrm{e}1}^\mathrm{s} = -\tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s} \frac{\tilde{c}_{31}}{\tilde{c}_{11}}; \omega_{\mathrm{e}2}^\mathrm{s} = -\tilde{\mathrm{v}}_{\mathrm{e}3}^\mathrm{s}; \omega_{\mathrm{e}3}^\mathrm{s} = -\tilde{W}_y^i + \tilde{\mathrm{v}}_{\mathrm{e}2}^\mathrm{s}. \quad (17)$$

By numerical solution of the quaternion differential equation $\dot{\boldsymbol{\Lambda}}_\mathrm{e}^\mathrm{s} = \boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s} \circ \boldsymbol{\omega}_\mathrm{e}^\mathrm{s}/2$ with regard to (17) one can obtain values $\boldsymbol{\lambda}_{\mathrm{e}s}^\mathrm{s} \equiv \boldsymbol{\lambda}_\mathrm{e}^\mathrm{s}(t_s)$ for the discrete time moments $t_s \in \mathrm{T}_n$ with period T_q, $s = 0 \div n_q$, $n_q = T_n/T_q$ when initial value $\boldsymbol{\Lambda}_\mathrm{e}^\mathrm{s}(t_\mathrm{i}^n)$ is given. Further solution is based on the elegant extrapolation of values $\boldsymbol{\sigma}_{\mathrm{e}s}^\mathrm{s} = \boldsymbol{\lambda}_{\mathrm{e}s}^\mathrm{s}/(1 + \lambda_{0\,\mathrm{e}s}^\mathrm{s})$ by the vector of Rodrigues' modified parameters and values $\boldsymbol{\omega}_{\mathrm{e}s}^\mathrm{s}$ by the angular rate vector. The extrapolation is carried out by these two sets of n_q coordinated 3-degree vector splines with analytical obtaining a high-precise approximation of the SRF $\boldsymbol{\mathcal{S}}$ guidance motion with respect to the GRF $\mathbf{E}_\mathrm{e}$ both on vector of angular acceleration and on vector of its local derivative. At last stage, required functions $\boldsymbol{\Lambda}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\dot{\boldsymbol{\varepsilon}}(t) = \boldsymbol{\varepsilon}^*(t) + \boldsymbol{\omega}(t) \times \boldsymbol{\varepsilon}(t)$ is calculated by *explicit* formulas.

6 Continuous Control Laws

For continuous forming the control torque $\mathbf{M}(t)$ the SC the simplified model is such:

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2; \;\; \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{G}^o = \mathbf{M}. \quad (18)$$

Let functions $\boldsymbol{\Lambda}^p(t)$, $\boldsymbol{\omega}^p(t)$ and $\boldsymbol{\varepsilon}^p(t) = \dot{\boldsymbol{\omega}}^p(t)$ represent the SC angular programmed motion. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\boldsymbol{\Lambda}}^p(t) \circ \boldsymbol{\Lambda}$, Euler parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 e_0 + [\mathbf{e}\times]$ with $\det(\mathbf{Q}_e) = e_0 \neq 0$. If error in the rate vector is defined as $\delta\boldsymbol{\omega} \equiv \tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \mathbf{C}_e \boldsymbol{\omega}^p(t)$, and required control torque vector $\mathbf{M}$ is formed as

$$\mathbf{M} = \boldsymbol{\omega} \times \mathbf{G}^o + \mathbf{J}(\mathbf{C}_e \dot{\boldsymbol{\omega}}^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}_e \boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}}), \quad (19)$$

then the simplest nonlinear model of the SC's attitude error is as follows:

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle/2; \;\; \dot{\mathbf{e}} = \mathbf{Q}_e \tilde{\boldsymbol{\omega}}/2; \;\; \dot{\tilde{\boldsymbol{\omega}}} = \tilde{\mathbf{m}}. \quad (20)$$

For model (20) a *non-local nonlinear* coordinate transformation is defined and applied at analytical synthesis by the exact feedback linearization technique. This results in the nonlinear continuous control law

$$\tilde{\mathbf{m}}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}}) = -(\mathbf{A}_0\, \mathbf{e}\, \mathrm{sgn}(e_0) + \mathbf{A}_1\, \tilde{\boldsymbol{\omega}}), \quad (21)$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2/2)/e_0)\mathbf{I}_3$; $\mathbf{A}_1 = a_1^* \mathbf{I}_3 - \mathbf{R}_{e\omega}$, $\mathrm{sgn}(e_0) = (1, \text{ if } e_0 \geq 0) \vee (-1, \text{ if } e_0 < 0)$, matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\boldsymbol{\omega}} \rangle \mathbf{Q}_e^\mathrm{t}[\mathbf{e}\times]/(2e_0)$, and parameters a_0^*, a_1^* are analytically calculated on spectrum $\mathrm{S}_{ci}^* = -\alpha_c \pm j\omega_c$ in each channel. Simultaneously the Lyapunov function $\boldsymbol{\upsilon}(\boldsymbol{\mathcal{E}}, \tilde{\boldsymbol{\omega}})$ is analytically constructed for close-loop continuous system (20) and (21).

7 Filtration of Discrete Measurements

At given digital control period T_u discrete frequency characteristics are computed via absolute pseudo-frequency $\lambda = (2/T_u)\mathrm{tg}(\omega T_u/2)$. For period's multiple n_q and a filtering period $T_q = T_u/\mathrm{n_q}$ applied filter have the discrete transfer function $\mathrm{W_f}(\mathrm{z_q}) = (1 + \mathrm{b}_1^\mathrm{f})/(1 + \mathrm{b}_1^\mathrm{f}\mathrm{z_q}^{-1})$, where $\mathrm{b}_1^\mathrm{f} \equiv -\exp(-T_q/T_\mathrm{f})$ and $\mathrm{z_q} \equiv \exp(\mathrm{s}T_q)$. Measured error quaternion and Euler parameters' vector are $\mathbf{E}_s = (e_{0s}, \mathbf{e}_s) = \tilde{\boldsymbol{\Lambda}}^p(t_s) \circ \boldsymbol{\Lambda}_s^\mathrm{m}$ and $\boldsymbol{\mathcal{E}}_s = \{e_{0s}, \mathbf{e}_s\}$, and the error filtering is executed by the relations

$$\tilde{\mathbf{x}}_{s+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{B}}\mathbf{e}_s; \quad \mathbf{e}_s^\mathrm{f} = \tilde{\mathbf{C}}\tilde{\mathbf{x}}_s + \tilde{\mathbf{D}}\mathbf{e}_s, \quad (22)$$

where matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ have diagonal form with $\tilde{a}_i = -\mathrm{b}_1^\mathrm{f}$; $\tilde{b}_i = \mathrm{b}_1^\mathrm{f}$; $\tilde{c}_i = -(1 + \mathrm{b}_1^\mathrm{f})$ and $\tilde{d}_i = 1 + \mathrm{b}_1^\mathrm{f}$.

8 Pulse-width and Digital Control Laws

At initial modes the OEU pulse-width control is applied with period $T_u = T_u^{\mathrm{e}} = 4\,s$ and filtering period $T_q = T_q^{\mathrm{e}} = 1\,s$. At initial damping a forming the discrete command signals v_{ir} for the OEU pulse-width control on channels is very simple: $\mathrm{v}_{ir} = \mathrm{k}_i^{\omega}(\omega_i^{\mathrm{c}} - \omega_{ir}^{\mathrm{f}})$. Here ω_{ir}^{f} are filtered measurements of angular rate components and k_i^{ω} are constant gain factors.

At initial the SC guidance simultaneously on the Sun and on the Earth the SC attitude filtered error vector $\mathbf{e}_r^{\mathrm{f}}$ is also applied for forming the OEU pulse-width control $\mathbf{v}_r$. In first, here the stabilizing vector component $\tilde{\mathbf{m}}_r$ is calculated by the relation $\tilde{\mathbf{m}}_r = \mathbf{K}^p\mathbf{e}_r^{\mathrm{f}} + \mathbf{K}^{\omega}\tilde{\boldsymbol{\omega}}_r^{\mathrm{f}}$ with constant diagonal matrixes $\mathbf{K}^p$ and $\mathbf{K}^{\omega}$. Than preliminary vector $\tilde{\mathbf{v}}_r = \{\tilde{\mathrm{v}}_{ir}\}$ is evaluated for forming a required control torque $\mathbf{M}_r(t)$ for $t \in [t_r, t_r + T_u^{\mathrm{e}})$. At last, the command vector $\mathbf{v}_r = \{\mathrm{v}_{ir}\}$ is calculated by next simple algorithm: $\mathrm{q}_r = \max|\tilde{\mathrm{v}}_{ir}|, i = 1 \div 3;\ if\ \mathrm{q}_r > 0\ then\ \mathrm{v}_{ir} = T_u^{\mathrm{e}}\tilde{\mathrm{v}}_{ir}/\mathrm{q}_r$. Some results on nonlinear dynamics of pulse-width attitude control by a flexible spacecraft were presented in (Somov, 2008*a*).

At the SC rotational maneuvering and the SC course motion, gyromoment digital control is applied with period $T_u = 0.25\,s$ and filtering period $T_q = T_u/4$, taking into account a time delay at incomplete measurement of state and onboard signal processing:

$$\tilde{\mathbf{m}}_k \equiv \mathbf{v}_k = -(\mathbf{K}_{\mathrm{d}}^{\mathrm{x}}\hat{\mathbf{x}}_k + \mathbf{K}_{\mathrm{d}}^{\mathrm{u}}\mathbf{u}_k);\ \mathbf{u}_{k+1} = \mathbf{v}_k; \quad (23)$$
$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}_{\mathrm{od}}\,\hat{\mathbf{x}}_k + \mathbf{B}_{\mathrm{od}}^{\mathrm{u}}\,\mathbf{u}_k + \mathbf{B}_{\mathrm{od}}^{\mathrm{v}}\,\mathbf{v}_k + \mathbf{G}_{\mathrm{d}}(\mathbf{e}_k^{\mathrm{f}} - (\mathbf{C}_{\mathrm{od}}\,\hat{\mathbf{x}}_k + \mathbf{D}_{\mathrm{od}}^{\mathrm{u}}\,\mathbf{u}_k + \mathbf{D}_{\mathrm{od}}^{\mathrm{v}}\,\mathbf{v}_k)).$$

Here $k \in \mathbb{N}_0, \hat{\mathbf{x}}_k = \{\hat{\mathbf{e}}_k, \hat{\tilde{\boldsymbol{\omega}}}_k\}$, matrices have conforming dimensions and some turning parameters are applied for ensuring robust properties by gyromoment control system of the flexible weak damping spacecraft. Here discrete information on only the SC attitude *filtered* error vector $\mathbf{e}_k^{\mathrm{f}}$ is applied for forming the digital stabilizing component $\tilde{\mathbf{m}}_k$ (23). Taking into account (4), (15) and $\mathbf{M} = \mathbf{M}^{\mathbf{g}}$ (19), we obtain the GMC's robust digital control law $\mathbf{u}_k^g = \{u_{pk}^g\}$ in analytic form.

9 Computer Simulation

Fig. 2 presents some results on computer simulation of a gyromoment ACS for Russian remote sensing SC by the *Resource-DK* type. Here the rate errors are represented at consequence of the SC spatial rotational maneuver for time $t \in [0, 45)$ sec and the SC spatial course motion for time $t \in [45, 90]$ sec. Applied digital robust nonlinear control law is flexible switched at the time moment $t = 45$ sec on astatic ones with respect to the acceleration.

10 CONCLUSION

New results were presented on optimizing the SC spatial rotational maneuvers, on a pulse-width control and nonlinear digital robust gyromoment control by the *3-SPE* scheme, applied for agile spacecraft.

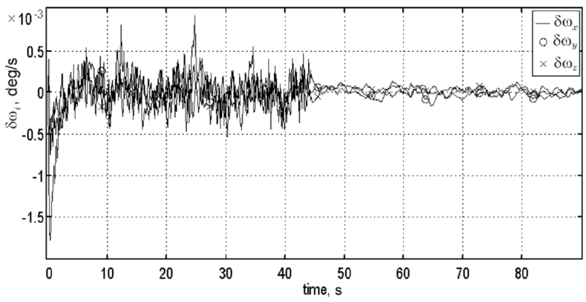

Figure 2. The rate errors for consequence of the SRM and the SCM

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