CYBERNETICS AND PHYSICS, VOL. 4, NO. 2, 2015 , 49–57

COMPARISON OF TWO BUMPLESS TRANSFER METHODS FOR A DISCRETE-TIME SWITCHED SYSTEM WITH ROBUST CONTROL

Peter Valach
Institute of Automotive Mechatronics
Faculty of Electrical Engineering
and Information Technology
Slovak University of Technology
in Bratislava
Bratislava,
Slovak Republic
peter.valach@stuba.sk

Danica Rosinová
Institute of Automotive Mechatronics
Faculty of Electrical Engineering
and Information Technology
Slovak University of Technology
in Bratislava
Bratislava,
Slovak Republic
danica.rosinova@stuba.sk

Abstract
Two novel schemes are presented for a bumpless transfer in a discrete-time switched system with robust controller. Transient effects produced by switching system modes are reduced by the proposed methods, which are based on pre-setting an off-line controller with reference model respective to an inactive mode. In this way, oscillations in output transient response are damped, when this mode is activated in a switching time. An off-line controller is driven using off-line closed loop in the first method and in the second method, an off-line controller is driven by a static feedback gain F. The equations determining static feedback gain F for the discrete-time system were developed using quadratic cost function to optimize transient behaviour. The obtained results from both approaches are illustrated and compared by simulation experiments. The proposed methods are verified and compared also by real plant experiments.

Key words
Bumpless transfer, switched systems, robust control, discrete-time system.

1 Introduction

Switching systems often appear in control practice, where several operation modes and switching between them are considered. Switching systems include the piecewise linear case, when several linear controllers are designed for a controlled plant, linearized in different operating points respective to different system modes, or cases with slow/fast controllers, changing a dynamics of system. Switching between several operation modes then introduces nonlinearity into the control loop [Campo, Morari, and Nett, 1989; Liberzon, 2003],which may cause undesirable transient effects. The suppression of these effects is called bumpless transfer. Important contribution on bumpless transfer in early stages was provided by Hanus, e.g. [Hanus, 1988] who proposed the conditioned controller to overcome transient effects when switching only the controller.

Various approaches have been proposed for a bumpless transfer in past decades. In [Turner and Walker, 2000; Cheong and Safonov, 2008; Zaccarian and Teel, 2002], bumpless transfer is proposed for the case of switching controllers, while the plant dynamics does not change. The authors employ the scheme, where the off-line controller is pre-set, so that on switching instant, the control variable does not change. In [Malloci *et al.*, 2009], a discrete-time switched system is considered, and the bumpless transfer is proposed to avoid step changes of control variable. The authors proposed additional controller, modifying the control signal for a determined period of time to reach the above aim. The latter bumpless transfer scheme, however, does not always provide satisfactory results, since after the determined period, oscillations can still appear at the system output.

In this paper, we propose the bumpless transfer (BT) schemes for a discrete-time switched system. We consider the system with robust control and assume that the control signal can be switched to the value respective to other mode within the sampling period. The aim of the proposed bumpless transfer schemes is to damp oscillations in output transient response, when new system mode is activated in switching time. The off-line controller is set up to produce control signal, which ensures the tracking of reference trajectory for system in off-line mode. The presented results further extend previously proposed solution presented by the authors in [Valach and Rosinová, 2015].

The paper is organized as follows. Section 2 introduces problem formulation and summarizes the results from literature on LMI robust stability conditions guaranteeing stability of the discrete-time switching system which are used in next sections. Section 3 provides robust PI controller design guaranteeing robust stability. The main result - bumpless transfer schemes 1, 2 and 3 for switching system are proposed in Section 4. The results obtained with and without the proposed schemes are shown and compared in Section 5, both for simulation and real switched laboratory DC motor system.

2 Problem Formulation and Preliminaries

Let us consider an uncertain switched system described by linear discrete-time state space model:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{N} \xi_i(k)(A_i x(k) + B_i u(k)), \\ y(k) &= \sum_{i=1}^{N} \xi_i(k)(C_i x(k)) \end{aligned} \tag{1}$$

where

$$\Omega_i = \left\{ \begin{array}{c} (A_i, B_i) = \sum\limits_{l(i)=1}^{M(i)} \alpha_{l(i)} \left[A_{l(i),i}, B_{l(i),i} \right], \\ \alpha_{l(i)} \geq 0, \sum\limits_{l(i)=1}^{M(i)} \alpha_{l(i)}(k) = 1 \end{array} \right\} \tag{2}$$

$C_i = C$ for $i = 1, ..., N$, $\xi_i(k)$ is the logic parameter and it has value 1, when the state matrix belongs into i-th domain; $x(k)$ is the state variable vector; $u(k)$ is the control variable vector; $y(k)$ is the output variable vector; i is the operation mode; N is number of system modes; in each mode, a polytopic uncertainty domain Ω_i with $M(i)$ vertices is considered for system matrices.

The main aim is to design robust stabilizing controller for a switched system (1), (2) so that the undesirable transient effect caused by switching between modes are suppressed.

To achieve the main aim, stabilizing robust static output feedback (SOF) control

$$u(k) = K_i C x(k) \tag{3}$$

depending on an active system mode is designed, based on results from literature, [Maherzi, Bernussou, and Mhiri, 2007], and the novel bumpless transfer schemes appropriate for the considered switched system are proposed as a main result.

In the first step, robust static output feedback control (3) is to be designed. The respective closed loop system matrix in the i-th mode is $A_i + B_i K_i C$. Output feedback gain matrices K_i for individual modes can be calculated via Linear matrix inequalities and equalities (LMIs and LMEs), using recent result summarized in Theorem 1.

Theorem 1 [Maherzi, Bernussou, and Mhiri, 2007] Discrete-time switching system (1) can be stabilized by a static output feedback if there exist N symmetric positive definite matrices $S_1, S_2, \ldots, S_N, N$ matrices $G_1, G_2, ..., G_N$, N matrices $U_1, U_2, ..., U_N$ of appropriate dimensions and N regular matrices $V_1, V_2, ..., V_N$ such that the following LMIs and LMEs hold:

$$\begin{bmatrix} G_i + G_i^T - S_i & (A_{l(i),i} G_i + B_{l(i),i} U_i C_i)^T \\ * & S_j \end{bmatrix} > 0 \tag{4}$$

$$V_i C_i = C_i G_i \tag{5}$$

for $i, j = 1, ..., N$.

The respective static output feedback gain is defined by:

$$K_i = U_i V_i^{-1}. \tag{6}$$

Theorem 1 provides sufficient stability condition for the system switching from i-th to j-th mode. Satisfying conditions (4) and (5) for all pairs (i, j) guarantees stability for switching between arbitrary system modes.

It should be noted that the original problem of SOF controller design yields bilinear matrix formulation of the respective robust stability condition which is computationally hard to solve. To linearize the term $(A_i + B_i K_i C) G_i$, the equality (5) is added to change the order of unknown matrices and thus enable substitution $K_i V_i = U_i$ which leads to transforming the static output feedback problem into LMI. Therefore the resulting LMIs (4), (5) are only sufficient for stabilization of the given switched system.

3 Robust PI Controller Design for a Switched System

This section describes application of Theorem 1 to design robust stabilizing PI controller for a switched system (1). In this case, the dynamics of PI controller will be included into the system state description, thus receiving the augmented system description as shown below.

A discrete-time PI controller is described by control algorithm:

$$u(k) = K_{P,i}e(k) + K_{I,i}\sum_{t=1}^{k} e(t) \quad (7)$$

where

$$e(k) = y(k) - r(k), \quad (8)$$

$e(k)$ is control error; $r(k)$ is reference variable.

The respective description of the discrete-time PI controller in state space is

$$\begin{aligned} q(k+1) &= [I]q(k) + [I]e(k) \\ u(k) &= K_{I,i}q(k) + (K_{P,i} + K_{I,i})e(k) \end{aligned} \quad (9)$$

or

$$\begin{aligned} q(k+1) &= A_{r,i}q(k) + B_{r,i}e(k) \\ u(k) &= C_{r,i}q(k) + D_{r,i}e(k) \end{aligned}$$

where I denotes identity matrix of the respective dimensions.

The resulting augmented system model comprises both controlled system (1) and PI controller (9) dynamics; the latter corresponds to the integral part of PI controller, which enables tracking reference trajectory $r(k)$. Below we assume that the reference $r(k)$ does not change within transient time (the aim is step response tracking), so it is considered as a constant. Integral action for set-point tracking is described by

$$q(k+1) = q(k) + y(k) - r(k). \quad (10)$$

Augmented system for i-th mode corresponding to (1) and (10) is

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ C_i & I \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ -I \end{bmatrix} r(k) \\ \begin{bmatrix} y(k) \\ q(k) \end{bmatrix} &= \begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}. \end{aligned} \quad (11)$$

Having the augmented system (11) including both system and controller dynamics, PI controller design is transformed into a static output controller design for augmented system:

$$u(k) = K_i \begin{bmatrix} e(k) \\ q(k) \end{bmatrix}, i = 1, ..., N. \quad (12)$$

The respective output feedback gain matrices K_i for all modes are calculated from LMIs (4), (5) and (6).

PI controller parameters are obtained as

$$K_i = \begin{bmatrix} K_i(1) & K_i(2) \end{bmatrix} = \begin{bmatrix} K_{P,i} + K_{I,i} & K_{I,i} \end{bmatrix}.$$

4 Bumpless Transfer for a Discrete-time Switched System

In this section, two bumpless transfer schemes for a switched system are proposed. The aim of bumpless transfer is to reduce the transient effects (bumps) caused by switching the controllers and system dynamics depending on the switching system modes. The output oscillations due to switching are caused by old state values (before changing dynamics) of controller or/and plant. The bumpless transfer schemes aims at appropriately modifying these state values. The problem of output transient effects when the system is switched, however, cannot be totally removed.

4.1 Simple Bumpless Transfer Scheme

We propose the bumpless transfer scheme for switching from i-th to j-th mode, inspired by the idea from [Turner and Walker, 2000]. The main idea is to drive the off-line (j-th) controller, so that the value of controlled variable is preset to the value, respective to the j-th mode. The j-th mode dynamics is provided by the respective reference model (off-line plant). The proposed Scheme 1 for switching from the first to the second mode is shown in Fig. 1, [Valach and Rosinová, 2015]. As shown in Fig. 1, the whole off-line

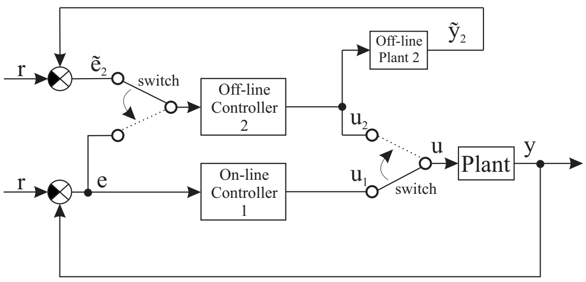

Figure 1. Bumpless transfer scheme 1 for switching system dynamics.

closed loop is used to appropriately drive the off-line controller.

4.2 Improved Bumpless Transfer Scheme

In this section, a bumpless transfer scheme for switching modes is proposed, extending results from literature, where bumpless transfer scheme for switching only controller was proposed.

Firstly, the basic idea for bumpless transfer is introduced. The bumpless transfer scheme for minimizing undesirable transient effects when switching controller from off-line to on-line control is provided in [Turner and Walker, 2000]. They proposed to suppress the transient effects by synthesizing a static feedback gain F_i (mode i), which drives the off-line controller as shown in Scheme 2 (Fig. 2). The off-line controller is described by:

$$q(k+1) = A_{r,i}q(k) + B_{r,i}\alpha(k)$$
$$u_{off}(k) = C_{r,i}q(k) + D_{r,i}\alpha(k).$$

The bumpless transfer scheme proposed in [Turner

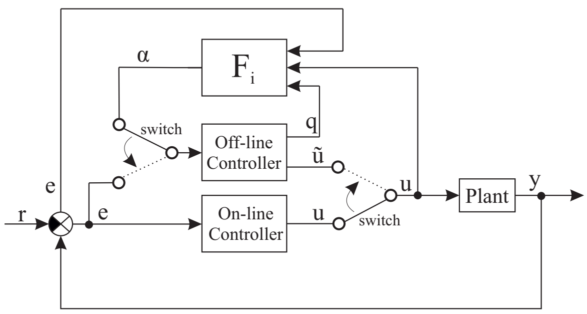

Figure 2. Bumpless transfer scheme 2.

and Walker, 2000], calculates the static feedback gain F_i using quadratic cost function minimization for the discrete-time system. The respective results are summarized below.

$$\alpha(k) = F_i \begin{bmatrix} q(k) \\ u(k) \\ e(k) \end{bmatrix} \tag{13}$$

$$F_i = (I - \Delta B_{r,i}^T \Pi B_{r,i})^{-1}\Delta \begin{bmatrix} (D_{r,i}^T W_u C_{r,i} + B_{r,i}^T \Pi A_{r,i})^T \\ -(D_{r,i}^T W_u + B_{r,i}^T (I-M)^{-1}\hat{U})^T \\ -(W_e + B_{r,i}^T (I-M)^{-1}\hat{E})^T \end{bmatrix}^T \tag{14}$$

where

$$\Delta = -(D_{r,i}^T W_u D_{r,i} + W_e)^{-1}$$
$$\tilde{A} = A_{r,i} + B_{r,i}\Delta D_{r,i}^T W_u C_{r,i}$$
$$\tilde{B} = B_{r,i}\Delta B_{r,i}^T$$
$$\tilde{C} = C_{r,i}^T W_u C_{r,i} + C_{r,i}^T W_u D_{r,i}\Delta D_{r,i}^T W_u C_{r,i}$$
$$M = \tilde{A}^T (I - \Pi\tilde{B})^{-1}$$
$$\hat{U} = M\Pi B_{r,i}\Delta D_{r,i}^T W_u + C_{r,i}^T W_u + $$
$$+ C_{r,i}^T W_u C_{r,i}\Delta C_{r,i}^T W_U$$
$$\hat{E} = M\Pi B_{r,i}\Delta D_{r,i}^T W_e + C_{r,i}^T W_u D_{r,i}\Delta W_e$$

W_u,W_e are weighting matrices and Π is a stabilizing solution to the discrete-time algebraic Riccati equation:

$$\tilde{A}^T (I - \Pi\tilde{B})^{-1}\Pi\tilde{A} - \Pi + \tilde{C} = 0.$$

Note that if $D = 0$, the expression for F_i reduces.

As already mentioned above, the described approach was developed for the case, when the controllers are switched without changing the dynamics of the controlled system, therefore it is not directly appropriate for switched systems.

In a switched system (1), the problem is different, since the dynamics of the system is also changed by switching between modes. In this case, we have modified the above Scheme 2 to Scheme 3 (Fig. 3). Scheme 3 includes new elements which are: off-line mode system model and the respective DC gain, to tailor the off-line controller output to the respective system dynamics corresponding to the reference. The augmented signal $\alpha(k)$ is formulated so that it forces the off-line controller output to reach the value respective to $\tilde{u}(k)$. Off-line control $\tilde{u}(k)$ provides reference tracking for off-line mode (system model + controller respective to the off-line mode). Matrix F_i in Scheme 3 is given by

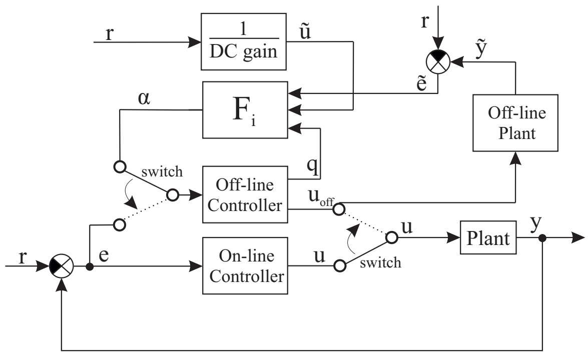

Figure 3. Bumpless transfer scheme 3.

(14) as in Scheme 2, however, augmented signal $\alpha(k)$ is formulated as follows:

$$\alpha(k) = F_i \begin{bmatrix} q(k) \\ \tilde{u}(k) \\ \tilde{e}(k) \end{bmatrix}. \tag{15}$$

Applying approaches, described by Schemes 1 and 3 in Figs 1 and 3, the bumpless control can be designed for switched systems, when the system has two or more modes that represent different system dynamics. Scheme in Fig. 2 is appropriate when the dynamics of system does not change, but controllers do. In the case when both the system dynamics and also PI controllers are switched, it is assumed that the control variable can be changed within the sampling period, but the output signal transient effect (due to switching) should be with minimal error.

When robust stabilizing controllers are designed for a switched system, stability is not influenced by implementing the proposed schemes for bumpless transfer.

Therefore in the next examples, it is not necessary to check robust stability.

5 Switched System — Simulation Results

The developed bumpless transfer schemes were tested on a laboratory system shown in Fig. 4, which consists of two DC motors and on/off switching rule. In each mode, one of the motors is active and the other one is active only when the switch is on. The input to this switched system is voltage [V] and output is electrical power. The main aim is to design robust controller with bumpless transfer for this laboratory switched system. Robustness against changing load is considered. The uncertain model respective to the real plant was obtained using identification of discrete-time linearized models in two defined modes which represent the switching. The respective ARMAX models were calculated from measured input and output data. Uncertainty of this system corresponds to changing the value of load (2-6), which is represented by two vertices of polytopic system for each operation mode, corresponding to upper and lower value of load. The resulting models for both modes, considering sample time $Ts = 0.1s$ are given below.

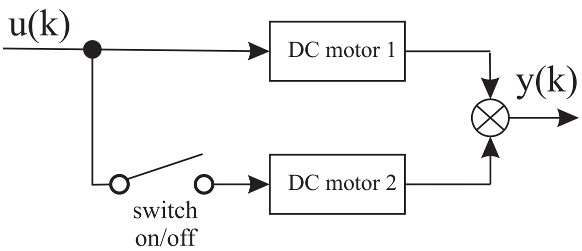

Figure 4. Scheme of switched DC motors.

Transfer functions for the first mode:
Value for load = 2

$$G_{1,1}(z) = \frac{-0.002738z^2 + 0.02474z + 0.05236}{z^3 - 1.654 + 0.756z - 0.05814}.$$

Value for load = 6

$$G_{2,1}(z) = \frac{-0.001168z^2 + 0.009795z + 0.02475}{z^3 - 1.731z^2 + 0.873z - 0.1205}.$$

The system dynamics is changed, when the switch position is changed.
Transfer functions for the second mode:
Value for load = 2

$$G_{1,2}(z) = \frac{-0.001049z^2 + 0.04709z + 0.1091}{z^3 - 1.704 + 0.8573z - 0.1069}.$$

Value for load = 6

$$G_{2,2}(z) = \frac{0.0002408z^2 + 0.03457z + 0.06102}{z^3 - 1.947z^2 + 1.289z - 0.3128}.$$

The robust PI controller was designed from LMIs (4), (5), (6) for the uncertain switched system (1) respective to the above transfer functions. The PI controller parameters respective to a state-space controller description (9) for each mode are:

$$A_{r,1} = 1; B_{r,1} = 1; C_{r,1} = 0.1642; D_{r,1} = 1.0949$$
$$A_{r,2} = 1; B_{r,2} = 1; C_{r,2} = 0.0859; D_{r,2} = 0.6681.$$

The designed robust controllers were verified by simulations, the respective results for both vertices of the polytopic system are shown in Figs. 5-8. Time responses of the system output when both system mode and a load value are changed at the same time are depicted in Figs. 7 and 8. Presented simulation results illustrate the fact that the designed controllers guarantee robust stability of the uncertain switched system during switching and dynamical load change. Note the system oscillations when the mode is switched (without changing the reference signal).

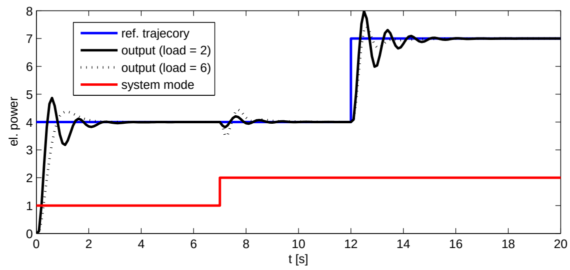

Figure 5. Time responses of the system output in different vertices of the polytopic system.

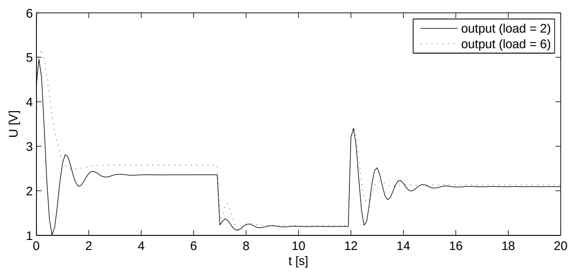

Figure 6. Time responses of the control variable in different vertices of the polytopic system.

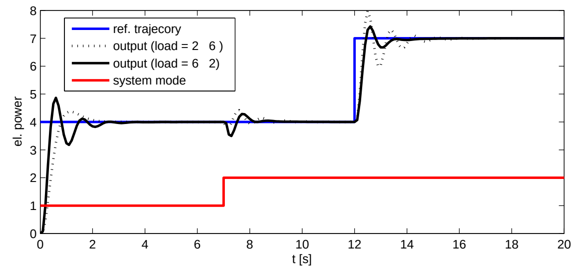

Figure 7. Time responses of the system output during switching, load change from 2 to 6 (solid line), load change from 6 to 2 (dotted line).

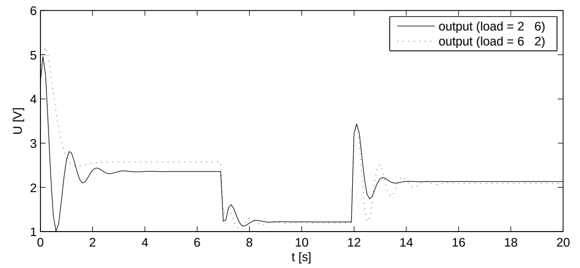

Figure 8. Time responses of the control variable during switching, load change from 2 to 6 (solid line), load change from 6 to 2 (dotted line).

5.1 Bumpless Transfer Application

In this section, the proposed bumpless transfer (BT) schemes are verified both by simulations and on the real plant. We compare simulation results and results for the real laboratory plant for several closed loop configurations:

- Closed loop without BT scheme
- Closed loop with BT Scheme 1 (Fig. 1) combined with Scheme 2 (Fig. 2), denoted as BTa
- Closed loop with BT Scheme 2 (Fig. 2) combined with Scheme 3 (Fig. 3) denoted as BTb.

The proposed bumpless transfer schemes (2 latter configurations) were tested for cases:

- Switching modes
- Switching only controller for the same mode.

Both proposed alternative BT schemes BTa and BTb were compared for

- Switching modes in steady state
- Switching modes within transient time.

In the following, we consider constant value of load, which represents one vertex of polytopic model. For illustrating the bumpless transfer in the case when the controllers are switched in the same mode, another robust PI controller was designed for the first mode with the following parameters:

$$A_{r,11} = 1; B_{r,11} = 1; C_{r,11} = 0.12; D_{r,11} = 0.52.$$

Fig. 9 shows a signal respective to changing the system modes and switching controllers (in mode 1) in the upper part, and reference trajectory behaviour in the lower part.

In the first case, simulation results for switched system without bumpless transfer are depicted in Figs. 10 and 11 (Fig. 10 reference trajectory tracking; Fig. 11 control variable response).

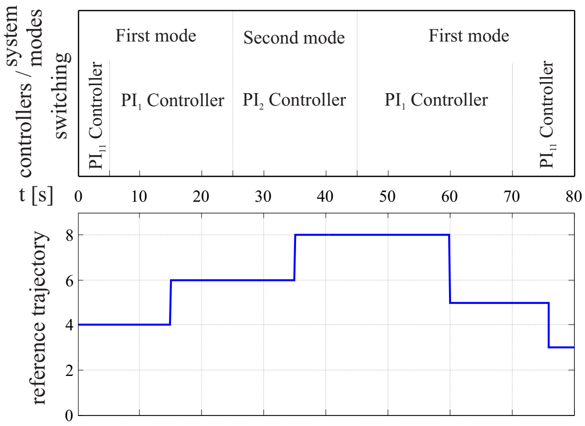

Figure 9. Graphical illustration of system modes, controllers switching and the reference trajectory

As can be seen, changing controllers parameters (in time 5s, 25s, 45s and 75s) and system modes (in time 25s and 45s) according to Fig. 9 brings some transient effects, caused by control discontinuities.

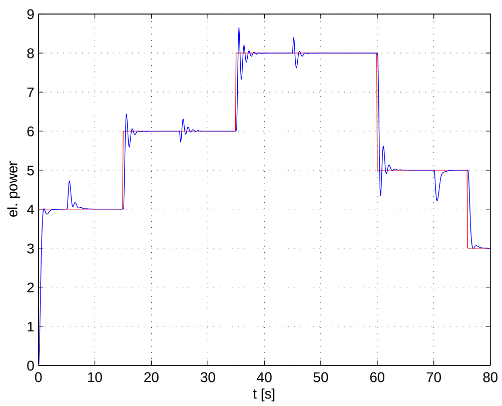

Figure 10. Time responses of the electric power and reference trajectory.

Next simulation results correspond to the proposed methods of bumpless transfer for switching systems (BTa or BTb). When the system mode is switched (in time 25s and 45s) in steady state, both BTa and BTb provide the same results. When only the controller is switched in time 5s and 75s (without switching a system mode), the approach from Scheme 2

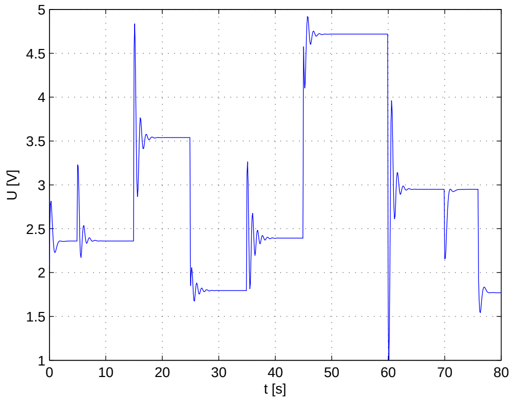

Figure 11. Time responses of the control variable.

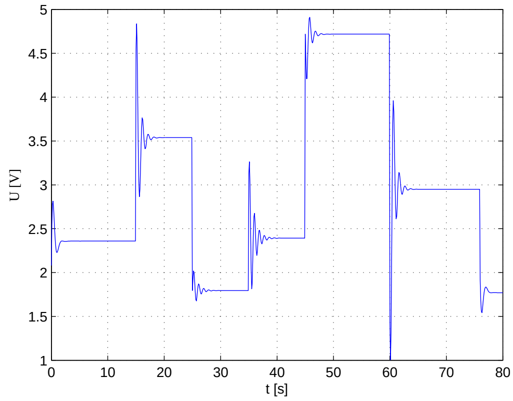

Figure 13. Time responses of the control variable (using both approaches: BTa or BTb).

is activated, because the parameters of controller are switched, while the system dynamics remains the same, as shown in Fig. 9. As illustrated by simulation results for switched system in Figs. 12 and 13, the proposed combined method (Scheme 1 with 2 or 2 with 3) outperforms results without BT scheme. In comparison with simulations in the first case without BT, the time responses seems to be very close to each other, when the system is switching, but as shown below, this is specific for the designed controller parameters.

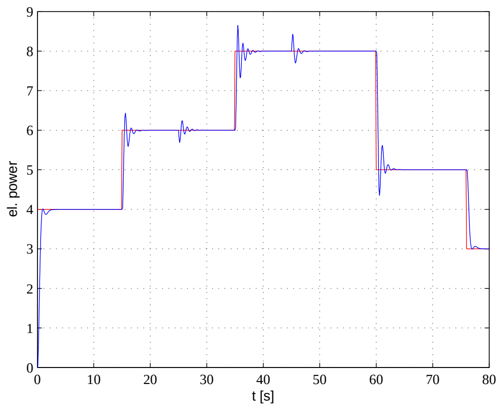

Figure 12. Time responses of the electric power and reference trajectory (using approaches: BTa or BTb).

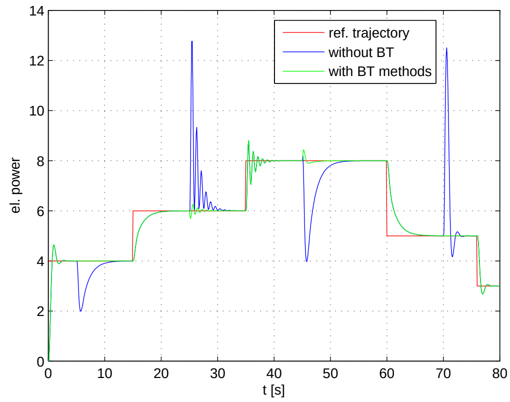

Figure 14. Time responses of the electric power and reference trajectory (with and without BT methods).

Let us now consider redesigned robust controller parameters:

$$
\begin{aligned}
A_{r,1} &= 1; B_{r,1} = 1; C_{r,1} = 0.045; D_{r,1} = 0.245 \\
A_{r,2} &= 1; B_{r,2} = 1; C_{r,2} = 0.08; D_{r,2} = 0.78 \\
A_{r,11} &= 1; B_{r,11} = 1; C_{r,11} = 0.16; D_{r,11} = 0.5.
\end{aligned}
$$

For redesigned controllers, simulation results (Figs. 14 and 15) show significant differences between the cases with and without the proposed BT scheme. Though the problem of output transient effects when the system is switched (at time 25s and 45s) cannot be totally removed, the oscillations due to switching are significantly reduced.

The next results show the differences between bumpless transfer Scheme 1 and 3. We set new switching times of system to obtain switching during the transient time. In the proposed Schemes 1 and 3, various presetting of the off-line controller is used, which leads to different results. Experiments considering the control with and without BT methods are performed also on real plant. Results are shown in figures below (Fig. 18 reference trajectory tracking; Fig. 19 control variable response).

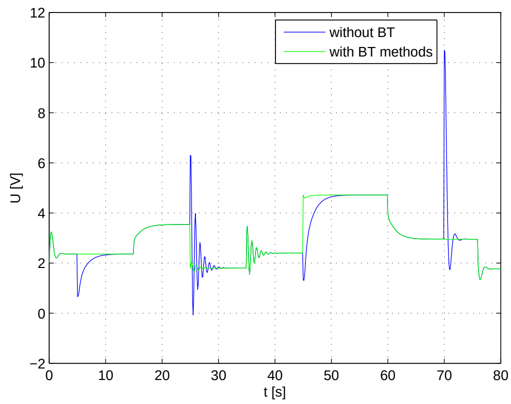

Figure 15. Time responses of the control variable (with and without BT methods).

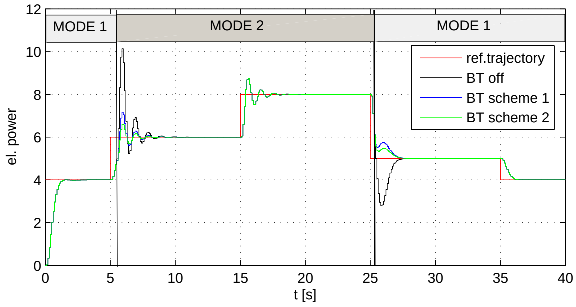

Figure 16. Time responses of the electric power and reference trajectory (with and without BT methods).

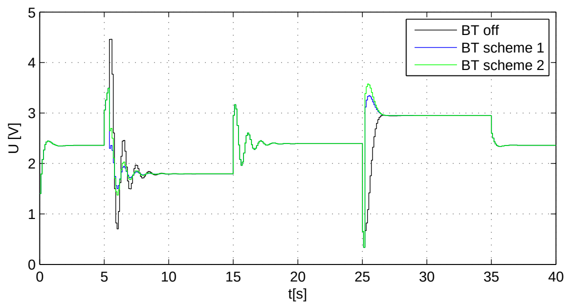

Figure 17. Time responses of the control variable (with and without BT methods).

6 Conclusion

The main aim of this paper was to propose bumpless transfer (BT) applicable for switched systems. The developed BT schemes were applied for a discrete-time switched system with robust PI controllers, designed using LMI approach. The proposed BT approach was inspired by previous results from literature, considering, however, the case when only a controller is switched. Both schemes 1 and 3 yield satisfactory results, damping the oscillations in output transient response caused by switching the modes. The difference between the developed two methods appears when switching is activated within system transient time. The results are illustrated both by simulations and on real laboratory plant experiments.

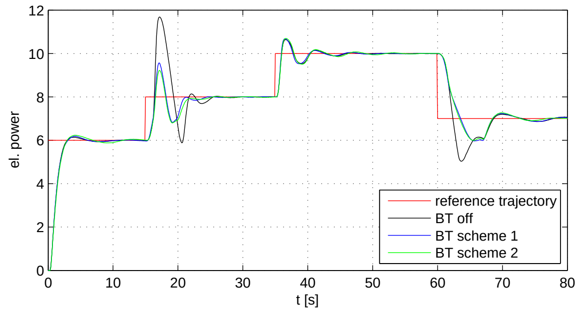

Figure 18. Time responses of the electric power and reference trajectory (with and without BT methods) on real plant.

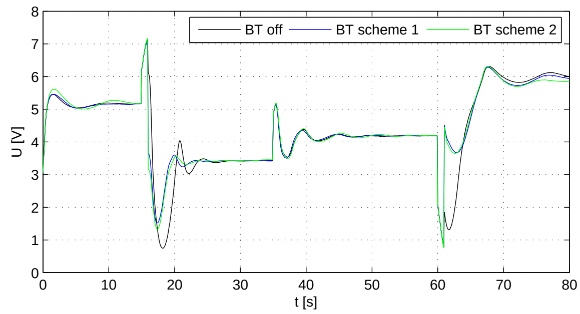

Figure 19. Time responses of the control variable (with and without BT methods) on real plant.

Acknowledgements

The work has been supported by Grant N 1/1241/12 of the Slovak Scientific Grant Agency and Slovak Research and Development Agency, grant No. APVV-077-12.

References

Campo, P.J., Morari, M., and Nett, C.N. (1989). Multivariable anti-windup and bumpless transfer. A general theory. *Proceedings of the 1989 American Control Conference* pp. 706-1711.

Cheong, S.Y., and Safonov, M.G. (2008). Bumpless transfer for adaptive switching controls. *Proceedings of the IFAC World Congress 2008*.

Geromel, J.C., and Colaneri, P. (2006). Stability and stabilization of discrete time switched systems. *International Journal of Control*. **79**. Issue 07, pp. 719728.

Hanus, R. (1988). Anti-windup and bumpless transfer: a survey.*In Proc. 12th IMACS World Congr. on Scientific Computing*, Paris, pp. 59–65.

Liberzon, D. (2003). *Switching in Systems and Control*, Boston, MA:Birkhauser.

Maherzi, E., Bernussou, J., and Mhiri, R. (2007). Stability and stabilization for uncertain switched systems, a polyquadratic approach. *Int. Journal on Science and Techniques of Automatic control*, **1**, No. 1.

Malloci, I., Hetel, L., Daafouz, J., Iung, C., and Bonidal, R. (2009). Bumpless transfer for discrete-time switched systems. *Proceedings of the 2009 American Control Conference*, pp. 1078–1083.

Turner, M.C., and Walker, D.J. (2000). Linear quadratic bumpless transfer.*Automatica*, **36**(8), pp. 1089–1101.

Valach, P., and Rosinová, D. (2015). Bumpless transfer for a discrete-time switched system with robust control. In:*Physcon 2015*, Turkey, Istanbul.

Zaccarian, L., and Teel, A.R. (2002). A common framework for anti-windup, bumpless transfer and reliable designs. *Automatica*, **38**(10), pp. 1735–1744