PID Adaptive Control Design Based on Singular Perturbation Technique: A Flight Control Example ⋆

Valery D. Yurkevich *,

* *Novosibirsk State Technical University, 20 K. Marx av., Novosibirsk, 630092, Russia (e-mail: yurkev@ac.cs.nstu.ru)*

Abstract: The paper treats a question of proportional-integral-derivative (PID) controller design for aircraft pitch attitude in the presence of uncertain aerodynamics. The presented design methodology guarantees desired pitch attitude transient performance indices by inducing of two-time-scale motions in the closed-loop system where the controller dynamics is a singular perturbation with respect to the system dynamics. Stability conditions imposed on the fast and slow modes and sufficiently large mode separation rate between fast and slow modes can ensure that the full-order closed-loop nonlinear system achieves the desired properties in such a way that the pitch attitude transient performances are desired and insensitive to external disturbances and variations of aerodynamic characteristics. The singular perturbation method is used throughout the paper in order to get explicit expressions for evaluation of the controller parameters. The high-frequency-gain online identification and gain tuning are incorporated in the control loop in order to maintain the stability of the fast-motion transients for a large range of aerodynamic characteristics variations. Numerical example and simulation results are presented.

Keywords: flight control; aircraft pitch attitude control; adaptive control; singular perturbations.

1. INTRODUCTION

Due to the complexity, variability, and uncertainties of aerodynamics, many different control techniques are used in order to design of aircraft and missile control systems aimed to maintain transient performances in the presence of uncertain aerodynamics.

There are numerous approaches to flight control system design, for instance, based on the Nonlinear Inverse Dynamics (NID) method, feedback linearization, nonlinear H_∞, linear matrix inequalities, l_1 optimal control, and LQR (see, for example, Petrov and Krutko (1981); Vukobratović and Stojić (1988); Lane and Stengel (1988); Wise (1995)). In particular, control systems with sliding mode discussed by Schumacher (1994) and control systems with the highest derivative in feedback treated by Błachuta *et al.* (1997), Czyba and Błachuta (2003), are very powerful tools for aircraft and missile control system design under uncertainties. Note that the design methodology discussed by Błachuta *et al.* (1997), Czyba and Błachuta (2003) leads to the structure of proportional-integral-derivative (PID) controller in case of systems with relative degree equals two.

It is well known that throughout the huge set of flight control applications, the PI (PID) controllers are extensively used. Problems of PI (PID) control system analysis and design are discussed, for instance, by O'Dwyer (2003). The majority of known design procedures of PI (PID) controllers are applicable merely for stable linear systems. For example, the well known tuning rules suggested by Ziegel and Nichols (1942), or its various modifications are widely used for selection of controller parameters. In order to fetch out the best PI and PID controllers in accordance with the assigned design objectives, a set of tuning rules, identification and adaptation schemes has been developed (see Åström *et al.* (1993)). The main disadvantage for the most part of the existing procedures for PI or PID controller design is that the desired transient performances can not be guaranteed in the presence of nonlinear plant parameter variations and unknown external disturbances. In order to overcome this disadvantage, the singular perturbation technique (see Kokotović et al. (1999), Naidu (2002)) may be used for PI or PID controller design as was shown by Yurkevich (2004) where desired output transients are guaranteed by inducing of two-time-scale motions in the closed-loop system. Stability conditions imposed on the fast and slow modes and sufficiently large mode separation rate between fast and slow modes can ensure that the full-order closed-loop nonlinear system achieves the desired properties in such a way that the output transient performances are desired and insensitive to external disturbances and parameter variations of the system. The stability of fast-motion transients in the closed-loop system is provided by proper selection of controller parameters, while slow-motion transients correspond to the stable reference model of desired mapping from reference input into controlled output.

⋆ This work was financially supported by Russian Foundation for Basic Research (RFBR) under grant no. 08-08-00982-a.

The critical point of the workability of the singular perturbation design methodology is that the controller parameters should be selected in accordance with the requirement on fast-motion transients stability and, moreover, the desired degree of time-scale separation between the fast and slow modes in the closed-loop system should be provided (see Yurkevich (2004)). However, the large variations of parameters of the system may give the disappearance of the two-time-scale structure of the trajectories in the closed-loop system. In order to maintain the two-time-scale structure of the trajectories in the closed-loop system and the desired damping of fast transients, an adaptive gain tuning scheme is proposed in this paper. As a result, the desired pitch attitude transient performance indices can be provided for significantly large range of aerodynamic characteristics variations.

The paper is organized as follows. First, a model of the aircraft longitudinal motion is defined. Second, the singular perturbation design methodology is highlighted for the purpose of aircraft pitch attitude control in the presence of uncertain aerodynamics. Third, the utmost importance of high-frequency-gain online identification and gain tuning in nonlinear control systems designed via singular perturbation technique is shown. Fourth, a gain tuning procedure and high-frequency-gain online identification procedure are introduced. Finally, simulation results of the aircraft pitch attitude adaptive control system are presented as well.

2. AIRCRAFT CONTROL PROBLEM

2.1 Aircraft Longitudinal Motion Model

The discussed approach to PID adaptive controller design for aircraft pitch attitude angle is treated based on the aircraft longitudinal motion model represented by Wojciechowski *et al.* (1989):

$$\begin{aligned}
\dot{\theta} &= q, \\
\dot{u} &= -wq - g\sin\theta + \frac{a_c}{m}\delta_c + \\
&\quad + \frac{1}{2m}\rho v_a^2\{S_x c_x(\alpha,\delta_h)\cos\alpha - S_z c_z(\alpha,\delta_h)\sin\alpha\}, \\
\dot{w} &= uq + g\cos\theta + \\
&\quad + \frac{1}{2m}\rho v_a^2\{S_x c_x(\alpha,\delta_h)\sin\alpha + S_z c_z(\alpha,\delta_h)\cos\alpha\}, \\
\dot{q} &= \frac{1}{2J_y}\rho v_a^2 L_y S_y m_y(\alpha,\delta_h),
\end{aligned} \tag{1}$$

where, in accordance with Fig. 1, we have that
v_a is an air velocity value, $v_a = (v_{ax}^2 + v_{az}^2)^{1/2}$;
v_{ax}, v_{az} are components of the air velocity,
$v_{ax} = u - v_{wx}\cos\theta + v_{wz}\sin\theta$;
$v_{az} = w - v_{wx}\sin\theta - v_{wz}\cos\theta$;
α is an angle of attack, $\alpha = \tan^{-1}(v_{az}/v_{ax})$;
m is an aircraft mass, $m = 2000$ kg;
J_y is a moment of inertia, $J_y = 5000\,\mathrm{kg\ m}^2$;
L_y is an equivalent arm force in the flow coordinate system, $L_y = 0.5$ m;
S_x, S_y, S_z are equivalent areas in the flow coordinate system, $S_x = 0.5\,\mathrm{m}^2$, $S_y = 2.0\,\mathrm{m}^2$, $S_z = 10.0\,\mathrm{m}^2$;
ρ is the air density, $\rho = 1.2\,\mathrm{kg/m}^3$;
g is the acceleration, $g = 9.81\ \mathrm{m/s}^2$;
a_c is a coefficient,$a_c = 20.0$ N/% ;
θ is an aircraft pitch attitude angle, $\theta \in [-0.3, 0.3]$ rad;
u is a longitudinal velocity, $u \in [50, 300]$ m/s;
w is a normal velocity, $w \in [-20, 20]$ m/s;
q is an aircraft pitch rate, $q \in [-0.15, 0.15]$ rad/s;
δ_h is an angle of elevator deflection, $\delta_h \in [-0.7, 0.7]$ rad;
δ_c is a thrust coefficient, $\delta_c \in [0, 400]$ % ;
v_{wx}, v_{wz} are components of wind velocity in the inertial system, $v_{wx} \in [-20, 20]$ m/s, $v_{wz} \in [-10, 10]$ m/s;

We assume also that the functions $c_x(\alpha,\delta_h)$, $c_z(\alpha,\delta_h)$, $m_y(\alpha,\delta_h)$ have the following form:

$$\begin{aligned}
c_x(\alpha,\delta_h) &= c_x^o + c_x^{\alpha 2}\alpha^2 + c_x^{h2}(\delta_h)^2 \\
c_z(\alpha,\delta_h) &= c_z^o + c_z^{\alpha}\alpha + c_z^{h}\delta_h \\
m_y(\alpha,\delta_h) &= m_y^{\alpha}\alpha + m_y^{h}\delta_h
\end{aligned}$$

where $c_x^o = -0.2$; $c_x^{\alpha 2} = -0.002$; $c_x^{h2} = -0.002$; $c_z^h = -10^{-4}$; $c_z^o = -0.15$; $c_z^{\alpha} = -8.6$; $m_y^{\alpha} = 0.057$; $m_y^h = -0.01$.

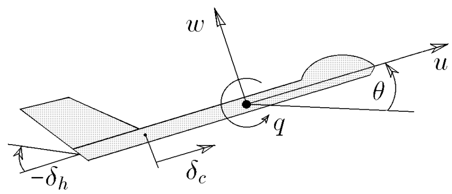

Fig. 1. Aircraft longitudinal motion model.

2.2 Problem of Aircraft Pitch Attitude Control

The control problem is to provide the following condition:

$$\lim_{t\to\infty} e_\theta(t) = 0 \tag{2}$$

where $e_\theta(t)$ is the error of the reference input realization; $e_\theta(t) = \theta^d(t) - \theta(t)$; $\theta^d(t)$ is the reference input. Moreover, the controlled transients $e_\theta(t) \to 0$ should have a desired behavior. These transients should not depend on the external disturbances and varying parameters of the aircraft model (1).

2.3 Input-Output Mapping of Aircraft Model

From (1) it follows that the second time derivative of $\theta(t)$ depends algebraically on the control variable δ_h, that is

$$d^2\theta/dt^2 = f_\theta(\cdot) + b_\theta(\cdot)\delta_h \tag{3}$$

where

$$f_\theta(\cdot) := \frac{1}{2J_y}\rho v_a^2 L_y S_y m_y^{\alpha}\alpha, \quad b_\theta(\cdot) := \frac{1}{2J_y}\rho v_a^2 L_y S_y m_y^{h}.$$

Hence, the second time derivative of $\theta(t)$ is the relative highest derivative of the output variable $\theta(t)$. Note, the condition $b_\theta(\cdot) < 0$ holds for the above defined parameters.

Remark 1. The expression (3) describes the input-output mapping of the aircraft model without taking into account the internal dynamics of aircraft (in particular case, that is zero-dynamics). It is assumed through the text that the internal dynamics is stable or at least is bounded. The analysis of the internal behavior of the system (1) was done by Yurkevich *et al.* (1991).

Remark 2. The parameter b_θ is called as the high-frequency gain of the input-output mapping defined by (3), where θ is considered as the output variable, while δ_h is considered as the input variable.

Remark 3. The high-frequency gain b_θ may undergo extensive variations depending on operating point and aerodynamic characteristics, in particular, due to the variation of the air velocity v_a.

3. CONTROLLER DESIGN VIA TIME-SCALE SEPARATION

3.1 PID Controller

Consider the controller given by

$$\mu^2\, \delta_h^{(2)} + d_1 \mu\, \delta_h^{(1)} = k[F(\theta^{(1)}, \theta, \theta^d) - \theta^{(2)}] \tag{4}$$

where μ is the small positive parameter, and

$$F(\theta^{(1)}, \theta, \theta^d) := -a_1\theta^{(1)} - a_0[\theta^d - \theta]$$

where $a_0 > 0$ and $a_1 > 0$. The parameters a_0, a_1 are selected such that the polynomial

$$s^2 + a_1 s + a_0 \tag{5}$$

has the desired root distribution inside the left part of the s-plane, where roots of the polynomial (5) are defined by the requirements imposed on the desired output transient performance indices of $\theta(t)$ in the system (1).

Remark 4. The control law (4) can be expressed in terms of transfer functions, that is the structure of the conventional PID controller given by

$$\delta_h(s) = \frac{k}{\mu(\mu s + d_1)} \left\{ \frac{a_0}{s}[\theta^d(s) - \theta(s)] - (s + a_1)\theta(s) \right\}$$

where the controller is proper and implemented without an ideal differentiation of $\theta(t)$ or $\theta^d(t)$.

The replacement of $\theta^{(2)}$ in (4) by the right member of (3) yields the closed-loop system equations in the form

$$\begin{aligned} \theta^{(2)} &= f_\theta(\cdot) + b_\theta(\cdot)\delta_h \\ \mu^2\, \delta_h^{(2)} + d_1\mu\, \delta_h^{(1)} + kb_\theta(\cdot)\delta_h &= k[F(\theta^{(1)}, \theta, \theta^d) - f_\theta(\cdot)]. \end{aligned} \tag{6}$$

Denote $\theta_1 = \theta$, $\theta_2 = \theta^{(1)}$, $\delta_{h1} = \delta_h$, $\delta_{h2} = \mu\, \delta_h^{(1)}$. Hence, from (6), the singularly perturbed differential equations

$$\begin{aligned} \dot\theta_1 &= \theta_2, \\ \dot\theta_2 &= f_\theta(\cdot) + b_\theta(\cdot)\delta_{h1} \\ \mu\, \dot\delta_{h1} &= \delta_{h2}, \\ \mu\, \dot\delta_{h2} &= -kb_\theta(\cdot)\delta_{h1} - d_1\delta_{h2} + k[F(\theta_2, \theta_1, \theta^d) - f_\theta(\cdot)] \end{aligned} \tag{7}$$

result as $\mu \to 0$.

If $\mu \to 0$, then fast and slow modes are artificially forced in the system (7) where the time-scale separation between these modes depends on the parameter μ. Hence, the properties of (7) can be analyzed on basis of the two-time-scale technique and, as a result, slow and fast motion subsystems are derived in the next subsection of the paper.

3.2 Two-Time-Scale Motions Analysis

Let us introduce the new fast time scale $t_0 = t/\mu$. Hence, from (7), the closed-loop system equations

$$\begin{aligned} \frac{d}{dt_0}\theta_1 &= \mu\theta_2, \\ \frac{d}{dt_0}\theta_2 &= \mu[f_\theta(\cdot) + b_\theta(\cdot)\delta_{h1}] \\ \frac{d}{dt_0}\delta_{h1} &= \delta_{h2}, \\ \frac{d}{dt_0}\delta_{h2} &= -kb_\theta(\cdot)\delta_{h1} - d_1\delta_{h2} + k[F(\theta_2, \theta_1, \theta^d) - f_\theta(\cdot)] \end{aligned} \tag{8}$$

result. If $\mu \to 0$, then from (8) the FMS equations

$$\begin{aligned} \frac{d}{dt_0}\delta_{h1} &= \delta_{h2}, \\ \frac{d}{dt_0}\delta_{h2} &= -kb_\theta(\cdot)\delta_{h1} - d_1\delta_{h2} + k[F(\theta_2, \theta_1, \theta^d) - f_\theta(\cdot)]. \end{aligned} \tag{9}$$

follow. Then, by returning to the primary time scale $t = \mu t_0$, we obtain the following FMS equations:

$$\begin{aligned} \mu\, \dot\delta_{h1} &= \delta_{h2}, \\ \mu\, \dot\delta_{h2} &= -kb_\theta(\cdot)\delta_{h1} - d_1\delta_{h2} + k[F(\theta_2, \theta_1, \theta^d) - f_\theta(\cdot)] \end{aligned} \tag{10}$$

where θ_1 and θ_2 are treated as the constant values during the transients in (10). This requirement can be easily satisfied for sufficiently small design parameter μ. Finally, the FMS equations (10) may by rewritten as

$$\mu^2\, \delta_h^{(2)} + d_1\mu\, \delta_h^{(1)} + kb_\theta(\cdot)\delta_h = k[F(\theta^{(1)}, \theta, \theta^d) - f_\theta(\cdot)] \tag{11}$$

where $f_\theta(\cdot)$ and $b_\theta(\cdot)$ are treated as the constant values during the transients in (11).

Assume that the sign of k is selected such that the condition $kb_\theta(\cdot) > 0$ holds and the desired fast damping of FMS transients is provided by selection of controller parameters d_1, μ, and k. Then , the quasi-steady state for the FMS (11) yields $\delta_h(t) = \delta_h^{id}(t)$ where

$$\delta_h^{id} = b_\theta^{-1}(\cdot)[F(\theta^{(1)}, \theta, \theta^d) - f_\theta(\cdot)]$$

and δ_h^{id} is exactly the inverse dynamics solution. Substitution of $\delta_h = \delta_h^{id}$ into (3) yields the SMS equation

$$\theta^{(2)} + a_1\theta^{(1)} + a_0\theta = a_0\theta^d. \tag{12}$$

By the another way, the SMS equation (12) can be directly derived from (7), by taking $\mu = 0$.

In accordance with the main qualitative property of the singularly perturbed systems (see Klimushchev and Krasovskii (1962); Hoppensteadt (1966); Kokotović et al. (1999); Naidu (2002)), we have, if an isolated equilibrium point of the FMS (11) exists and one is exponentially stable, then there exists $\mu^* > 0$ such that for all $\mu \in (0, \mu^*)$ the trajectories of the singularly perturbed system (7) approximate to the trajectories of the SMS (12). So, if a sufficient time-scale separation between the fast and slow modes in the system (7) and exponential convergence of FMS transients to equilibrium are provided, then, after the damping of fast transients, the desired output behavior prescribed by (12) is fulfilled despite that $f_\theta(\cdot)$ and $b_\theta(\cdot)$ are unknown complex functions. Thus, the output transient performance indices are insensitive to parameter variations of the nonlinear system and external disturbances, by that the solution of the discussed control problem (2) is maintained.

4. PID ADAPTIVE CONTROLLER

4.1 Problem of High-Frequency-Gain Online Identification and Gain Tuning

The critical point of the workability of the discussed design methodology via singular perturbation technique is that the controller parameters should be selected in accordance with the requirement on FMS stability and, moreover, the desired degree of time-scale separation between the fast and slow modes in the closed-loop system (7) should be provided.

From (11), the FMS characteristic polynomial

$$A_{fms}(s) = \mu^2 s^2 + d_1\mu, s + \gamma \tag{13}$$

follows, where $\gamma = kb_\theta(\cdot)$.

The main disadvantage is that the decrease of time-scale separation degree and loss of the FMS transient performances may occur in case of large variations of the high-frequency gain $b_\theta(\cdot)$.

In order to overcome the disadvantage caused by variations of b_θ, two problems have to be solved. The first one is online identification of b_θ. The second one is the adaptive gain tuning based on the knowledge of an estimate $\hat{b}_\theta$ for b_θ. Then, by taking

$$k = k_1\bar{k}_0 \text{ and } \bar{k}_0 = \hat{b}_\theta^{-1} \tag{14}$$

the condition $\gamma \approx k_1$ holds where $k_1 > 0$. As a result of gain tuning given by (14), the variations of the high-frequency gain b_θ do not alter the FMS transient performance indices.

4.2 Adaptive Gain Tuning

The block diagram of the proposed aircraft pitch attitude control system with adaptive gain tuning is shown in Fig. 2. Take $\bar{k}_0 = \bar{k}k_0$ and

$$\delta_h = \bar{k}k_0\hat{\delta_h} \tag{15}$$

where $\hat{\delta_h}$ is the new control variable, $\bar{k} = \mathrm{sgn}(b_\theta) = -1$, k_0 is the tuning gain, and $k_0 > 0$.

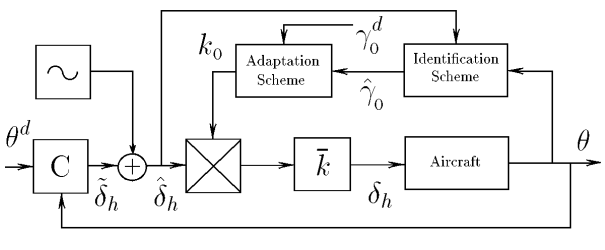

Fig. 2. Block diagram of the aircraft pitch attitude control system with adaptive gain tuning

Consider the method of online identification based on a high-frequency probing signal (see, for example, Eykchoff (1974)). Let

$$\hat{\delta_h}(t) = \tilde{\delta_h}(t) + \delta_h^0 \sin(\omega t) \tag{16}$$

where $\delta_h^0 \sin(\omega t)$ is the high-frequency probing signal with small value of amplitude δ_h^0. Next, let us rewrite the controller given by (4) as

$$\mu^2\,\tilde{\delta}_h^{(2)} + d_1\mu\,\tilde{\delta}_h^{(1)} = k_1[F(\theta^{(1)}, \theta, \theta^d) - \theta^{(2)}] \tag{17}$$

Take, for example, the tuning rule in the following form

$$\frac{dk_0}{dt} = \alpha_\gamma[\gamma_0^d - \hat{\gamma}_0], \quad k_0(0) = k_0^0 \tag{18}$$

where γ_0^d is the desired value of γ_0 and $\hat{\gamma}_0$ is the estimate of $\gamma_0 := \bar{k}k_0b_\theta(\cdot)$. It is clear, in the case of steady-state when $dk_0/dt = 0$, from (18) the condition $k_0 = \gamma_0^d/(\bar{k}\hat{b}_\theta(\cdot))$ results where $\hat{b}_\theta(\cdot)$ is an estimate of $b_\theta(\cdot)$.

4.3 High-Frequency-Gain Online Identification

The high-frequency small oscillations are forced in $\hat{\delta}_h(t)$ and $\theta(t)$ due to the high-frequency probing signal $\delta_h^0 \sin(\omega t)$ what was incorporated in the control system. Let $A_{\hat{\delta}_h}$ be the amplitude of oscillations with frequency ω that are forced in the control $\hat{\delta}_h(t)$ and A_θ be the amplitude of oscillations with frequency ω that are forced in the output $\theta(t)$. From (3) and (15) it follows that

$$\lim_{\omega\to\infty} \frac{A_\theta(\omega)}{A_{\hat{\delta}_h}(\omega)}\omega^2 = \gamma_0. \tag{19}$$

Hence, the high-frequency-gain online identification via relation (19) involves two amplitude detectors for $A_\theta(\omega)$ and $A_{\hat{\delta}_h}(\omega)$. For example, the amplitude detector based on the relationship $A_\xi = \sqrt{\xi^2 + (\dot{\xi}/\omega)^2}$ can be used when $\xi(t) = A_\xi \sin(\omega t)$. The approach by the use of (19), which is additionally supplemented by the low-pass and high-pass filtering, is used in this paper in order to get the estimates $\hat{A}_\theta(\omega)$ and $\hat{A}_{\hat{\delta}_h}(\omega)$, where the amplitude detector for $\hat{A}_{\hat{\delta}_h}(\omega)$ is described by

$$\begin{aligned}
\tau_0^2\frac{d^2\bar{\delta}_h}{dt^2} + 2\tau_0\frac{d\bar{\delta}_h}{dt} + 1 &= \tau_0^2\frac{d^2\hat{\delta}_h}{dt^2} \\
\tau_f^2\frac{d^2u_1}{dt^2} + 2\tau_f\frac{du_1}{dt} + 1 &= \bar{\delta}_h \\
\tau_f^2\frac{d^2u_2}{dt^2} + 2\tau_f\frac{du_2}{dt} + 1 &= \frac{d\bar{\delta}_h}{dt} \\
\hat{A}_{\hat{\delta}_h} &= k_f\sqrt{u_1^2 + (u_2/\omega)^2} \\
k_f &= \sqrt{[1-(\tau_f\omega)^2]^2 + (2\tau_f\omega)^2}
\end{aligned} \tag{20}$$

The amplitude detector for $\hat{A}_\theta(\omega)$ is the same as (20). Then, with the help of the low-pass filter given by

$$\tau_1\frac{d\hat{\gamma}_0}{dt} + \hat{\gamma}_0 = \frac{\hat{A}_\theta}{\hat{A}_{\hat{\delta}_h} + \epsilon}\omega^2, \quad \hat{\gamma}_0(0) = \hat{\gamma}_0^0, \tag{21}$$

the estimate $\hat{\gamma}_0$ results where ϵ is the small positive parameter in order to avoid the singularity condition in the right member of (21).

5. SIMULATION RESULTS

The simulation results of the closed-loop system are based on the aircraft longitudinal motion model (1) with (15). The high-frequency probing signal is incorporated in the system as shown in (16). In accordance with (17), consider the feedback controller (C) in Fig. 2 in the following form:

$$\mu^2\,\tilde{\delta}_h^{(2)} + d_1\mu\,\tilde{\delta}_h^{(1)} = k_1[-a_1\theta^{(1)} - a_0[\theta^d - \theta] - \theta^{(2)}]. \tag{22}$$

The adaptive gain tuning for k_0 is provided in accordance with the rule (18) while the high-frequency-gain online identification scheme consists of (20)-(21). The controller parameters are selected as $k_1 = 10$, $a_0 = 0.1145$, $a_1 = 0.4$, and $\mu = 0.6609$ s. The parameters of the adaptive gain tuning scheme and the high-frequency-gain online identification scheme are selected as $\gamma_0^d = 1$, $\hat{\gamma}_0(0) = 1$, $\epsilon = 10^{-5}$. $\alpha_\gamma = 0.2$, $k_0(0) = 65$, $\omega = 100$ rad/s, $A_\omega = 0.0003$, $\tau_f = 0.003$ s, $\tau_0 = 0.01$ s, $\tau_1 = 0.3$ s, and $\delta_c(t) = 150$ % for all t. The simulation results of the closed-loop system are shown in Figs. 3–12. Figure 3 contains the plots of $\theta^d(t)$ and $\theta(t)$ showing the output step response in the closed-loop system where the air velocity $v_a(t)$ variations are displayed in Figure 4. From Figure 9 it can be observed that the condition $\hat{\gamma}_0(t) \to \gamma_0^d$ is kept due to the tuning of the gain $k_0(t)$ as seen in Figure 10.

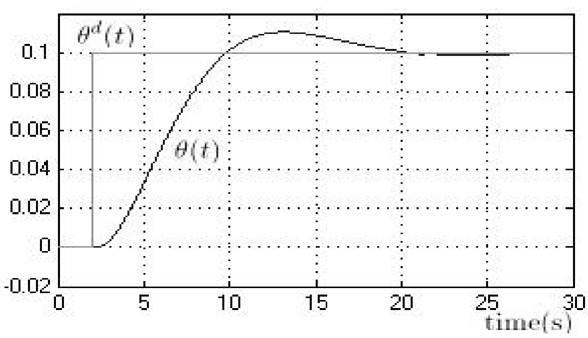

Fig. 3. Plots of the aircraft pitch attitude $\theta(t)$ (rad) and $\theta^d(t)$ (rad).

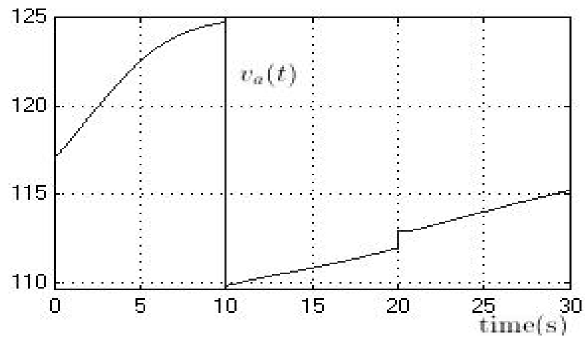

Fig. 4. Plot of the air velocity $v_a(t)$ (m/s).

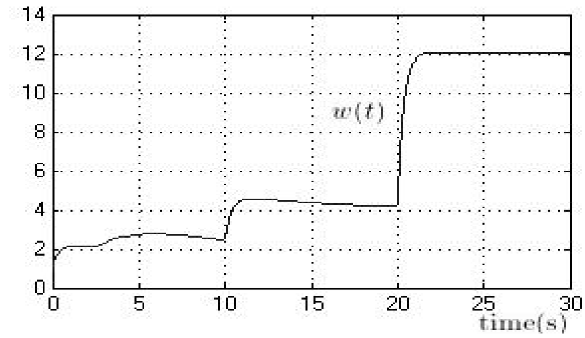

Fig. 5. Plot of the normal velocity $w(t)$ (m/s).

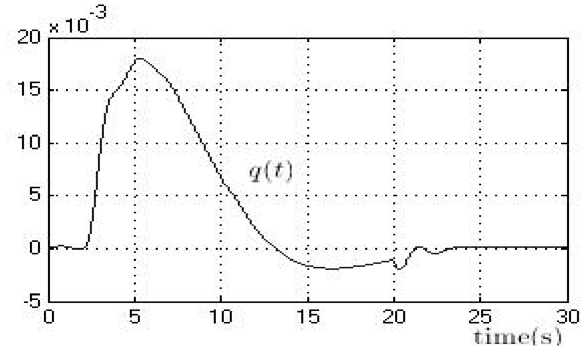

Fig. 6. Plot of the aircraft pitch rate $q(t)$ (rad/s).

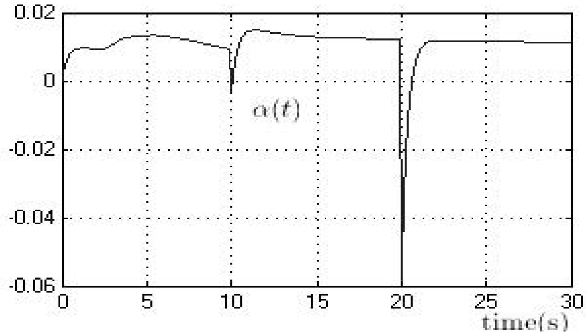

Fig. 7. Plot of the angle of attack $\alpha(t)$ (rad).

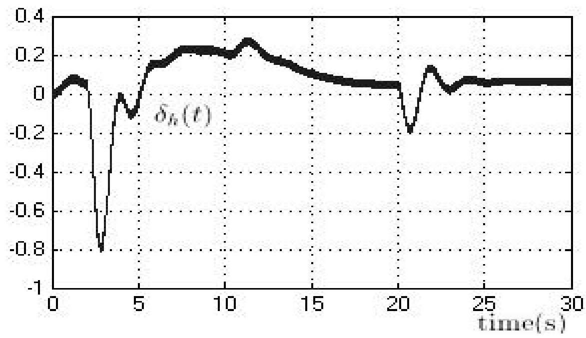

Fig. 8. Plot of the elevator deflection $\delta_h(t)$ (rad).

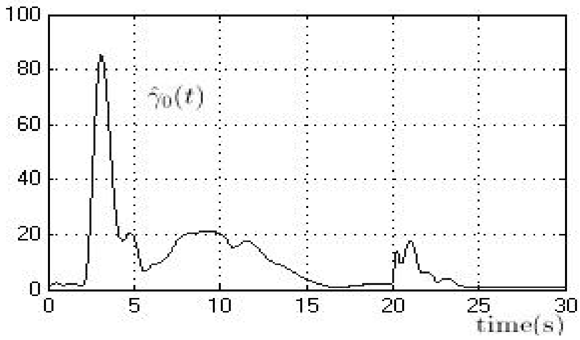

Fig. 9. Plot of $\hat{\gamma}_0(t)$ where $\hat{\gamma}_0(t)$ is the estimate of $\gamma_0(t)$.

6. CONCLUSION

The main advantage of the discussed singular perturbation technique for control system analysis and design is that the parameters of the PID adaptive controller for nonlinear aircraft model can be analytically derived in

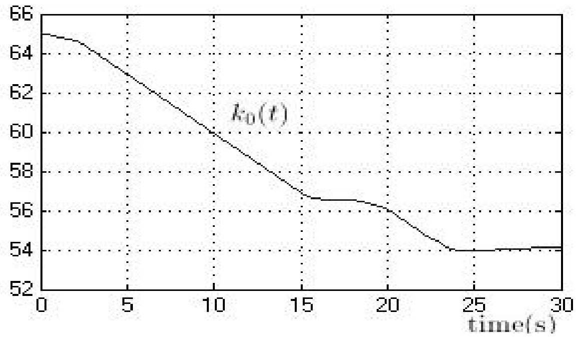

Fig. 10. Plot of the tuning gain $k_0(t)$.

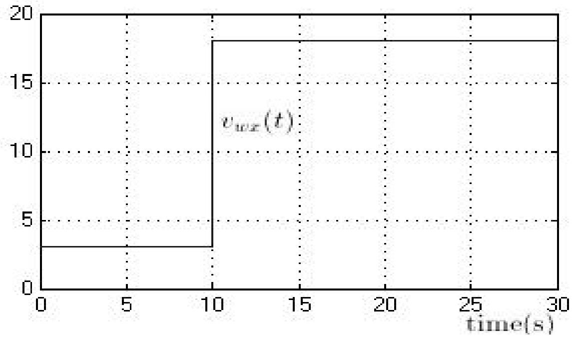

Fig. 11. Plot of wind gusts $v_{wx}(t)$ (m/s).

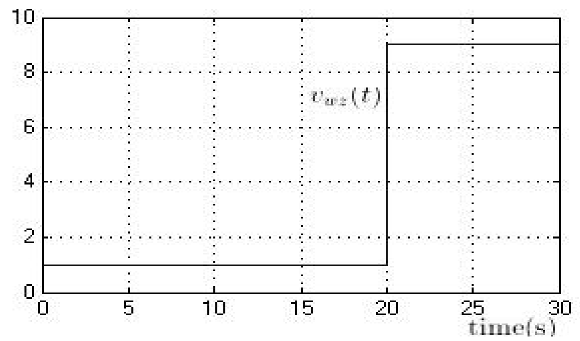

Fig. 12. Plot of wind gusts $v_{wz}(t)$ (m/s).

accordance with such indirect performance objectives as the desired root distribution of the reference model characteristic polynomial, while the desired root distribution is defined by such direct aircraft pitch attitude performance objectives as the settling time and overshoot.

The application of the singular perturbation technique in the presented design methodology allows to get desired aircraft pitch attitude transient performance indices for nonlinear aircraft model under uncomplete knowledge about external disturbances and aerodynamic characteristics. It has been shown, the high-frequency-gain online identification and gain tuning in the discussed adaptive control system allow to maintain the two-time-scale structure of the trajectories of the closed-loop system and desired transient performance indices of fast transients in case of significantly large range of aerodynamic characteristics variations. Simulation results show the effectiveness of the proposed control laws.

REFERENCES

Åström, K. J., Hagglund, T., Hang,C. C., and Ho, W. K. (1993). Automatic tuning and adaptation for PID controllers - A survey, *Contr. Eng. Pract.*, volume 1, no. 4, 699–714.

Błachuta, M. J., Yurkevich, V. D., and Wojciechowski, K. (1997). Design of analog and digital aircraft flight controllers based on dynamic contraction method. *Proc. of the AIAA Guidance, Navigation, and Control Conference.* New Orleans, LA, volume 3, 1719-1729.

Czyba, R. and Błachuta, M. J. (2003). Robust longitudinal flight control design: dynamic contraction method. *Proc. of American Control Conf.*, Denver, Colorado, 1020–1025.

O'Dwyer, A. (2003). *Handbook of PI and PID Tuning Rules*, London, UK: Imperial College Press.

Eykchoff, P. (1974). *System Identification*, Wiley.

Hoppensteadt, F. C. (1966). Singular perturbations on the infinite time interval. *Trans. of the American Mathematical Society*, volume 123, 521–535.

Klimushchev, A. I. and Krasovskii, N. N. (1962). Uniform asymtotic stability of systems of differential equations with a small parameter in the derivative terms. *J. Appl. Math. Mech.*, volume 25, 1011–1025.

Kokotović, P. V., Khalil, H. K., and O'Reilly, J. (1999). *Singular Perturbation Methods in Control: Analysis and Design*, Philadelphia, PA: SIAM.

Lane, S. and Stengel, R. (1988). Flight control design using non-linear inverse dynamics, *Automatica*, volume 24, 471–483.

Naidu, D. S. (2002). Singular perturbations and time scales in control theory and applications: An overview. *Dynamics of Continuous, Discrete & Impulsive Systems (DCDIS), Series B: Applications & Algorithms*, volume 9, no. 2, 233–278.

Petrov, B. N. and Krutko, P. D. (1981). Inverse problems of controlled-flight dynamics: lateral motion, *Sov. Phys.-Dokl.*, USA. volume 26, no. 2, 147–149.

Schumacher, D. A. (1994). Rapid response missile pitch to trim flight control using sliding modes, *Proc. of the 33rd IEEE Conf. on Decision and Control*, volume 4, 3854–3859.

Vukobratović, M. and Stojić, R. (1988). *Modern aircraft flight control. Lecture Notes in Control and Information Sciences*, Springer-Verlag, volume 109.

Wise, K. A. (1995). Applied controls research topics in the aerospace industry, *Proc. of the 34th Conf. on Decision and Control*, New Orleans, LA, 751–756.

Wojciechowski, K., Ordys, A., and Polańska, J. (1989). Model Przestrzennego Ruchu Samolotu da Celow Symulacji i Sterowania, *Zeszyty Naukowe Politechniki Slaskiej*, 1989. (in Polish)

Yurkevich, V. D., Blachuta, M. J., and Wojciechowski, K. (1991). Stabilization system for the aircraft longitudinal motion, *Archiwum Automatyki i Robotyki*, Warsaw, volume 36, no. 3-4, 517–535. (in Polish)

Yurkevich, V. D. (2004). *Design of Nonlinear Control Systems with the Highest Derivative in Feedback*, World Scientific.

Ziegel, G. and Nichols, N. B. (1942). Optimum settings for automatic controllers, *Trans. ASME*, volume 64, no. 8, 759–768.