OPEN-LOOP CONTROL ON THE EFFICIENCY OF QUANTUM BATTERY WITH RESERVOIR ENGINEERING

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Abstract

We discuss the case of a harmonic oscillator-based quantum battery strongly coupled to a highly non-Markovian thermal reservoir via the quantum charger described by the Caldeira–Leggett model. The coupling between the reservoir and the battery serves as a control parameter for the system. We consider the system to stay in the strongly underdamped regime. Within the framework of the open-loop approach, we determine the optimal shape of control for the battery charging work and then restore the control coupling characteristics of QB in the Hamiltonian for the alternative cases of low and high temperatures. Ultimately, we discuss some possible ways to develop our model for the feedback algorithms.

Key words

Quantum battery, reservoir engineering, Caldeira–Leggett model, underdamped regime, open-loop control.

1 Introduction: Control Algorithms for Quantum Batteries

Quantum batteries are capable of storing and transferring energy at a quantum level. In recent years, we have observed a boom in theoretical and experimental research on such devices and their practical implementation. The work principles of a quantum battery (QB) cannot be reduced to its classical analog, and specifically, quantum concepts, such as the quantum many-body approach in an open environment, must be developed to model such devices [Campaioli, Gherardini, et al., 2024].

The charging and discharging processes in quantum batteries, together with the possible energy loss during its storage, in their inorganic (quantum dots and wells, perovskite systems, superconductors) and even organic (microcavities [Quach, McGhee, et al., 2022]) realizations are based on the thermodynamic principles of open quantum systems [Camposeo, Virgili, et al., 2025]. As a rule, the theoretical QB protocol includes the model for the quantum battery itself, and its charger placed in the external reservoir [Caravelli, Yan, et al., 2021]. Some models study in detail the energy storage lifetime [Liu, Zhang, et al., 2025] and dissipative processes during the charging procedure [Pokhrel and Gea-Banacloche, 2025].

The set of control methods used for QBs can be algorithmically divided into two categories: open-loop and feedback algorithms.

1.1 The feedback approaches

Usually, the *feedback methods* are supposed to be more efficient in controlling a physical system, especially because of their possibilities to design an optimal and suboptimal control. For similar problems of the qubit state control [Borisenok and Gogoleva, 2024] and qubit energy control [Pechen and Borisenok, 2015; Pechen, Borisenok, Fradkov, 2022] it was successfully applied in the form of speed gradient [Fradkov, 2007]. Quantum-inspired machine learning has been used for unravelling the decoding operation in the non-ideal quantum communication channel [Tomilin and Il'ichov, 2023].

Optimal control for QBs can be applied both to direct charging from the environment and to mediated charging via a charger [Mazzoncini, Cavina et al., 2023]. The comparison of two techniques has been studied for coupled qubits and quantum harmonic oscillators. The optimal control is usually formulated in the form of Pontryagin's Minimum Principles [Mazzoncini, Cavina, et al., 2023], but may also cover the linear feedback [Morrone, Talarico, et al., 2024; Rodríguez, Ahmadi, et al., 2024; Hadipour and Haseli, 2025], homodyne-based feedback [Song, Wang, et al., 2025], exploiting dark states [Rodríguez, Ahmadi, et al., 2024], and so-called 'bang-bang' control (expending external fields with Heaviside step functions) [Mazzoncini, Cavina, et al., 2023]. The noise perturbation from the environment and the effects originated in nonzero environmental temperatures can be covered with the iterative numerical approach for drive optimization [Rodríguez, Ahmadi, et al., 2024].

An interesting example of such a technique uses local and non-local quantum measurement parameters as a control, enhancing charging energy and ergotropy [Du et al., 2025]. This approach is based on introducing new degrees of freedom in the system that originates in the measurement schemes.

1.2 The open-loop approaches

For the *open-loop control* in QBs, we can mention time-dependent classical pulses [Gemme et al., 2024] and Floquet engineering applying periodic driving fields [Du et al., 2025; Song, Wang, et al., 2025] to drive dissipative models. In [Cavaliere, Gemme, et al., 2025], rectangular pulses are applied to the leakage of energy by dynamical blockade.

The choice of particular control methods depends on the specific type of QB, its charging mechanism, and the desired performance characteristics [Shastri, Jiang, et al., 2025].

In [Borisenok, 2020], we already studied an open-loop control approach for the Dicke quantum battery, where the controlled coupling between the cavity and quantum two-level system. The model was formulated for the Tavis–Cummings Hamiltonian. In this paper, we discuss the case of *quantum harmonic oscillator* battery strongly coupled to a highly non-Markovian thermal reservoir via the quantum charger described by the Caldeira–Leggett model [Caldeira and Leggett, 1983]. Such a model has been developed in [Cavaliere, Gemme, et al., 2025] for driving the QB with a set of short-time rectangular pulses.

We follow here a different approach, finding the optimal shape of control for the charging work of the battery, and then restoring the control coupling characteristics of QB in the Hamiltonian.

2 Model for the Reservoir Engineering

Some studies consider the interaction of quantum batteries with the standard thermal reservoir [Elghaayda, Ali, et al., 2025].

Nevertheless, the more advanced technique involving the manipulation by the environment that the quantum battery interacts with to drive its charging dynamics is called a *reservoir engineering*. It presents diverse strategies for optimizing the charging processes of QBs [Ahmadi, Mazurek, Horodecki, et al., 2024; Lu, Tian, et al., 2025].

2.1 Reservoir engineering

One way to induce a directed energy flow from the charger to the battery is introducing nonreciprocity [Ahmadi, Mazurek, Horodecki, et al., 2024; Lu, Tian, et al., 2025] or Floquet engineering [Song, Wang, et al., 2025] to enhance charging efficiency and energy storage. Additionally, it can help in suppressing the self-discharging of QB [Che, Tan, et al., 2025].

Structured reservoir engineering can be used to improve the ergotropy by coupling QBs to a topological photonic waveguide in the long-time limit [Lu, Tian, et al., 2025].

The other alternative approach replaces the direct coherent interaction between a charger and a battery with a dissipative interaction mediated by an engineered shared reservoir to redistribute the energy optimally during the charging process [Ahmadi, Mazurek, Barzanjeh, et al., 2025].

2.2 The Caldeira-Leggett model

To describe the reservoir control for QB, we focus here on the semi-empirical Caldeira–Leggett model for the quantum system coupled to an engineered environment [Caldeira and Leggett, 1983].

The model demonstrates a few advantages: it involves a relatively simple model Hamiltonian, is analytically easily treatable, mimics all basic features of real physical systems, and covers the processes of dissipation [Ferialdi, 2017].

The Hamiltonian of the Caldeira–Leggett model is represented with the three terms [Cavaliere, Gemme, et al., 2025]:

$$\hat{H} = \hat{H}_B + \hat{H}_R + \theta(t)\hat{H}_C \ , \qquad (1)$$

with the components of the battery (B), reservoir (R), and the coupling (C) between B and R, which plays a role of charger. The unitless coupling parameter θ is off at the initial moment: $\theta(0) = 0$, and then it belongs to the interval [0,1].

According to the charging protocol [Cavaliere, Gemme, et al., 2025], the charging time is short, and the coupling must be switched off after that.

2.3 Energy characteristics of QB charging

The charging process described by (1) has two main energy characteristics. The first one is given by:

$$\Delta E_B(t) =< \hat{H}_B(t) > - < \hat{H}_B(0) > , \qquad (2)$$

with $< ... >=$ Tr$[..., \hat{\rho}(0)]$ with $\hat{\rho}(0)$ is the density matrix of the system (1) at the initial moment of time. In (2), $< \hat{H}_B(t) >$ represents the *energy stored in QB* at the moment t to compare with the initial quantity $< \hat{H}_B(0) >$.

The second important characteristic is the *ergotropy*, i.e., the maximum amount of energy that can be extracted from the battery through unitary processes [Koshihara and Yuasa. 2023; Hoang, Metz. et al., 2024]:

$$E(t) =< \hat{H}_B(t) > -\omega_0 \sqrt{\det \sigma_B(t)}. \qquad (3)$$

Here ω_0 is a basic frequency scale for the given QB, and $\sigma_B(t)$ is the covariance matrix; for details see [Cavaliere, Gemme, et al., 2025].

Correspondingly, based on (2) and (3), the performance of QB is represented with two ratios: The energy ratio

$$\eta_B(t) = \frac{E(t)}{< \hat{H}_B(t) >} , \qquad (4)$$

and the work ratio

$$\eta_W(t) = \frac{E(t)}{W(t)} , \qquad (5)$$

where $W(t)$ is the work of charging.

For the high efficiency of QB, both ratios (4) and (5) should be as close to 1 as possible.

2.4 Typical time scales and damping regimes in the model

Apart from the eigenfrequency ω_0, the model (1) possesses two typical inverse time scales: the coupling strength γ_0 and the Drude act-off, i.e. the inverse memory time scale ω_D, here we follow the notations in [Cavaliere, Gemme, et al., 2025]. Based on them, one can define the frequency:

$$\Omega = \sqrt{\gamma_0 \omega_D}. \qquad (6)$$

.

There are two distinct asymptotic regimes in the system, the *underdamped*:

$$\gamma_0 << \omega_0, \omega_D, \qquad (7)$$

and the *overdamped* one:

$$\gamma_0 >> \omega_0, \omega_D. \qquad (8)$$

In this paper, we study the underdamped regime (7), where the dissipation decreases and decoherence slows down as the reservoir damping is increased [Buxton, Russo, et al., 2023]. In such a case, the ergotropy (3) is already extremely close to the optimal one:

$$E(t) \simeq < \hat{H}_B(t) > , \qquad (9)$$

such that for the control of the QB efficiency control, one can focus on (5).

3 Control in the Undamped Regime

We discuss here the open-loop control for the strongly underdamped regime (7) in the model (1). For further representation, we use the energy units with the Boltzmann constant $k = 1$ and the Planck constant $\hbar = 1$.

3.1 Control on the work of charging

In the strong underdamped regime (7) at the *low temperature* $T << \omega_D$, we can present the ergotropy (3) as:

$$< \hat{H}_B(t) > \simeq E(t) \simeq \frac{\Omega^2}{4\omega_0} u(t), \qquad (10)$$

and the charging work as:

$$W(t) = \simeq \frac{\Omega^2}{4\omega_0} \left[1 - e^{-\omega_D t} + u(t)\right] . \qquad (11)$$

Here $u(t)$ is a unitless non-negative control signal, such that $u(0) = 0$. The control is applied at the time scale $1/\omega_D$.

Unlike [Cavaliere, Gemme, et al., 2025], we first engineer the shape of the signal $u(t)$, and then restore the shape of the coupling parameter $\theta(t)$ via the auxiliary functions:

$$\chi(t) = \frac{1}{\Omega} \sqrt{u(t)}, \qquad (12)$$

and

$$\gamma(t) = \gamma_0 e^{-\omega_D t} \theta(t), \qquad (13)$$

which are expressed one with another with their images in the Laplace domain s [Cavaliere, Gemme, et al., 2025]:

$$\tilde{\chi}(s) = \frac{1}{s^2 + \omega_0^2 + s\tilde{\gamma}(s)} . \qquad (14)$$

In the similar manner, at the *high temperature* $T >> \omega_D$ one can get: for the ergotropy:

$$< \hat{H}_B(t) > \simeq E(t) \simeq \qquad (15)$$
$$\simeq \frac{T}{2} \left[u(t) + \left(1 - e^{-\omega_D t}\right)\right] ,$$

and for the charging work:

$$W(t) \simeq \frac{T}{2}\left[u(t) + \frac{\Omega^2}{\omega_0^2}\left(1 - e^{-\frac{2\omega_0^2\omega_D}{\Omega^2}t}\right)\right] . \quad (16)$$

Then we can again apply Eqs (12)-(14).

The coupling in the model is short-time and must be switched off at the scale $1/\omega_D$. The typical scales for the frequencies are: γ_0 belongs to the interval from 10 to 100 s^{-1}, ω_D - from 10 to 200 s^{-1}, and $\omega_0 = \omega_D/3\sqrt{3}$ [Cavaliere, Gemme, et al., 2025]. Let's chose $\gamma_0 = 49$ s^{-1}, and $\omega_D = 100\ s^{-1}$, then by (6) $\Omega = 70\ s^{-1}$.

3.2 Engineering the control signal

The control is done at the time scales $1/\Omega$ for $t \geq 1$, such that:

$$u(t) = \left(e^{\Omega t} - 1\right)^2 . \quad (17)$$

Then by (12):

$$\chi(t) = \frac{1}{\Omega}\left(e^{\Omega t} - 1\right) , \quad (18)$$

with the Laplace image:

$$\tilde{\chi}(s) = \frac{1}{s(s-\Omega)} . \quad (19)$$

Then by (14):

$$\tilde{\gamma}(s) = -\Omega - \frac{\omega_0^2}{s} , \quad (20)$$

with

$$\gamma(t) = -\Omega^2\delta(\Omega t) - \omega_0^2 , \quad (21)$$

and, by (13) the coupling parameter:

$$\theta(t) = -\frac{1}{\gamma_0}\left[\Omega^2\delta(\Omega t) + \omega_0\right]e^{\omega_D t} . \quad (22)$$

The negative sign corresponds to the charging procedure, i.e., transfer of energy from the reservoir to the battery. The control must be switched off at the time scale $1/\omega_D$.

4 The Low and High Temperature Cases

Now we can study the work ratio (5) for different temperature regimes [Cavina and Esposito, 2024; Zhang, Tan, et al., 2024]. The case of medium temperatures, which demands more accurate modification of the damping terms in the corresponding Lindblad equation, is not covered here. see [Diósi, 1993].

4.1 The law temperature case

For the law temperature (LT) case, substituting (10) and (11) to (5), one obtains:

$$\eta_W^{\mathrm{LT}}(t) = \frac{u(t)}{u(t) + 1 - e^{-\omega_D t}} , \quad (23)$$

that by (17) implies:

$$\eta_W^{\mathrm{LT}}(t) = \left[1 + \frac{1 - e^{-\omega_D t}}{\left(e^{\Omega t} - 1\right)^2}\right]^{-1} . \quad (24)$$

Obviously, the function $\eta_W^{\mathrm{LT}}(t)$ tends to 1 exponentially fast at the scale $1/\Omega \simeq 0.014\ s$, while the control time scale is evaluated as $1/\omega_D \simeq 0.01\ s$.

4.2 The high temperature case

In the same manner, for the high temperature (HT) case by (15)-(16):

$$\eta_W^{\mathrm{HT}}(t) = \frac{u(t) + 1 - e^{-\omega_D t}}{u(t) + \frac{\Omega^2}{\omega_0^2}\left(1 - e^{-\frac{2\omega_0^2\omega_D}{\Omega^2}t}\right)} , \quad (25)$$

one gets:

$$\eta_W^{\mathrm{HT}}(t) = \left[1 + \frac{1 - e^{-\omega_D t}}{\left(e^{\Omega t} - 1\right)^2}\right] \times \quad (26)$$
$$\times \left[1 + \frac{\Omega^2}{\omega_0^2}\frac{\left(1 - e^{-\frac{2\omega_0^2\omega_D}{\Omega^2}t}\right)}{\left(e^{\Omega t} - 1\right)^2}\right]^{-1} ,$$

which also tends to 1 exponentially fast.

Thus, for both cases, the low and the high temperature, the control (17) provides the achievability of the goal.

5 Conclusions

The Caldeira–Leggett model demonstrates an extremely rich variety of damping regimes caused by different physical factors [Hagstrom and Morrison, 2011].

In the underdumped regime, our open-loop control approach allows us to formulate a quite efficient way to engineer the coupling between the reservoir and the battery to optimize the charging work.

The optimisation demonstrates an exponential character. The algorithm is simple and does not demand sufficient computational costs.

Nevertheless, we believe that the feedback control can work much more efficiently, as we observed already in other models for quantum batteries [Borisenok, 2020].

6 Discussions

The feedback control in the form of speed gradient [Fradkov, 2007] works well in other quantum systems like qubits [Pechen, Borisenok, Fradkov, 2022; Borisenok and Gogoleva, 2024], and we consider it to be a prominent candidate for the application to Caldeira–Leggett model. The speed gradient algorithm is robust and not very sensitive to the system perturbations, which can be extremely important in the case of a quantum noisy environment. The main problem with this method is related to the violation of the Fradkov–Pogromsky theorem conditions [Fradkov and Pogromsky, 1998], without which the control goal may not be achieved. Sometimes this occurs in models of quantum systems. In any case, for physical systems, it works better than the alternative target attractor feedback [Kolesnikov, 2013]: it often demands too strong energy control, and also creates many computational costs.

The additional noisy effects can be studied in Caldeira–Leggett Hamiltonians, like Gaussian noise [Onofrio and Sundaram, 2022; Christie, Bolhuis, et al., 2024], and the noise time-correlation functions in different regimes [Pelargonio and Zaccone, 2023].

Above all, for the further development of the control models, the influence of external magnetic fields on Caldeira–Leggett systems may be a subject of great research interest and prominent perspectives [Jauffred, Onofrio, et al., 2017; Matevosyan and Allahverdyan, 2023].

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