ROBUST CONTROL OF A TIME-VARYING CHAIN MULTI-AGENT PLANT

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Article history:
Received 05.04.2025, Accepted 05.05.2025

Abstract

A novel robust time-varying chain multi-agent control system is obtained. To solve the problem, control actions are formed in each agent of the network on the basis of the auxiliary loop method taking into account the information about the preceding agent. The signal from the leading subsystem is received only in the first agent of the network. The connection is one-way. The control system in the agent is built using measured data on the outputs of the agent itself and the agent preceding it. Compensation of perturbations in each agent of the network is realized by forming a special signal carrying information about all perturbations acting on the agent, and then its damping by means of an auxiliary loop. The construction of such a control system requires information about the derivatives of the intermediate signals of the system, for which two Halil observers are used, in a special way. To illustrate the performance of the proposed chain network system, we consider a numerical example of control a multi-agent plant consisting of six agents whose dynamic processes are described by nonstationary equations. The agents are subject to the action of uncontrolled external disturbances. Modeling in MATLAB Simulink has been carried out. The simulation results confirmed the theoretical conclusions and showed good performance of the chain system under conditions of uncertainty and nonstationarity of the network agents' models.

Key words

robust control, time-varying, chain, dynamic plant, disturbances, dynamic accuracy, multi-agent plant, auxiliary loop, observer, parametric uncertainty

1 Introduction

A great number of publications are devoted to control methods networks of agents of different nature. When solving network problems in power engineering, neurobiology, production, ecology, it is required to achieve various control objectives: synchronization, desynchronization, consensus, swarming, etc. For example, in [Bobtsov et al., 2024], leaderless consensus problems are considered for networks of fully actuated Euler-Lagrangian agents perturbed by unknown additive disturbances. The network is an undirected weighted graph with time delays. In the paper [Furtat et al., 2014] the problem of robust synchronisation of a network of interconnected agents with a leader is solved, in which each local subsystem of the network is described by a linear differential equation with time-varying parametric and functional uncertainty. The paper [Semenov and Fradkov, 2021] is devoted to the problem of adaptive synchronization in heterogeneous Hindmarsh-Rose neural networks. In [Olfati-Saber, 2006], an algorithm for the swarming behaviour of control agents is proposed. In [Xianwei et al., 2020], consensus of linear multi-agent systems on undirected graphs is investigated. In the paper [Jian et al., 2024], an adaptive method with state observers is applied to achieve consensus in multiple random Euler-Lagrangian mechanical systems. The approach is applied to the control of a real multi-joint robot, where each joint of the robot is treated as a mechanical system. In [Li et al., 2018] the problem of $H\infty$ consensus for multiagent-based supply chain systems under switching topology and uncertain demands. Also models of chain plants are used in the papers [Xia and Li, 2023; Brinkman et al., 2022]. An example of a multi-agent control plant model is a cell model, in which the material flow is represented as a series of connected cells, and each cell assumes that the flow has a perfect mixing structure and there is no mixing between cells.

In the paper [Baytimerova et al., 2008] a mathematical model of a cascade of R-reactors is used to solve the problem of technological optimization of the dimerization process of α-methylstyrene in the presence of $NaHY$ zeolite.

An important problem of multi-agent plant control is the influence of variable parameters of the plant model on the functioning of the control system. This is due to the fact that the parameters of, for example, technological plants and the processes under which they function are not always constant: the quality of the supplied raw materials changes, units wear out, technological equipment becomes obsolete, etc. Besides, technological control plants function in conditions of uncertainty [Polyak et al., 2021; Furtat and Putov, 2013], as well as constantly acting perturbations [Andrievsky and Furtat, 2020a, 2020b; Nikiforov, 2003; Tsykunov, 2009]. Obtaining effective control laws that compensate the influence of time-varying parameters, as well as the effect of controlled and uncontrolled disturbances, is one of the important tasks in the design of control systems for nonstationary plants. Different solutions to the problems of control of unsteady plants have been obtained [Alexandrov, 2023; Pyrkin et al., 2023; Mitrishkin et al., 2022]. Examples of unsteady plants include a tokamak reactor for magnetic control of plasma position, shape and current [Mitrishkin et al., 2022].

In this paper, a novel chain network control structure using the auxiliary loop method is derived to solve the network problem. It is proposed to form control actions in each agent of the network using the auxiliary loop method. In each agent of the network, the output of the preceding agent is monitored, and the signal from the leading subsystem is received only in the first agent of the network. The control systems of each agent are built using measured data on the output of the agent itself and its predecessor. Compensation of the action of disturbances in each agent of the network is realized by forming a special signal carrying information about all disturbances acting on the agent, and then its damping with the help of an auxiliary loop. The construction of such a chain system requires information about the derivatives of the intermediate signals of the system, for which two Halil observers [Atassi and Khalil, 1999] are used, in a special way.

2 Problem statement

Consider a plant model in the form

$$
\begin{array}{l}
Q_1(p,t)y_1(t) = k_1R_1(p,t)u_1(t) + f_1(t),\\
Q_l(p,t)y_l(t) = k_lR_l(p,t)u_l(t) + \overline{N}(p)y_{l-1}(t)+\\
+f_l(t), p^iy_1(0) = y_{1i},\ p^iy_l(0) = y_{li},\\
i = \overline{0,n-1}, l = \overline{2,r},
\end{array}
\tag{1}
$$

where $x \in \mathbb{R}^n$, $y_l(t)$ and $u_l(t)$ are outputs and inputs of agents, $y_l(t) \in R$, $u_l(t) \in R$, $p = d/dt-$ differentiation operator, $Q_l(p,t) = p^n + q_{l1}(t)p^{n-1} + ... + q_{ln}(t), R_l(p,t) = p^m + r_{l1}(t)p^{m-1} + ... + r_{lm}(t)-$ differential operators of orders n and m respectively, deg$\overline{N}_l(p) \leq n$, $f_l(t)-$ external disturbances, coefficients $k_1 > 0$, $k_l > 0$, $y_{1i}, y_{li}-$ known initial conditions.

The leading agent of a chain multi-agent plant is described by the equation

$$
Q_m(p)y_m(t) = k_mg(t), \tag{2}
$$

where $g(t)-$ the setting influence, $k_m > 0$, $y_m(t)-$ the scalar output of the leading agent, deg$Q_m(p) = n - m$.

The proposed control law should ensure fulfillment of the goal

$$
|y_r(t) - y_m(t)| \leq \delta,\ \ t \geq T_0, \tag{3}
$$

$\delta > 0$ is a required accuracy, $T_0-$the time after which from the beginning the target condition must be met for the system to function.

But since the value $y_r(t) - y_m(t)$ is not controlled, the target conditions for each agent in the chain are inequalities

$$
|y_1(t) - y_m(t)| \leq \delta_1, |y_l(t) - y_{l-1}(t)| \leq \delta_l,\ \ l = \overline{2,r}.
$$

It is not difficult to see that in order to ensure condition (3), the sum of δ_1 and δ_l, $l = \overline{2,r}$ must be less than the required accuracy δ.

Assumptions

1. Coefficients $q_{li}(t)$, $r_{lj}(t)$ of operators $Q_l(p,t)$, $R_l(p,t)$ such that $q_{li} = q_{li}0\Delta q_{li}(t)$, $i = \overline{1,n}$; $r_{lj} = r_{lj}0 + \Delta r_{lj}(t), j = \overline{1,m}$, $|\Delta q_{li}(t)| < \gamma_{li}^{'}$, $|\Delta r_{lj}(t)| < \gamma_{lj}^{''}$, $\gamma_{li}^{'}, \gamma_{lj}^{''}$ — some positive numbers, $l = \overline{1,r}$.
2. The values q_{li0}, r_{lj0} and the value k_l depend on the vector of unknown parameters $\xi_l \in \Xi$, where Ξ is a known bounded set of possible values of vector ξ_l, $l = \overline{1,r}$.
3. The external disturbance $f_l(t)$ and the setting influence $g(t)$ are bounded functions, $l = \overline{1,r}$.
4. The polynomials $R_l(\lambda;t)$ are Hurwitz at any fixed t, $\lambda-$ is a complex variable in the Laplace transform, $l = \overline{1,r}$.
5. deg$Q_l(p,t) = n$, deg$R_l(p,t) = m, l = \overline{1,r}$. $Q_m(\lambda)-$ the Hurwitz polynomial.
6. The scalar inputs $u_l(t)$ and outputs $y_l(t)$ of agents are available for measurement in the control system, $l = \overline{1,r}$.

Let represent the operators $Q_l(p,t)$ and $R_l(p,t)$ in the form of sums of stationary and time-varying summands $Q_l(p,t) = Q_{l0}(p) + \Delta Q_l(p,t), R_l(p,t) = R_{l0}(p) + \Delta R_l(p;t)$, where $Q_{l0}(p)$, $R_{l0}(p)$ are differential operators with constant unknown coefficients depending on vectors of unknown parameters $\xi_l \in \Xi$. $\Delta Q_l(p,t)$ and $\Delta R_l(p,t)$ are nonstationary operators whose coefficients are bounded continuous functions of time such that $\Delta Q_l(p,t) = \Delta q_{l1}(t)p^{n-1} + ... + \Delta q_{ln}(t)$, $\Delta R_l(p,t) = \Delta r_{l1}(t)p^{m-1} + ... + \Delta r_{lm}(t)$. $\deg Q_{l0}(p) = n$, $\deg\Delta Q_l(p,\ t) = n-1$, $\deg R_{l0}(p) = m$, $\deg\Delta R_l(p,t) = m-1$.

Then equations (1) will take the form

$$\begin{gathered} Q_{10}(p)y_1(t) = k_1R_{10}(p)u_1 - \Delta Q_1(p,t)y_1 + \\ +k_1\Delta R_1(p,t)u_1 + f_1(t), \\ Q_{l0}(p)y_l(t) = k_lR_{l0}(p)u_l - \Delta Q_l(p,t)y_l + \\ +k_l\Delta R_l(p,t)u_l + \overline{N}_l(p)y_{l-1}(t) + f_l(t), \\ l = \overline{2,r}. \end{gathered} \tag{4}$$

Let us apply the known parameterization [Feuer and Morse, 1978], we obtain

$$\begin{gathered} Q_{1m}(p)y_1(t) = k_1u_1 + \frac{N_{11}(p)}{M_1(p)}u_1(t) + \\ +\frac{N_{12}(p)}{M_1(p)}y_1(t) + \frac{S_1(p)\Delta Q_1(p,t)}{M_1(p)}y_1(t) \\ +\frac{k_1S_1(p)\Delta R_1(p,t)}{M_1(p)}u_1(t) + \frac{S_1(p)}{M_1(p)}f_1(t) + \varepsilon_1(t), \\ Q_{lm}(p)y_l(t) = k_lu_l + \frac{N_{l1}(p)}{M_l(p)}u_l(t) + \\ +\frac{N_{l2}(p)}{M_l(p)}y_l(t) + \frac{S_l(p)\overline{N}_l(p)}{M_l(p)}y_{l-1}(t) - \\ -\frac{S_l(p)\Delta Q_l(p,t)}{M_l(p)}y_l(t) + \frac{k_lS_l(p)\Delta R_l(p,t)}{M_l(p)}u_l(t) + \\ +\frac{S_l(p)}{M_l(p)}f_l(t) + \varepsilon_l(t), \end{gathered} \tag{5}$$

where $M_1(\lambda), M_l(\lambda), S_1(\lambda), S_l(\lambda)-$ Hurwitz polynomials, $\deg M_1(p)= \deg M_l(p) = n-1$, $\deg S_1(p)= = \deg S_l(p) = n-m-1$, $\deg N_{11}(p)= \deg N_{l1}(p) = n-2$, $\deg N_{12}(p)= \deg N_{l2}(p) = n-1$, $\deg \overline{N}_l(p) \le n$, $\varepsilon_1(t), \varepsilon_l(t)-$ exponentially decaying functions determined by initial conditions, $l = \overline{2,r}$. The multi-agent plant consists of identical agents, so $Q_{1m} = Q_{lm} = Q_m$.

Let an equations for the error $e_1(t) = y_1(t) - y_m(t)$, $e_l(t) = y_l(t) - y_{l-1}(t)$:

$$\begin{gathered} Q_m(p)e_1(t) = k_1u_1(t) + \frac{N_{11}(p)}{M_1(p)}u_1(t) + \\ +\frac{N_{12}(p)}{M_1(p)}y_1(t) + \frac{S_1(p)\Delta Q_1(p,t)}{M_1(p)}y_1(t) + \\ +\frac{k_1S_1(p)\Delta R_1(p,t)}{M_1(p)}u_1(t) + \frac{S_1(p)}{M_1(p)}f_1(t) + \\ +\varepsilon_1(t) - k_mg(t), \\ Q_m(p)e_l(t) = k_lu_l(t) + \frac{N_{l1}(p)}{M_l(p)}u_l(t) + \\ +\frac{N_{l2}(p)}{M_l(p)}y_l(t) + \frac{S_l(p)\overline{N}_l(p)}{M_l(p)}y_{l-1}(t) - \\ -\frac{S_l(p)\Delta Q_l(p,t)}{M_l(p)}y_l(t) + \frac{k_lS_l(p)\Delta R_l(p,t)}{M_l(p)}u_l(t) + \\ +\frac{S_l(p)}{M_l(p)}f_l(t) + \varepsilon_l(t) - k_mg(t), \\ l = \overline{2,r}. \end{gathered} \tag{6}$$

Let us write equations (6) in the form

$$\begin{gathered} Q_m(p)e_1(t) = k_1u_1(t) + \psi_1(t), \\ Q_m(p)e_l(t) = k_lu_l(t) + \psi_l(t), \end{gathered} \tag{7}$$

where

$$\begin{gathered} \psi_1(t) = \frac{N_{11}(p)}{M_1(p)}u_1(t) + \frac{N_{12}(p)}{M_1(p)}y_1(t) + \\ +\frac{S_1(p)\Delta Q_1(p,t)}{M_1(p)}y_1(t) + \frac{k_1S_1(p)\Delta R_1(p,t)}{M_1(p)}u_1(t) + \\ +\frac{S_1(p)}{M_1(p)}f_1(t) + \varepsilon_1(t) - k_mg(t), \end{gathered}$$

$$\begin{gathered} \psi_l(t) = \frac{N_{l1}(p)}{M_l(p)}u_l(t) + \frac{N_{l2}(p)}{M_l(p)}y_l(t) + \\ +\frac{S_l(p)\overline{N}_l(p)}{M_l(p)}y_{l-1}(t) - \frac{S_l(p)\Delta Q_l(p,t)}{M_l(p)}y_l(t) + \\ +\frac{k_lS_l(p)\Delta R_l(p,t)}{M_l(p)}u_l(t) + \frac{S_l(p)}{M_l(p)}f_l(t) + \\ +\varepsilon_l(t) - k_mg(t), l = \overline{2,r}. \end{gathered}$$

Let us apply the inverse Laplace transform to equation (7), and represent the obtained equations in vector-matrix form

$$\begin{gathered} \dot{\Delta}_l(t) = A_m\Delta_l(t) + D_0k_lu_l(t) + D_0\psi_l(t), \\ e_l(t) = L\Delta_l(t), \end{gathered} \tag{8}$$

where

$$A_m = \begin{pmatrix} -q_{m1} & I_{n-m-1} \\ . & \\ . & \\ . & \\ -q_{m(n-m)} & 0 \end{pmatrix},$$

$q_{m1}, q_{m2}, ..., q_{m(n-m)}-$ coefficients of the polynomial Q_m from assumption 5, $I_{n-m-1}-$ unit matrices of the corresponding dimensions. The differential operator $T_l(\lambda)$ is such that the condition is satisfied

$$T_l(\lambda)/Q_{lm}(\lambda) = 1/(\lambda + a_{lm}),\ a_{lm} > 0.$$

Then equations (7) can be transformed, resulting in

$$\begin{gathered} (p + a_{1m})e_1(t) = \beta_1v_1(t) + \varphi_1(t), \\ (p + a_{lm})e_l(t) = \beta_lv_l(t) + \varphi_l(t), \end{gathered} \tag{9}$$

where

$$\begin{gathered} \varphi_1(t) = \frac{N_{11}(p)}{T_1(p)M_l(p)}u_l(t) + \frac{N_{12}(p)}{T_1(p)M_1(p)}y_1(t) - \\ -\frac{S_1(p)\Delta Q_1(p,t)}{T_1(p)M_1(p)}y_1(t) + \frac{k_1S_1(p)\Delta R_1(p,t)}{T_1(p)M_1(p)}u_1(t) + \\ +\frac{S_1(p)}{T_1(p)M_1(p)}f_1(t) + \frac{1}{T_1(p)}(\varepsilon_1(t) - k_mg(t)) + \\ +(k_1\alpha_1 - \beta_1)v_1(t), \\ \varphi_l(t) = \frac{N_{l1}(p)}{T_l(p)M_l(p)}u_l(t) + \frac{N_{l2}(p)}{T_l(p)M_l(p)}y_l(t) + \\ +\frac{S_l(p)\overline{N}_l(p)}{T_l(p)M_l(p)}y_{l-1}(t) - \frac{S_l(p)\Delta Q_l(p,t)}{T_l(p)M_l(p)}y_l(t) + \\ +\frac{k_lS_l(p)\Delta R_l(p,t)}{T_l(p)M_l(p)}u_l(t) + \frac{S_l(p)}{T_l(p)M_l(p)}f_l(t) + \\ +\frac{1}{T_l(p)}(\varepsilon_l(t) - k_mg(t)) + (k_l\alpha_l - \beta_l)v_l(t), \\ l = \overline{2,r} \end{gathered}$$

The functions $\varphi_1(t),\ \varphi_l(t),\ l = \overline{2,r}$ describe the signals that carry information about uncertainty and time-varying parameters of the model, the outputs of previous agents, and external uncontrolled disturbances of the multi-agent plant. We will build control system so as to compensate the negative influence of these signals on the whole plant. For this purpose, we will introduce in each agent auxiliary loop

$$(p + a_{lm})\overline{e}_l(t) = \beta_l v_l(t), \quad (10)$$

and get the equations for the mismatches $\zeta_l(t) =_l (t) - _l(t)$

$$(p + a_{lm})\zeta_l(t) = \varphi_l(t), l = \overline{1,r}. \quad (11)$$

If we form the control action $v_l(t)$ in the form of

$$v_l(t) = -\frac{1}{\beta_l}(p+a_{lm})\zeta_l(t) = -\frac{1}{\beta_l}\varphi_l(t), l = \overline{1,r}, \quad (12)$$

then from (10) we obtain

$$(p + a_{lm})e_l = 0, l = \overline{1,r}. \quad (13)$$

It follows from (14) that $\lim_{t\to\infty} e_l(t) = 0, l = \overline{1,r}$. Let us prove the boundedness of all signals of the designed system. Substituting $\varphi_l(t)$ into (13) , we obtain

$$\begin{array}{l} v_l(t) = -\frac{1}{\beta_l}(k_l\alpha_l - \beta_l)v_l(t) - \\ -\frac{N_{l1}(p)}{\beta_l T_l(p) M_l(p)}u_l(t) - \frac{\varphi_{l1}(t)}{\beta_l}, \end{array} \quad (14)$$

where

$$\begin{array}{c} \varphi_{11}(t) = \frac{N_{12}(p)}{T_1(p)M_1(p)}y_1 - \frac{S_1(p)\Delta Q}{T_1(p)M_1(p)}y_1(t) + \\ +\frac{k_1 S_1(p)\Delta R_1(p,t)}{T_1(p)M_1(p)}u_1(t) + \frac{S_1(p)}{T_1(p)M_1(p)}f_1(t) + \\ +\frac{1}{T_1(p)}(\varepsilon_1(t) - k_m g(t)). \\ \varphi_{l1}(t) = \frac{N_{l2}(p)}{T_l(p)M_l(p)}y_l + \frac{S_l(p)\overline{N_l}(p)}{T_l(p)M_l(p)}y_{l-1}(t) - \\ -\frac{S_l(p)\Delta Q_l(p,t)}{T_l(p)M_l(p)}y_l(t) + \frac{k_l S_l(p)\Delta R_l(p,t)}{T_l(p)M_l(p)}u_l(t) + \\ +\frac{S_l(p)}{T_l(p)M_l(p)}f_l(t) + \frac{1}{T_l(p)}(\varepsilon_l(t) - k_m g(t)),\ l = \overline{2,r}. \end{array}$$

Let us express from equation (15) the variable $v_l(t)$, and substitute the obtained expression into (9)

$$\begin{array}{l} u_l(t) = -\frac{1}{k_l}(\frac{N_{l1}(p)}{M_l(p)}u_l(t) + \frac{N_{l2}(p)}{M_l(p)}y_l(t) + \\ +\frac{S_l(p)\overline{N_l}(p)}{T_l(p)M_l(p)}y_{l-1}(t) - \frac{S_l(p)\Delta Q_l(p,t)}{M_l(p)}y_l(t) + \\ +\frac{k_l S_l(p)\Delta R_l(p,t)}{M_l(p)}u_l(t) + \frac{S_l(p)}{M_l(p)}f_l(t) + \varepsilon_l(t) - \\ -k_m g(t)),\ l = \overline{1,r}. \end{array} \quad (15)$$

Let us substitute (16) into (6), as a result we obtain

$$Q_m(p)e_l(t) = 0, \quad (16)$$

whence follows the boundedness not only of the quantities $e_l(t)$, but also of $n-m$ their derivatives, and, hence, of the variables $y_l(t)$ and their derivatives by virtue of assumptions 3 and 5. Let us represent equation (16) in the following form

$$\begin{array}{l} (k_l M_l(p) + N_{l1}(p) + k_l S_l(p)\Delta R_l(p;t)u_l(t) = \\ = -(N_{l2}(p)y_l(t) + \overline{N}(p)y_{l-1}(t) - \\ -S_l(p)\Delta Q_l(p;t)y_l(t) + S_l(p)f_l(t) + \\ +M_l(p)\varepsilon_l(t) - M_l(p)k_m g(t)). \end{array} \quad (17)$$

Let

$$k_l M_l(p) + N_{l1}(p) + k_l S_l(p)\Delta R_l(p;t) = k_l S_l(p) R_l(p;t),$$

where the polynomials $S_l(\lambda)-$ are Hurwitzian, and $R_l(p;t)-$ are stable by assumption 4. Furthermore, $y_l(t),\ f_l(t),\ \varepsilon_l(t),\ g(t)-$ bounded functions, hence $u_l(t)-$ bounded functions. It follows which follows from the boundedness of the signals $\varphi_l(t),\ l = \overline{1,r}$. Then from the expression (12) follows boundedness of the variables $\zeta_l(t)$ and their derivatives.

$$u_l(t) = \overline{T_l}\xi_l(t), l = \overline{1,r} \quad (18)$$

where $\overline{T_l} = [s_{l0}, s_{l1}, ..., s_{l(n-m-1)}]$, $s_{l0}, s_{l1}, s_{l1}, ..., s_{l(n-m-1)}-$ coefficients of the polynomials $\overline{T_l}(\lambda)$, $\xi_l(t)-$ state vectors obtained from observers [Atassi et al., 1999], which are represented as

$$\begin{array}{l} \dot{\xi}_l = F_{l0}\xi_l(t) + B_{l0}(v_l(t) - \overline{v}_l(t)), \\ \overline{v}_l(t) = L\xi_l(t),\ l = \overline{1,r}. \end{array} \quad (19)$$

Here $\xi_l(t) \in R^{n-m}$, $F_{l0}-$ matrix in Frobenius form with zero lower row, $L = [1, 0, ..., 0]$, $B_{l0}^{\mathrm{T}} = \left[\frac{b_{l1}}{\mu_l}, ..., \frac{b_{l(n-m)}}{\mu_l^{n-m}}\right]$. The parameters $b_{l1}, ..., b_{l(n-m)}$ are chosen so that the matrices $F_l = F_{l0} + B_l L$ are Hurwitzian, $B_l^{\mathrm{T}} = [b_{l1}, ..., b_{l(n-m)}]$.

Let us substitute (19) into (6), and by choosing the polynomials $T_l(\lambda),\ l = \overline{1,r}$, so that the transfer functions of the functions satisfy the condition

$$(p + a_{lm})e_l(t) = \beta_l v_l(t) + \overline{\varphi_l}(t), l = \overline{1,r}, \quad (20)$$

where $\overline{\varphi}_l(t) = \varphi_l(t) + \beta_l(v_l(t) - \overline{v}_l(t))$.

The functions $\overline{\varphi}_l(t),\ l = \overline{1,r}$ describe information about the uncertainties of the parameters of the models control agents, external uncontrolled disturbances, errors in the estimates of variables $v_l(t)$ and their $n - m - 1$ derivatives.

The system is built under assumption 6, so we will form the intermediate signals $v_l(t)$ in the form of

$$v_l(t) = -\frac{1}{\beta_l}(p + a_{lm})\overline{\zeta}_l(t), l = \overline{1,r}, \quad (21)$$

where $\overline{\zeta}_l(t)-$ is the estimate obtained from the observer (20) in the form of

$$\begin{array}{l} \dot{z}_l = \overline{F}_{l0}z_l(t) + \overline{B}_{l0}(\zeta_l(t) - \overline{\zeta}_l(t)), \\ \overline{\zeta}_l(t) = L_2 z_l(t), \end{array} \quad (22)$$

where $z_l(t) \in R^2$, $\overline{F}_{l0} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\overline{B}_{l0} = \left[\frac{d_{l1}}{\mu_l}, \frac{d_{l2}}{\mu_l^2} \right]$, $L_2 = [1, 0]$. The parameters d_{l1}, d_{l2} are chosen similarly as in observers (19), $l = \overline{1, r}$.

Let us introduce the composite vectors

$$\begin{array}{l} \Delta(t) = col(\Delta_1, \ldots, \Delta_r),\ k = col(k_1, ..., k_r), \\ u = col(u_1, \ldots, u_r),\ \psi = col(\psi_1, \ldots, \psi_r), \\ e = col(e_1, \ldots, e_r),\ \overline{e} = col(\overline{e}_1, \ldots, \overline{e}_r), \\ v = col(v_1, \ldots, v_r),\ \overline{v} = col(\overline{v}_1, \ldots, \overline{v}_r), \\ \zeta = col(\zeta_1, \ldots, \zeta_r),\ \overline{\zeta} = col(\overline{\zeta}_1, \ldots, \overline{\zeta}_r), \\ \varphi = col(\varphi_1, \ldots, \varphi_r),\ \overline{\varphi} = col(\overline{\varphi}_1, \ldots, \overline{\varphi}_r), \\ \xi = col(\xi_1, \ldots, \xi_r),\ z = col(z_1, \ldots, z_r) \end{array}$$

and block-diagonal matrices

$A_m = \text{diag}\{A_{m1}, \ldots, A_{\text{mr}}\}$, $D = \text{diag}\{D_0, \ldots, D_0\}$, $\beta = \text{diag}\ \{\beta_1, \ldots, \beta_r\}$, $L_0 = \text{diag}\ \{L, \ldots, L\}$, $\alpha = \text{diag}\ \{\alpha_1, \ldots, \alpha_r\}$, $F_0 = \text{diag}\ \{F_{10}, \ldots, F_{r0}\}$, $B_0 = \text{diag}\ \{B_{10}, \ldots, B_{r0}\}$, $\overline{T} = \text{diag}\ \{\overline{T_1}, \ldots, \overline{T_r}\}$. We transform equations (9),(11),(13),(19),(20),(22), (23) into vector-matrix equations

$$\begin{array}{l} \dot{\Delta}(t) = A_m \Delta(t) + D_0 k u(t) + D_0 \psi(t), \\ e(t) = L\Delta(t), \end{array} \tag{23}$$

$$\overline{e} + a_m \dot{\overline{e}} = \beta v(t), \tag{24}$$

$$\beta v(t) = -(\dot{\zeta}_l(t) + a_m \zeta(t)), \tag{25}$$

$$u_l(t) = \overline{T_l} \xi_l(t), l = \overline{1, r}, \tag{26}$$

$$\begin{array}{l} \dot{\xi} = F_0 \xi(t) + B_0(v(t) - \overline{v}(t)), \\ \overline{v}(t) = L\xi(t), \end{array} \tag{27}$$

$$\beta v(t) = -(\dot{\overline{\zeta}} + a_m \overline{\zeta}_l(t)), \tag{28}$$

where $\overline{\zeta}_l(t)-$ estimates obtained from observer

$$\begin{array}{l} \dot{z} = \overline{F}_0 z(t) + \overline{B}_0(\zeta(t) - \overline{\zeta}(t)), \\ \overline{\zeta}(t) = L_2 z(t), \end{array} \tag{29}$$

Theorem 1. Let the conditions of assumptions 1-6 hold. Then for any $\delta > 0$ in (3) there are numbers $\mu > 0$, $T > 0$ such that for $\mu \le \mu_0$ and $t \ge T$ the system (24)-(29) the target condition (3) is satisfied and all variables in the system are bounded.

Proof. Let us introduce two vectors

$$\sigma^T(t) = \left(v(t), \dot{v}(t), ..., v^{(n-m-1)}(t)\right), z_0^T = (\zeta(t), \dot{\zeta}(t))$$

and normalized mismatch vectors

$$\overline{\eta}(t) = \Gamma_1^{-1}(\sigma(t) - \xi(t)),\ \overline{w}(t) = \Gamma_2^{-1}(z_0(t) - z(t)),$$

where $\Gamma_1 = \text{diag}\{\mu^{n-m-1}, ..., \mu, 1\}$, $\Gamma_2 = \text{diag}\{\mu, 1\}$. Then from (27) and (29) we have

$$\begin{array}{l} \dot{\overline{\eta}}(t) = \frac{1}{\mu} F \overline{\eta} - b_0 v^{(n-m)}(t),\ \theta(t) = \mu^{n-m-1} L \overline{\eta}(t), \\ \dot{\overline{w}}(t) = \frac{1}{\mu} \overline{F}\,\overline{w}(t) - \overline{b}_0 \ddot{\zeta}(t), \tau(t) = \mu L_2 \overline{w}(t), \end{array} \tag{30}$$

where $\overline{F} = \overline{F_0} + \overline{B_0} L_2$, $b_0^T = [0, ..., 1]$, $\overline{b_0}^T = [0, 1]$, $\theta(t) = v(t) - \overline{v}(t)$, $\tau(t) = \zeta(t) - \overline{\zeta}(t)$. Let transform equations (17) into equivalent equations with respect to the outputs $\theta(t)$ and $\tau(t)$

$$\begin{array}{l} \dot{\eta}(t) = \frac{1}{\mu} F \eta(t) - b \dot{v}(t),\ \theta(t) = \mu^{n-m-1} L \eta(t), \\ \dot{w}(t) = \frac{1}{\mu} \overline{F} w(t) - \overline{b} \dot{\zeta}(t),\ \tau(t) = \mu L_2 w(t), \end{array} \tag{31}$$

where $b^T = [1, 0, ..., 0]$, $\overline{b}^T = [1, 0]$.

Equations (17) and (18) are equivalent with respect to the outputs $\theta(t)$ and $\tau(t)$, since they are vector-matrix forms of the same equations

$$\begin{array}{l} \theta^{(n-m)}(t) + \frac{b_1}{\mu} \theta^{(n-m-1)}(t) + ... + \frac{b_{n-m}}{\mu^{n-m}} \theta(t) = \\ = v^{(n-m)}(t), \\ \ddot{\tau}(t) + \frac{d_1}{\mu} \dot{\tau}(t) + \frac{d_2}{\mu_2} \tau(t) = \ddot{\zeta}(t). \end{array}$$

Taking into account (9) and (14), equation (17) takes the form

$$(p + a_m) e(t) = -\mu(p + a_m) L_2 w(t). \tag{32}$$

From where we have $y(t) = -\mu L_2 w(t)$.

Let take Lyapunov function in the form

$$V(t) = \eta^T(t) H \eta(t) + w^T(t) H_1 w(t), \tag{33}$$

where the positive-definite matrices H and H_1 are solutions of the equations

$$HF + F^T H = -2\rho_1 I,\ \ H_1 \overline{F} + \overline{F}^T H_1 = -2\rho_2 I.$$

Calculating the derivative of $V(t)$ along the trajectories of the closed-loop system (17), one gets

$$\begin{array}{l} \dot{V}(t) = -2\frac{\rho_1}{\mu} |\eta(t)|^2 - 2\frac{\rho_2}{\mu} |w(t)|^2 - \\ -2\eta^T(t) H \dot{v}(t) - 2w^T(t) H_1 \dot{\zeta}(t). \end{array} \tag{34}$$

Let us rewrite the equations (17) in the form

$$\begin{array}{l} \mu_1 \dot{\eta}(t) = F \eta(t) - \mu_2 b \dot{v}(t), \\ \mu_1 \dot{w}(t) = \overline{F} w(t) - \mu_2 \overline{b} \dot{\zeta} \\ e = -\mu_2 L_2 w(t). \end{array} \tag{35}$$

Let us use the lemma [Brusin, 1995].

Lemma [Brusin, 1995]. If the system is described by the equations $\dot{x} = f(x, \mu_1, \mu_2)$, where $x \in \mathbb{R}^n$, $f(x, \mu_1, \mu_2)$ is a continuous Lipschitz function with respect to x and for $\mu_2 = 0$ has a bounded closed domain of dissipativity $\Omega = \{x : F(x) < C\}$, where $F(x)$ is positive-definite, a continuous, piecewise smooth function, then there exists $\mu_0 > 0$ such that for $\mu_1 < \mu_0$ and $\mu_2 < \mu_0$ the original system has the same Ω dissipativity domain if for some numbers $C,\ \overline{\mu}_1,\ \ \mu_2 = 0$ the condition is met

$$\sup_{|\mu_1|\le\overline{\mu}_1}\left(\left(\frac{\partial F(x)}{\partial x}\right)^T f(x,\mu_1,0)\right) \le -C,\ F(x) = \overline{C}.$$

In this case, if $\mu_2 = 0$ in (22), this is equivalent to the fact that all derivatives are measured and two exponentially stable systems are added $\mu_1\dot{\eta}(t) = F\eta(t), \dot{w}(t) = \overline{F}w(t)$. As already proved, in this case all the variables in the system are limited and the conditions of the lemma are fulfilled. In other words, in the area of Ω $e(t)\to 0,\ \ |v(t)| < k_1,\ \ |\zeta| < k_2$, and from (13) and (6) it follows that $|\dot{v}| < k_3, |\dot{\zeta}| < k_4$, where $k_1,\ k_2,\ k_3,\ k_4$ are some positive constants.

Let put $\mu_1 = \mu_2 = \mu$ in (22) and, substituting them in (21), use the estimates

$$\begin{array}{c}-2\eta^T(t)H\dot{v}(t) \le \frac{1}{\mu}|\eta(t)|^2+\\ +\mu\|H\|^2|\dot{v}(t)|^2 \le \frac{1}{\mu}|\eta(t)|^2 + \mu\|H\|^2k_3^2,\\ -2w^T(t)H_1\dot{\zeta}(t) \le \frac{1}{\mu}|w(t)|^2 + \mu\|H_1\|^2k_4^2.\end{array} \quad (36)$$

Substituting these estimates in (21), we obtain the inequality

$$\dot{V}(t) \le -\frac{\rho_1}{\mu}|\eta(t)|^2 - \frac{\rho_2}{\mu}|w(t)|^2 - \frac{1}{\mu}(\rho_1-1)|\eta(t)|^2 - -\frac{1}{\mu}(\rho_2-1)|w(t)|^2 + \mu(\|H\|^2k_3^2 + \|H_1\|^2k_4^2).$$

By selecting $\rho_1 > 1$ and $\rho_2 > 1$ one gets

$$\dot{V}(t) \le -\frac{\rho_1}{\mu}|\eta(t)|^2 - \frac{\rho_2}{\mu}|w(t)|^2 + \mu\beta, \quad (37)$$

where $\beta = \|H\|^2k_3^2 + \|H_1\|^2k_4^2$. From where it follows

$$\dot{V}(t) \le -\beta_1 V(t) + \mu\beta, \quad (38)$$

where $\beta_1 = \min\left\{\frac{\rho_1}{\mu\overline{\lambda}(H)}; \frac{\rho_2}{\mu\overline{\lambda}(H_1)}\right\}$, $\overline{\lambda}(\cdot)$ is the maximum eigenvalue of the corresponding matrix. From (25) we have $V(t) \le \frac{\mu\beta}{\beta_1}$.

Taking into account the inequality $|w(t)|^2 \le \frac{1}{\underline{\lambda}(H_1)}V(t) \le \frac{\mu\beta}{\beta_1}$ from the third equation (22) we have $|e(t)| = \mu|w(t)| \le \mu\sqrt{\frac{\mu\beta}{\beta_1}}$. From where it can be seen that for any $\delta > 0$ in (3) there exists μ_0 such that the target condition (3) will be fulfilled.

Let us introduce the matrix

$$\Psi = \begin{pmatrix} -\frac{2\rho_1}{\mu}I + \beta_1 H & O & -H & O\\ * & -\frac{2\rho_2}{\mu}I + \beta_1 H_1 & O & -H_1\\ * & * & O & O\\ * & * & * & O\end{pmatrix}. \quad (39)$$

Theorem 2. Consider the control system (23)-(29). Suppose that for given numbers $\beta > 0, \mu > 0$ there exist coefficients $\rho_1 > 1, \rho_2 > 1$ and matrices $H,\ H_1$ such that the linear matrix inequality

$$\begin{pmatrix} -\frac{2\rho_1}{\mu}I + \beta_1 H & O & -H & O\\ * & -\frac{2\rho_2}{\mu}I + \beta_1 H_1 & O & -H_1\\ * & * & O & O\\ * & * & * & O\end{pmatrix} \le 0$$

are valid. Then the closed-loop system is stable and the target condition (3) is satisfied, where $\delta = \mu\sqrt{\frac{\mu\beta}{\beta_1}}, \beta = \|H\|^2k_3^2 + \|H_1\|^2k_4^2$, $\beta_1 = \min\left\{\frac{\rho_1}{\mu\overline{\lambda}(H)}; \frac{\rho_2}{\mu\overline{\lambda}(H_1)}\right\}$.

Proof. Let us find the conditions under which the inequality (38) is satisfied

$$\dot{V}(t) + \beta_1 V(t) - \mu\beta \le 0. \quad (40)$$

Let us substitute the Lyapunov function (34) and its derivative in (40)

$$\begin{array}{c}-2\frac{\rho_1}{\mu}|\eta(t)|^2 - 2\frac{\rho_2}{\mu}|w(t)|^2 - 2\eta^T(t)H\dot{v}(t)-\\ -2w^T(t)H_1\dot{\zeta}(t) + \beta_1\eta^T(t)H\eta(t)+\\ +\beta_1 w^T(t)H_1 w(t) - \mu\beta \le 0.\end{array} \quad (41)$$

Since the value $\beta = \|H\|^2k_3^2 + \|H_1\|^2k_4^2$ is positive, $\mu > 0$ from Theorem 1 the inequality (41) will be satisfied when the next inequality is fulfilled

$$\begin{array}{c}-2\frac{\rho_1}{\mu}|\eta(t)|^2 - 2\frac{\rho_2}{\mu}|w(t)|^2 - 2\eta^T(t)H\dot{v}(t)-\\ -2w^T(t)H_1\dot{\zeta}(t) + \beta_1\eta^T(t)H\eta(t)+\\ +\beta_1 w^T(t)H_1 w(t) \le 0.\end{array} \quad (42)$$

Introduce the vector

$$z = col(\eta(t), w(t), \dot{v}(t), \dot{\zeta}(t)).$$

Then condition (42) can be written in the form

$$z^T\Psi z \le 0, \quad (43)$$

or in matrix form

$$(\eta^T\ w^T\ \dot{v}^T\ \dot{\zeta}^T)\begin{pmatrix}\psi_{11} & O & -H & O\\ * & \psi_{22} & O & -H_1\\ * & * & O & O\\ * & * & * & O\end{pmatrix}\begin{pmatrix}\eta\\ w\\ \dot{v}\\ \dot{\zeta}\end{pmatrix} \le 0, \quad (44)$$

$\psi_{11} = -\frac{2\rho_1}{\mu}I + \beta_1 H, \psi_{22} = -\frac{2\rho_2}{\mu}I + \beta_1 H_1$.

From inequality (44) follows the inequality

$$\begin{pmatrix} -\frac{2\rho_1}{\mu}I + \beta_1 H & O & -H & O\\ * & -\frac{2\rho_2}{\mu}I + \beta_1 H_1 & O & -H_1\\ * & * & O & O\\ * & * & * & O\end{pmatrix} \le 0, \quad (45)$$

where for any dynamic accuracy $\delta = \mu\sqrt{\frac{\mu\beta}{\beta_1}}$ from Theorem 1, we can find the values of the controller parameters μ and β, satisfying the following conditions $\beta_1 = \min\left\{\frac{\rho_1}{\mu\overline{\lambda}(H)}; \frac{\rho_2}{\mu\overline{\lambda}(H_1)}\right\}, \beta = \|H\|^2k_3^2 + \|H_1\|^2k_4^2$.

3 Example

Consider a homogeneous network consisting of six time-varying agents given by the following systems of differential equations with variable coefficients

$$\begin{cases} \dot{x}_{11} = x_{12}, \\ \dot{x}_{12} = x_{13}, \\ \dot{x}_{13} = x_{14} + c_{10}u_1 \\ \dot{x}_{14} = -q_{14}(t)x_{11} - q_{13}(t)x_{12} - q_{12}(t)x_{13} - \\ -q_{11}(t)x_{14} + c_1(t)u_1 + f_1(t), \\ y_1 = x_{11} \end{cases}$$

$$\begin{cases} \dot{x}_{l1} = x_{l2} + n_{l1}y_{l-1}, \\ \dot{x}_{l2} = x_{l3} + n_{l2}y_{l-1}, \\ \dot{x}_{l3} = x_{l4} + c_{l0}u_l + n_{l3}y_{l-1}, \\ \dot{x}_{l4} = -q_{l4}(t)x_{l1} - q_{l3}(t)x_{l2} - \\ -q_{l2}(t)x_{l3} - q_{l1}(t)x_{l4} + \\ +c_l(t)u_l + +n_{l4}y_{l-1} + f_l(t), \\ y_l = x_{l1}, l = \overline{2,6}. \end{cases}$$

Let us move from canonical forms of describing agents to operator forms of description. Then the equations in operator forms will take the form (1)

$$(p^4 + q_{11}(t)p^3 + q_{12}(t)p^2 + q_{13}(t)p + q_{14}(t))y_1(t) = \\ = (r_{10}p + r_{11}(t))u_1(t) + f_1(t),$$

$$(p^4 + q_{l1}(t)p^3 + q_{l2}(t)p^2 + q_{l3}(t)p + q_{l4}(t))y_l(t) = \\ = (r_{l0}p + r_{l1}(t))u_l(t) + \overline{N}_l(p)y_{l-1}(t) + f_l(t), \;\; l = \overline{2,6},$$

where the coefficients
$r_{10} = c_{10}, r_{l0} = c_{l0}, r_{11}(t) = q_{11}(t)c_{10} + + c_1(t), r_{l1}(t) = q_{l1}(t)c_{l0} + c_l(t), \overline{N}_l(p) = n_{l1}p^3 + + n_{l2}p^2 + n_{l3}p + n_{l4}, \; l = \overline{2,6}.$

Representing the coefficients $q_{li}(t), r_{l1}(t), \; l = \overline{1,6}$ as sums of the stationary and nonstationary components $q_{li}(t) = q_{li0}+\Delta q_{li}(t), i = \overline{1,4}, r_{l1}(t) = r_{l10}+\Delta r_{l1}(t)$, $l = \overline{1,6}$, we obtain equation (4):

$$(p^4 + q_{110}p^3 + q_{120}p^2 + q_{130}p + q_{140})y_1(t) = (r_{10}p+ \\ +r_{110})u_1(t) - (\Delta q_{11}(t)p^3 + \Delta q_{12}(t)p^2 + \Delta q_{13}(t)p+ \\ +\Delta q_{14}(t))y_1(t) + \Delta r_{11}(t)u_1(t) + f_1(t),$$

$$(p^4 + q_{l10}p^3 + q_{l20}p^2 + q_{l30}p + q_{l40})y_l(t) = (r_{l0}p+ \\ +r_{l10})u_l(t) - (\Delta q_{l1}(t)p^3 + \Delta q_{l2}(t)p^2 + \Delta q_{l3}(t)p+ \\ +\Delta q_{l4}(t))y_l(t) + \Delta r_{l1}(t)u_l(t) + \overline{N}(p)y_{l-1}(t)+ \\ +f_l(t), l = \overline{2,6}.$$

Note that $r_{l0} = k_l, \; l = \overline{1,6}$.

The reference model equation: $(p + 3)^3y_m(t) = 10r(t)$.

Suppose that we know the set Ξ of possible values of the parameters of the agents' models:
$-4 \le q_{li0} \le 4, -6 \le \Delta q_{li}(t) \le 6, i = \overline{1,4}$,
$1 \le r_{l0} \le 20, \; -7 \le \Delta r_{l1}(t) \le 25, 4 \le r_{l10} \le 15$,
$l = \overline{1,6}$. The external disturbances in each agent in the multi-agent plant are not controlled, under assumption 3 satisfy the condition $|f_l(t)| < 10, l = \overline{1,6}$.

We choose polynomials in each agent

$$T_l(\lambda) = (\lambda + 3)^2.$$

Let us introduce the auxiliary loop as

$$(p + 3)\overline{e}_l(t) = 20v_l(t), \beta_l = 20, \; l = \overline{1,6}$$

then equations of observers (19), (22) will take the form

$$\begin{cases} \dot{\xi}_{l1}(t) = \xi_{l2}(t) + \frac{6}{\mu_l}(v_l(t) - \xi_{l1}(t)), \\ \dot{\xi}_{l2}(t) = \frac{8}{\mu_l^2}(v_l(t) - \xi_{l1}(t)), \\ \overline{v}_l(t) = \xi_{l1}(t), l = \overline{1,6}. \end{cases}$$

$$\begin{cases} \dot{z}_{l1}(t) = \frac{3}{\mu_l}(\zeta_l(t) - z_{l1}(t)), \\ \overline{\zeta}_l(t) = z_{l1}(t), l = \overline{1,6}. \end{cases}$$

The control law is introduced in the form

$$u_l(t) = \xi_{l1}(t) + 6\xi_{l2}(t) + 9\dot{\xi}_{l2}(t), \\ v_l(t) = -\frac{1}{20}(3z_{l1}(t) + \dot{z}_{l1}(t)), l = \overline{1,6}.$$

The chain system was simulated with the following values of the coefficients of the agents' model equations: $q_{1i0} = 3, \; q_{2i0} = 4, \; q_{3i0} = 2, \; q_{4i0} = -1, \; i = \overline{1,4}, \; \Delta q_{l1}(t) = 3\cos 4t, \; \Delta q_{l2}(t) = 5\cos 4t, \Delta q_{l3}(t) = 3\sin t, \; \Delta q_{l4}(t) = \sin 2t, \;\; c_{l1}(t) = 5 + \sin 5t, \; c_{l0} = r_{l0} = k_l = 4, \; r_{l10} = 3, \Delta r_{l1}(t) = \sin t, \; l = \overline{1,6}$. $n_{12} = n_{21} = n_{31} = n_{41} = 1, \;\; n_{13} = n_{23} = n_{32} = n_{42} = 2, \; n_{14} = n_{24} = n_{34} = = n_{43} = 3$. The setting effect in the leader equation agent is $g(t) = 1 + \sin 3t, \; k_m = 10. \; f_1(t) = 9\sin 1.7t, f_2(t) = 8,5\sin 3t, \; f_3(t) = 7\sin 5t, \; f_4(t) = 2\sin t, f_5(t) = 3\sin t.f_6(t) = 5\sin 2t$. Regulator parameters: $\mu_l = 0.01, \; \beta_l = 20, \mathrm{a_{lm}} = 3, \; l = \overline{1,6}$. The initial conditions are zero.

The solution of inequality (45) are matrices
$H = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}, H_1 = 0.01, \rho_1 = \rho_2 = 4.$

Figures 1–6 show transients of the tracking errors.

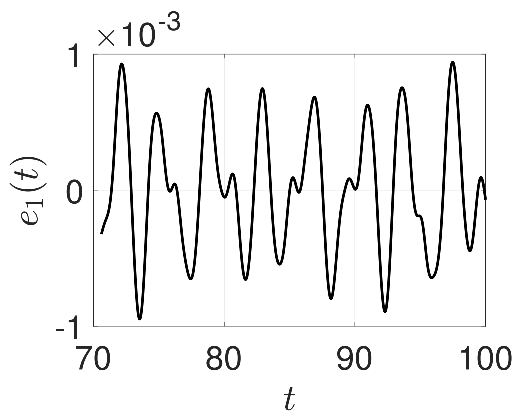

Figure 1. The transients of the tracking error $e_1(t)$.

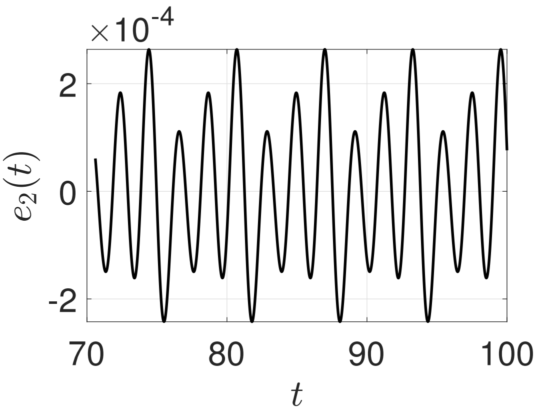

Figure 2. The transients of the tracking error $e_2(t)$.

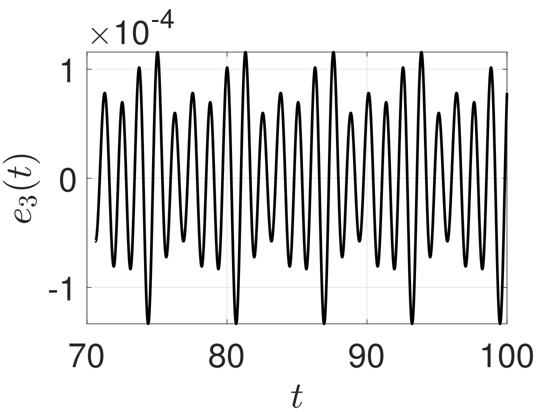

Figure 3. The transients of the tracking error $e_3(t)$.

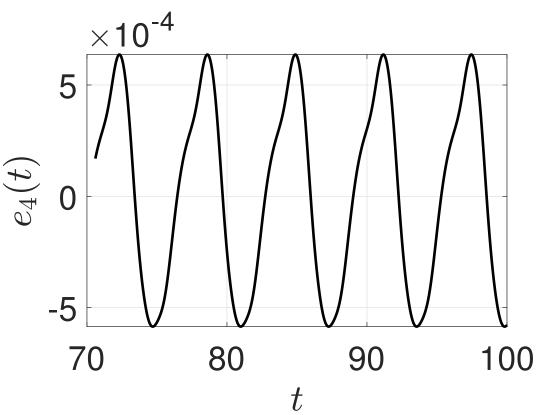

Figure 4. The transients of the tracking error $e_4(t)$.

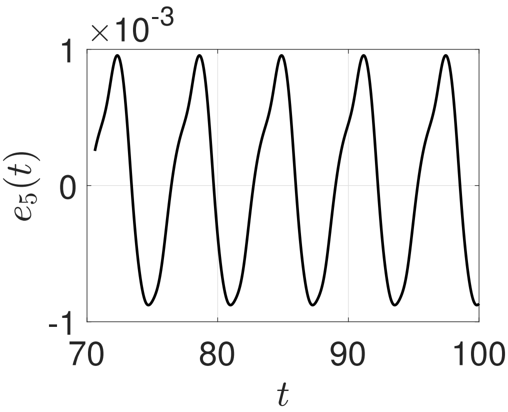

Figure 5. The transients of the tracking error $e_5(t)$.

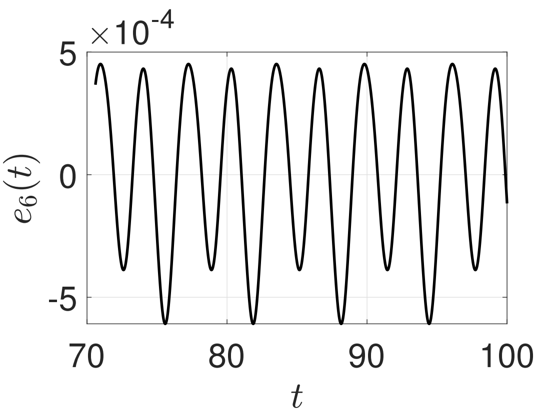

Figure 6. The transients of the tracking error $e_6(t)$.

4 Conclusion

The proposed paper develops a robust control system for a multi-agent chain plant, the dynamic processes in which are described by a system of non-stationary equations with disturbances under conditions of uncertainty of model parameters.It is proposed to form control actions in each agent of the network using the auxiliary loop method. In each agent of the network, the output of the previous agent is controlled. The signal from the leading subsystem is received only in the first agent of the network. The control systems of each agent are built based on the measured outputs of the agent itself and its predecessor. Simulation of the control system of the network consisting of six agents was carried out in Simulink Matlab package. The simulation results confirmed the theoretical conclusions and demonstrated the effectiveness of the chain control system under conditions of parametric uncertainty and external disturbances.

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