ENOC 2008, Saint Petersburg, Russia, June, 30 - July, 4 2008

Adaptive Tuning of Feedback Gain in Time-Delayed Feedback Control

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We study the possibility to adaptively tune the feedback gain K in the well-known time-delayed feedback control. This adaptive modification of time-delayed feedback control is applied to the stabilization of an unstable fixed point and the stabilization of an unstable periodic orbit embedded ina chaotic attractor. The adaptation algorithm is constructed using the speed-gradient method. For the stabilization of a fixed point, a generic two-variable normal form is used. The linear stability analysis for the control applied to both one and two system variables is given. It is found by computer simulations that the adaptation algorithm can tune the feedback gain K from the initial value $K = 0$ to some appropriate value in the domain of successful control. This final value may depend on the initial conditions for the system variables and adaptation-algorithm parameter. The adaptation algorithm is developed for both the standard and the extended time-delay autosynchronization control schemes. We find that in both cases the speed-gradient based adaptation algorithm can ensure the successful control by finding an appropriate feedback gain K.

I. INTRODUCTION

Since the last decade, the stabilization of unstable and chaotic systems has been a field of extensive research. A variety of control schemes have been developed to control periodic orbits as well as steady states [1–3]. A simple and efficient scheme, introduced by Pyragas [4], is known as *time delay autosynchronization* (TDAS). This control method generates a feedback from the difference of the current state of a system to its counterpart some time units τ in the past. Thus, the control scheme does not rely on a reference system and has only a small number of control parameters, i.e., the feedback gain K and time delay τ. It has been shown that TDAS can stabilize both unstable periodic orbits, e.g., embedded in a strange attractor [4, 5], and unstable steady states [6–8]. In the first case, TDAS is most efficient if τ corresponds to an integer multiple of the minimal period of the orbit. In the latter case, the method works best if the time delay is related to an intrinsic characteristic timescale given by the imaginary part of the system's eigenvalue [8]. A generalization of the original Pyragas scheme, suggested by Socolar et al. [9], uses multiple time delays. This *extended time delay autosynchronization* (ETDAS) introduces a memory parameter R, which serves as a weight of states further in the past. A variety of analytic results about time-delayed feedback control are also known [10–13], for instance, in the case of long time delays [14] or the odd number limitation [15], which was refuted recently [16, 17].

Although there has been strong effort on the research on the original Pyragas method [18, 19], much less is known in the case of extended time-delayed feedback [20–25]. Recently it was shown that the additional memory parameter introduces a second timescale which leads to a strong improvement of the stabilization ability, for instance, arbitrary large correlations of stochastic oscillations without inducing a bifurcation [26]. In Ref. [27] it is shown that extended time-delayed feedback can be used to stabilize unstable steady states of focus type. By introduction of an additional memory parameter, this method is able to control a larger range of unstable fixed points compared to the original TDAS scheme.

In the present paper, we apply the speed gradient method [28, 29] to adaptively tune the feedback gain K, which is used in both TDAS and ETDAS control methods, and use this scheme to stabilize an unstable focus in a generic model and an unstable periodic orbit embedded in a chaotic attractor. The first system can be seen as a system close to a Hopf bifurcation. The speed-gradient method is a well known adaptive control technique which minimizes a predefined goal function by changing an accessible system parameter appropriately. The adaptation of the feedback gain may be useful for systems with unknown or slowly changing parameters.

This paper is organized as follows: In Sec. II, we develop the adaptation algorithm using the example of an unstable focus. In Sec. III, we apply the adaptive control scheme to stabilize an unstable periodic orbit embedded in the chaotic attractor of the Rössler system. Finally, we conclude with Sec. IV.

II. STABILIZATION OF AN UNSTABLE FIXED POINT

Following the many publications studying the UFP problem with constant feedback gain [8, 27], we consider a general dynamic system given by a vector field $\mathbf{f}$:

$$\dot{\mathbf{X}}(t) = \mathbf{f}[\mathbf{X}(t)],\ \mathbf{X}(t) = \mathrm{col}\{x(t), y(t)\} \qquad (1)$$

with an unstable fixed point $\mathbf{X}^*$ solving $\mathbf{f}(\mathbf{X}^*) = 0$. The stability of this fixed point is obtained by linearizing the vector field around $\mathbf{X}^*$. Without loss of generality, let us assume $\mathbf{X}^* = 0$. In the following we will consider the generic case of an unstable focus for which the linearized equations in center manifold coordinates x, y can be written as

$$\dot{x} = \lambda x + \omega y \tag{2a}$$
$$\dot{y} = -\omega x + \lambda y, \tag{2b}$$

where λ and ω are positive real numbers. They may be viewed as parameters governing the distance from the instability threshold, e.g., a Hopf bifurcation of system (1), and the intrinsic eigenfrequency, respectively. For notational convenience, Eq. (2) can be rewritten as

$$\dot{\mathbf{X}}(t) = \mathbf{A}\,\mathbf{X}(t). \tag{3}$$

The eigenvalues Λ_0 of the matrix $\mathbf{A}$ are given by $\Lambda_0 = \lambda \pm i\omega$, so that for $\lambda > 0$ and $\omega \neq 0$ the fixed point is an unstable focus. We shall now apply time-delayed feedback control [4] in order to stabilize this fixed point:

$$\dot{x}(t) = \lambda x(t) + \omega y(t) - K[x(t) - x(t-\tau)] \tag{4a}$$
$$\dot{y}(t) = -\omega x(t) + \lambda y(t) - K[y(t) - y(t-\tau)], \tag{4b}$$

where the feedback gain K and the time delay τ are real numbers. The goal of the control method is to change the sign of the real part of the eigenvalue.

Since the control force applied to the i-th component of the system involves only the same component, this control scheme is called diagonal coupling [23] and is suitable for an analytical treatment. Note that the feedback term vanishes if the USS is stabilized since $x^*(t-\tau) = x^*(t)$ and $y^*(t-\tau) = y^*(t)$ for all t, indicating the noninvasiveness of the TDAS method. First we consider the restricted scheme with time-delay feedback only in the first equation:

$$\dot{x} = \lambda x(t) + \omega y(t) - K[x(t) - x(t-\tau)] \tag{5a}$$
$$\dot{y} = -\omega x(t) + \lambda y(t), \tag{5b}$$

We assume that the value of τ is known and appropriately chosen.

To obtain an adaptation algorithm for feedback gain K according to the standard procedure of the speed-gradient method [28–30], let us choose the goal function as follows: $Q(x) = [x(t) - x(t-\tau)]^2/2$. Note that speed-gradient method can ensure the limit $Q(x(t)) \to 0$ as t goes to infinity. Then we need to calculate the partial derivative (in the scalar case) of dQ/dt with respect to the feedback gain K. After that the speed-gradient algorithm in the differential form is given by: $\dot{K} = -\gamma \nabla_K \dot{Q}$, where $\gamma > 0$ is the adaptation gain. For the system (5) we obtain:

$$\dot{Q} = (x(t) - x(t-\tau))(\dot{x}(t) - \dot{x}(t-\tau)), \tag{6}$$
$$\dot{x}(t) = \lambda x(t) + \omega y(t) - K[x(t) - x(t-\tau)]$$
$$\dot{x}(t-\tau) = \lambda x(t-\tau) + \omega y(t-\tau) - K[x(t-\tau) - x(t-2\tau)]$$
$$(\dot{x}(t) - \dot{x}(t-\tau)) = < \text{terms without K} > -\mathrm{K}[\mathrm{x(t)} - 2\mathrm{x}(\mathrm{t}-\tau) + \mathrm{x}(\mathrm{t}-2\tau)]. \tag{7}$$

Finally, the equation for the feedback gain reads

$$\dot{K}(t) = \gamma(x(t) - x(t-\tau))[x(t) - 2x(t-\tau) + x(t-2\tau)]. \tag{8}$$

Figure 1 depicts the phase portrait and the time series of y and K according to Eqs. (5) and (8). The parameters are chosen as $\lambda = 0.5$, $\omega = \pi$, and $\tau = 1.0$. These computer simulations show that this adaptation algorithm (with initial condition $K(0) = 0$) converges to some appropriate value of K.

In order to determine the stability of the fixed point, we perform a linear stability analysis for the system (5) and (8). This system has the fixed point $(0, 0, K^*)$ for any $K^* = const$. Linearization around the fixed point and the ansatz col$\{\delta x\, \delta y\, \delta K\} \propto \exp(\Lambda t)$ yields a transcendental eigenvalue equation which can be solved numerically:

$$0 = \det \begin{bmatrix} \lambda - K(1 - e^{-\Lambda\tau}) - \Lambda & \omega & 0 \\ -\omega & \lambda - \Lambda & 0 \\ 0 & 0 & -\Lambda \end{bmatrix} \tag{9}$$
$$= -\Lambda[(\lambda - \Lambda)(\lambda - K(1 - e^{-\Lambda\tau}) - \Lambda) + \omega^2]. \tag{10}$$

Note that the complex eigenvalues Λ of the original Pyragas control case, i.e., Eq. (5) are conserved.

Now, we come back to the original scheme with time-delay feedback in both equations (so called diagonal coupling):

$$\dot{x}(t) = \lambda x(t) + \omega y(t) - K[x(t) - x(t-\tau)] \tag{11a}$$
$$\dot{y}(t) = -\omega x(t) + \lambda y(t) - K[y(t) - y(t-\tau)]. \tag{11b}$$

In order to introduce a speed-gradient adaptation algorithm for the feedback gain K, we choose the goal func-

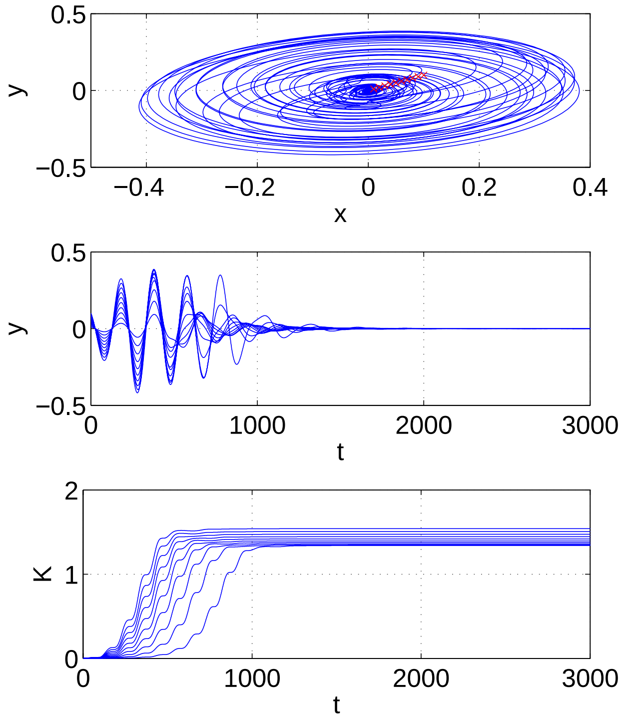

FIG. 1: (Color online) Phase portrait and time series $y(t)$ and $K(t)$ according to Eqs. (5) and (8). The red crosses mark different initial conditions. Parameters: $\lambda = 0.5$, $\omega = \pi$, and $\tau = 1.0$.

tion as follows: $Q(x) = [(x(t) - x(t-\tau))^2 + (y(t) - y(t-\tau))^2]/2$. Similar to the previous case of Eq. (11) we then obtain:

$$\begin{aligned} \dot{K}(t) \;&= \gamma\{(x(t) - x(t-\tau))[x(t) - 2x(t-\tau) + x(t-2\tau)] \\ &\quad + (y(t) - y(t-\tau))[y(t) - 2y(t-\tau) \\ &\quad + y(t-2\tau)]\}. \end{aligned} \tag{12}$$

Computer simulations show that this adaptation algorithm converges more quickly than algorithm (8), and the maximum values of x and y during adaptation are smaller in this case.

Performing a linear stability analysis for the system (11, 12), we obtain a characteristic equation. This system has still the fixed point $(0, 0, K^*)$ for any $K^* = const.$ Linearization near this fixed point yields again a transcendental eigenvalue equation which can be solved numerically:

$$0 = \det \begin{bmatrix} \lambda - K(1-e^{-\Lambda\tau}) - \Lambda & \omega & 0 \\ -\omega & \lambda - K(1-e^{-\Lambda\tau}) - \Lambda & 0 \\ 0 & 0 & -\Lambda \end{bmatrix} \tag{13}$$

$$= -\Lambda[(\lambda - K(1-e^{-\Lambda\tau}) - \Lambda)^2 + \omega^2] \tag{14}$$

$$= -\Lambda[\lambda + i\omega - K(1-e^{-\Lambda\tau}) - \Lambda][\lambda - i\omega - K(1-e^{-\Lambda\tau}) - \Lambda]. \tag{15}$$

This equation is equal to the case of Pyragas control considered in Ref. [8] except for the factor Λ, so our controlled system has an additional eigenvalue at $\Lambda = 0$.

Next, we consider the ETDAS scheme

$$\dot{\mathbf{X}}(t) = \mathbf{A}\,\mathbf{X}(t) - \mathbf{F}(t), \tag{16}$$

where the ETDAS control force $\mathbf{F}$ can be written in three equivalent forms

$$\mathbf{F}(t) = K \sum_{n=0}^{\infty} R^n \left[\mathbf{X}(t - n\tau) - \mathbf{X}(t - (n+1)\tau)\right] \tag{17a}$$

$$= K\left[\mathbf{X}(t) - (1-R)\sum_{n=1}^{\infty} R^{n-1}\mathbf{X}(t - n\tau)\right] \tag{17b}$$

$$= K\left[\mathbf{X}(t) - \mathbf{X}(t-\tau)\right] + R\mathbf{F}(t-\tau). \tag{17c}$$

Here, K and τ denote the (real) feedback gain and the time delay, respectively. $R \in (-1, 1)$ is a memory parameter that takes into account those states that are delayed by more than one time interval τ. Note that $R = 0$ yields the TDAS control scheme introduced by Pyragas [4].

The control force applied to the ith component of the system consists only of contributions of the same component. Thus, this scheme realizes diagonal coupling. The first form of the control force, Eq. (17a), indicates the noninvasiveness of the ETDAS method because $\mathbf{X}^*(t-\tau) = \mathbf{X}^*(t)$ if the fixed point is stabilized. The third form, Eq. (17c), is suited best for an experimental implementation since it involves states further than τ in the past only recursively.

First, we again consider the restricted scheme with

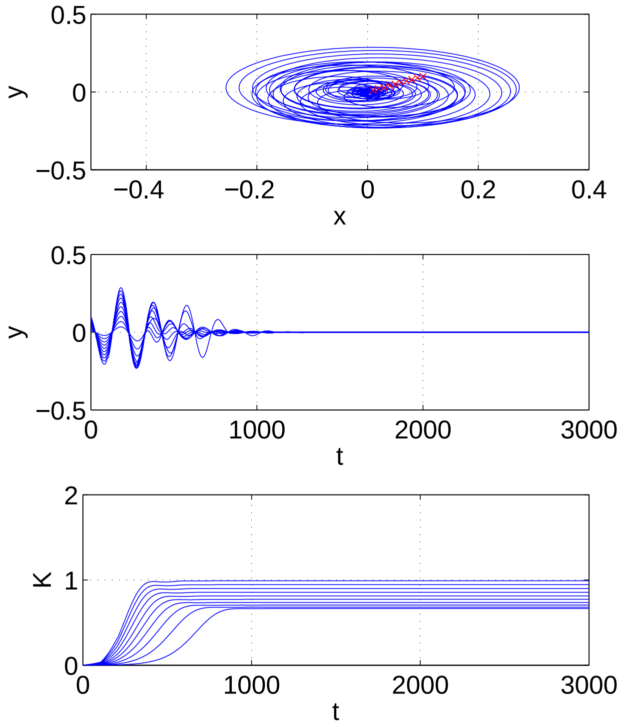

FIG. 2: (Color online) Phase portrait and time series $y(t)$ and $K(t)$ according to Eqs. (11) and (12). The red crosses mark different initial conditions. Parameters: $\lambda = 0.5$, $\omega = \pi$, and $\tau = 1.0$.

time-delay feedback only in the first equation: $\dot{\mathbf{X}}(t) = A\mathbf{X}(t) - \mathrm{col}\{F(t), 0\}$.

$$F(t) = K\,[x(t) - x(t-\tau)] + RF(t-\tau). \tag{18}$$

Thus, the first equation of controlled system becomes:

$$\begin{aligned} \dot{x}(t) &= \lambda\, x(t) + \omega\, y(t) \\ &\quad -K \sum_{n=0}^{\infty} R^n\,[x(t-n\tau) - x(t-(n+1)\tau)] \end{aligned} \tag{19}$$

For application of the speed-gradient adaptation algorithm for the feedback gain K, we choose the goal function again as $Q(x) = [x(t) - x(t-\tau)]^2/2$. This yields the following speed-gradient adaptation algorithm for the control applied to the x-component:

$$\begin{aligned} \dot{K}(t) &= \gamma(x(t) - x(t-\tau)) \sum_{n=0}^{\infty} R^n \\ &\quad \times [x(t-n\tau) - 2x(t-(n+1)\tau) + x(t-(n+2)\tau)] \end{aligned} \tag{20}$$

To obtain the recursive form of adaptation algorithm let us define the abbreviation

$$\begin{aligned} S(t) &= \sum_{n=0}^{\infty} R^n [x(t-n\tau) \\ &\quad -2x(t-(n+1)\tau) + x(t-(n+2)\tau)] \\ &= [x(t) - 2x(t-\tau) + x(t-2\tau)] + RS(t-\tau). \end{aligned} \tag{21}$$

Then, the recursive version of adaptation algorithm becomes:

$$\begin{aligned} \dot{K}(t) &= \gamma(x(t) - x(t-\tau))\{[x(t) - 2x(t-\tau) + x(t-2\tau)] \\ &\quad +RS(t-\tau)\}. \end{aligned} \tag{22}$$

Note that $\gamma(x(t-\tau) - x(t-2\tau))S(t-\tau)$ is equal to $\dot{K}(t-\tau)$. Assuming that $(x(t)-x(t-\tau))$ may be replaced by $(x(t-\tau) - x(t-2\tau))$ in the second term in Eq. (22), we obtain a simple approximate algorithm for $\dot{K}(t)$:

$$\begin{aligned} \dot{K}(t) &\approx \gamma(x(t) - x(t-\tau))[x(t) - 2x(t-\tau) + x(t-2\tau)] \\ &\quad +R\dot{K}(t-\tau). \end{aligned} \tag{23}$$

Computer simulations show that the adaptation algorithm (23) converges to appropriate K and stabilizes the fixed point for parameters where the TDAS algorithm fails.

We now come back to the system with control in both variables x and y. To apply a speed-gradient adaptation algorithm for feedback gain K, we follow the same strategy as before and choose the goal function as $Q(x) = [(x(t) - x(t-\tau))^2 + (y(t) - y(t-\tau))^2]/2$. Then, the speed-gradient adaptation algorithm for both components control and its recursive version are given by:

$$\begin{aligned} \dot{K}(t) = \gamma\{&(x(t) - x(t-\tau))[(x(t) - 2x(t-\tau) + x(t-2\tau)) + RS_x(t-\tau)] \\ &+(y(t) - y(t-\tau))[(y(t) - 2y(t-\tau) + y(t-2\tau)) + RS_y(t-\tau)]\} \end{aligned} \tag{24}$$

with the abbreviations

$$\begin{aligned} S_x(t) &= \sum_{n=0}^{\infty} R^n [x(t-n\tau) - 2x(t-(n+1)\tau) + x(t-(n+2)\tau)] = [x(t) - 2x(t-\tau) + x(t-2\tau)] + RS_x(t-\tau) \\ S_y(t) &= \sum_{n=0}^{\infty} R^n [y(t-n\tau) - 2y(t-(n+1)\tau) + y(t-(n+2)\tau)] = [y(t) - 2y(t-\tau) + y(t-2\tau)] + RS_y(t-\tau) \end{aligned} \tag{25}$$

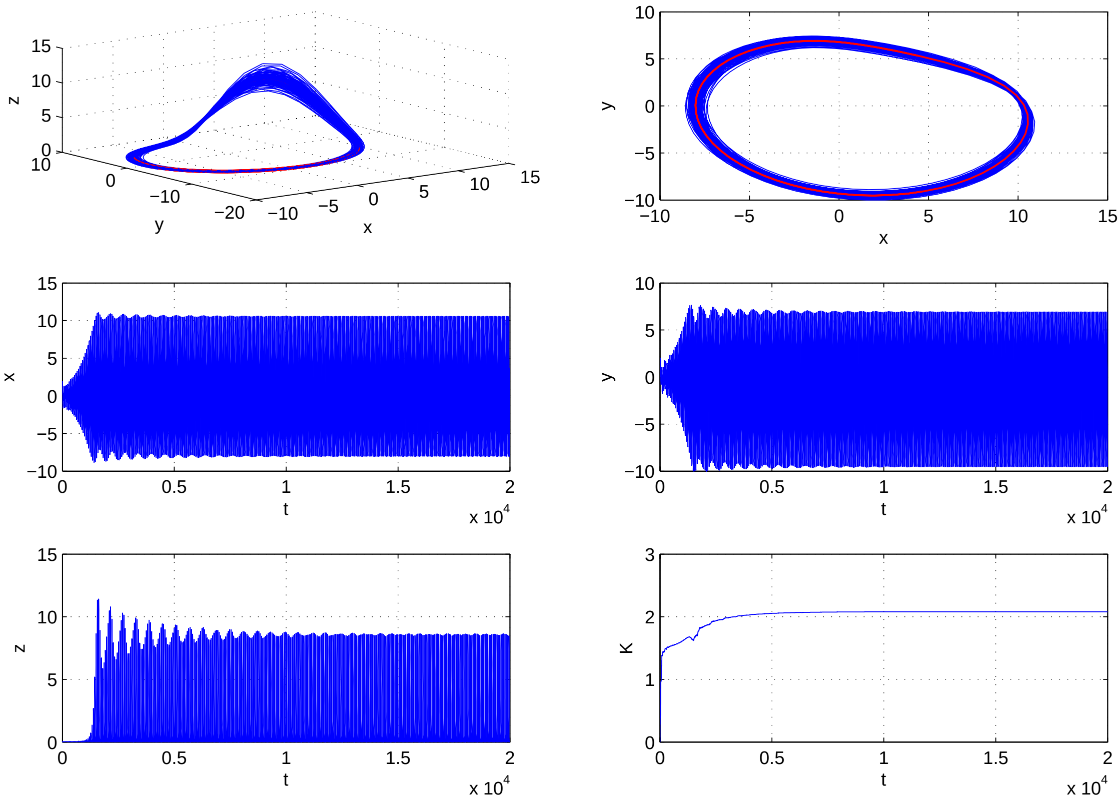

FIG. 3: (Color online) Phase portrait, projection onto the (x, y) plane and time series of $x(t)$, $y(t)$, $z(t)$, and $K(t)$ for controlled Rössler system (27) with adaptive control given by Eq. (28). Parameters: $a = 0.2$, $b = 0.2$, and $\mu = 6.5$.

Using the above mentioned approximation that terms $(x(t) - x(t - \tau))$ and $(y(t) - y(t - \tau))$ may be replaced by $(x(t-\tau) - x(t-2\tau))$ and $(y(t-\tau) - y(t-2\tau))$ in the second and fourth terms in Eq. (24), we again obtain a simple approximate algorithm for $\dot{K}(t)$:

$$\begin{aligned}\dot{K}(t) \approx\ & \gamma\{(x(t) - x(t-\tau))[x(t) - 2x(t-\tau) + x(t-2\tau)] \\ & +(y(t) - y(t-\tau))[y(t) - 2y(t-\tau) + y(t-2\tau)]\} \\ & +R\dot{K}(t-\tau) \qquad (26)\end{aligned}$$

III. STABILIZA TION OF AN UNSTABLE PERIODIC ORBIT IN THE RÖSSLER SYSTEM

In this section we apply the adaptive delayed feedback control to the Rössler system which is a a paradigmatic model for chaotic systems. The system exhibits chaotic oscillations born via a cascade of period-doubling bifurcations:

$$\begin{aligned}\dot{x}(t) &= -y(t) - z(t) - K[x(t) - x(t-\tau)] \qquad (27a)\\ \dot{y}(t) &= x(t) + ay(t) \qquad (27b)\\ \dot{z}(t) &= b + z(t)[x(t) - \mu]. \qquad (27c)\end{aligned}$$

In the following, we fix the parameter values as $a = 0.2$, $b = 0.2$, and $\mu = 6.5$. All quantities used in this paper are dimensionless. Unstable periodic orbits with periods $T_1 \approx 5.91679$ ("period-1 orbit") and $T_2 \approx 11.82814$ ("period-2 orbit") that are embedded into the chaotic attractor are shown by gray and black (red and black online) lines, respectively. As shown in Ref. [5], application of the delayed feedback of Pyragas type with $\tau = T_1$ and $0.24 < K < 2.3$ stabilizes the period-1 orbit, and it becomes the only attractor of the system. In Ref. [11] it has been predicted analytically by a linear expansion that control is realized only in a finite range of the values of K: at the lower control boundary the limit cycle should undergo a period-doubling bifurcation, and at the upper boundary a Hopf bifurcation generating a stable or an unstable torus from a limit cycle (Neimark-Sacker bifurcation).

As in the previous Section we obtain the speed-gradient adaptation algorithm for K:

$$\dot{K}(t) = \gamma(x(t)-x(t-\tau))[x(t)-2x(t-\tau)+x(t-2\tau)] \quad (28)$$

with the initial value $K(0) = 0$. Fig.3 depicts computer simulations for a time-delay $\tau = T_1$, which show that this adaptation algorithm converges to some appropriate value of K (see for example). Thus the chaotic dynamics of controlled system cannot prevent the successful adaptation.

IV. CONCLUSION

In summary, we have suggested a adaptation algorithm based on the speed-gradient method, to tune the feedback gain in the widely used time-delayed feedback control. We have shown that the adaptation algorithm can find appropriate values for the feedback gain and thus stabilize the desired orbit or fixed point. This has been demonstrated for the stabilization of an unstable focus in a generic model and the stabilization of an unstable periodic orbit embedded in chaotic attractor. We stress that this adaptive algorithm may especially be useful for systems with unknown or slowly changing parameters where the domains in parameter space of successful control are unknown.

Acknowledgments

This work was supported by Deutsche Forschungsgemeinschaft in the framework of Sfb 555 and Russian Foundation for Basic Research (grant 08-01-00775). P. Guzenko also thanks the DAAD program "Michail Lomonosov (B)" for the support of this work.

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