MIMO ADAPTIVE OUTPUT FEEDBACK SLIDING MODE CONTROL WITH PARALLEL FEEDFORWARD COMPENSATOR FOR LINEAR SYSTEMS

Kazuki Eguchi * **Zenta Iwai** * **Ikuro Mizumoto** *
Toshirou Funasako ** **Makoto Kumon** *

* *Department of Mechanical Engineering and Materials Science, Kumamoto University, 2-39-1 Kurokami, Kumamoto, Japan, 860-8555, E-mail:058d9001@gsst.stud.kumamoto-u.ac.jp*
** *Department of Aeronautics and Astronautics, Kyushu University*

Abstract: This paper describes a procedure for adaptive output feedback sliding mode control(SMC) based on Almost Strictly Positive Real(ASPR)-ness for a class of multi-input/multi-output linear time invariant systems. Stability of the control system is achieved by using the parallel feedforward compensator(PFC), which guarantees ASPR-ness of the plant. Effectiveness of the proposed method is confirmed through numerical simulations.

Keywords: sliding mode control, multi input multi output, strictly positive realness, output feedback, linear systems

1. INTRODUCTION

Sliding mode control(SMC) scheme has been applied to many industrial fields because of its superiority concerning robust control performance. The design procedure of SMC system is divided into two stages. The first phase is to choose a set of switching surface such that the original system restricted to the intersection of the switching surfaces. The second phase is to determine a switched control law that forces the system's trajectory to and maintains it on the sliding surface(Utkin, 1978; Edwards and Spurgeon, 1998). Unfortunately, most conventional methods in SMC use either full state or estimated state feedback so that they may be impractical or over complicated to implement. To improve the situation, there have been several proposals concerning static or dynamic output feedback SMC(Diong and Medanic, 1992; El-Khazali and DeCarlo, 1995; Yan *et al.*, 2004).

Recently, Ohtsuka et al. (Ohtsuka *et al.*, 2004; Ohtsuka *et al.*, 2006) have proposed a different design procedure using parallel feedforward compensator(PFC). This method is based on the almost strictly positive real(ASPR)ness of the controlled plant and PFC is used if the original plant does not satisfy the APSR condition(Iwai *et al.*, 1994). In this case, the sliding mode switching surface can be specified by an augmented plant output with PFC which guarantees the ASPR characteristics of the augmented plant.

However, the design of PFC, especially so called ladder network PFC, in MIMO system becomes extremely complicated(Iwai and Mizumoto, 1994). To improve the situation, Iwai et al. proposed quite simple new PFC design scheme(Iwai *et al.*, 2006).

In steady state, existence of measurement noise induces chattering phenomenon depending on magnitude of switching gain matrix. This problem can be alleviated by adaptive tuning of switching gain matrix. So, we also propose adaptive tuning rule of switching gain matrix.

Numerical simulation results are included to demonstrate the effectiveness of the proposed method.

2. SYSTEM DESCRIPTION AND PROBLEM SETUP

Consider the following observable and measurable n-th order m input/output system of the form

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) + B_1 d(t) \\ \boldsymbol{y}(t) = C\boldsymbol{x}(t) \end{cases} . \tag{1}$$

where, $\boldsymbol{x}(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}(t), \boldsymbol{y}(t) \in \boldsymbol{R}^m$ and $d(t) \in \boldsymbol{R}^1$ are state vector, control vector, output vector and disturbance vector, respectively.

[**Assumption 1**] Disturbance $d(t)$ and its derivatives are bounded.

$$|\boldsymbol{d}^{(i)}(t)| \leq d_i,\ i = 1, 2, \cdots, \rho, \quad d_i > 0 \ , \tag{2}$$

where $d^{(i)}(t)$ denotes i-th derivative of $d(t)$.

Let us introduce a reference vector $\boldsymbol{r}(t) \in \boldsymbol{R}^m$ defined by the reference model equation

$$D(s)[\boldsymbol{r}(t)] = \boldsymbol{0} \ , \tag{3}$$

$$D(s) = s^\rho + g_{\rho-1}s^{\rho-1} + \cdots + g_1 s + g_0 \ , \tag{4}$$

where 's' denotes the differential operator. The control objective is to realize the tracking between output $\boldsymbol{y}(t)$ and reference $\boldsymbol{r}(t)$ by using the output feedback sliding mode control(SMC). To facilitate the control law development, an appropriate system form will be employed. Let us define $\boldsymbol{z}(t)$, $\boldsymbol{v}(t)$ and tracking error $\boldsymbol{e}(t)$ as follows:

$$\boldsymbol{z}(t) = D(s)[\boldsymbol{x}(t)] \ , \tag{5}$$

$$\boldsymbol{v}(t) = D(s)[\boldsymbol{u}(t)] \ , \tag{6}$$

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{r}(t) \ , \tag{7}$$

where $\boldsymbol{z}(t) \in \boldsymbol{R}^n$, $\boldsymbol{v}(t), \boldsymbol{e}(t) \in \boldsymbol{R}^m$.

Multiplying $D(s)$ from the both side of (7) leads to

$$\begin{aligned} \boldsymbol{e}^{(\rho)}(t) &= C\boldsymbol{z}(t) - g_0\boldsymbol{e}(t) \\ &\quad - g_1\boldsymbol{e}^{(1)}(t) - \cdots - g_{\rho-1}\boldsymbol{e}^{(\rho-1)}(t) \ . \end{aligned} \tag{8}$$

From (1),(5),(6),(8), we have the following $\bar{n} = n + m \times \rho$-th order system with the input $\boldsymbol{v}(t)$ and the output $\boldsymbol{e}(t)$:

$$\begin{cases} \dot{\bar{\boldsymbol{x}}}(t) = \bar{A}\bar{\boldsymbol{x}}(t) + \bar{B}\boldsymbol{v}(t) + \boldsymbol{g}(t) \\ \boldsymbol{e}(t) = \bar{C}\bar{\boldsymbol{x}}(t) \end{cases} , \tag{9}$$

$$\bar{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{z}(t) \\ \boldsymbol{e}(t) \\ \boldsymbol{e}^{(1)}(t)(t) \\ \vdots \\ \boldsymbol{e}^{(\rho-1)}(t)(t) \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} A & 0 & \cdots\cdots & 0 \\ 0 & 0 & I_m \ \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots\cdots & I_m \\ C & -g_0 I_m & \cdots\cdots & -g_{\rho-1}I_m \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 0 & I_m & 0 & \cdots 0 \end{bmatrix},$$

$$\boldsymbol{g}(t) = \begin{bmatrix} B_1\bar{d}(t) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$\bar{d}(t) = \sum_{i=0}^{\rho} g_i d^{(i)}(t), \quad g_\rho = 1 \ .$$

In this report, the sliding hyperplanes are assumed to be constructed by outputs of the system. However the equation $\bar{C}\bar{B} = 0$ holds. It means that the outputs can not be controlled by any control inputs. To avoid such a situation, here, we introduce the following parallel feedforward compensator(PFC).

$$\begin{cases} \dot{\boldsymbol{x}}_f(t) = A_f\boldsymbol{x}_f(t) + B_f\boldsymbol{v}(t) \\ \boldsymbol{y}_f(t) = C_f\boldsymbol{x}_f(t) \end{cases} \tag{10}$$

where $\boldsymbol{x}_f(t) \in \boldsymbol{R}^{n_f}, \boldsymbol{v}(t), \boldsymbol{y}_f(t) \in \boldsymbol{R}^m$.

By adding the PFC to the system described by the equation (9), the following $n_a = n + m \times \rho + n_f$-th extended system is obtained.

$$\begin{cases} \dot{\boldsymbol{x}}_a(t) = A_a\boldsymbol{x}_a(t) + B_a\boldsymbol{v}(t) + \boldsymbol{g}_a(t) \\ \boldsymbol{\sigma}(t) = C_a\boldsymbol{x}_a(t) = \boldsymbol{e}(t) + \boldsymbol{y}_f(t) \end{cases} . \tag{11}$$

$$\boldsymbol{x}_a(t) = \begin{bmatrix} \bar{\boldsymbol{x}}(t) \\ \boldsymbol{x}_f(t) \end{bmatrix}, \ A_a = \begin{bmatrix} \bar{A} & 0 \\ 0 & A_f \end{bmatrix}, B_a = \begin{bmatrix} \bar{B} \\ B_f \end{bmatrix},$$

$$C_a = \begin{bmatrix} \bar{C} & C_f \end{bmatrix}, \ \boldsymbol{g}_a(t) = \begin{bmatrix} \boldsymbol{g}(t) & 0 \end{bmatrix}^T \ .$$

In the following, we assume that the extended system satisfies the following assumption.

[**Assumption 2**] Extended system (11) satisfies the following assumptions.

(1) The system is minimum phase.
(2) Relative MacMillan degree is $(n_a - m)/n_a$.
(3) High frequency gain matrix C_aB_a is positive definite.

The system which satisfies the above mentioned assumption is called as almost strictly positive real(ASPR) system. From the condition (3) of Assumption 2, the high frequency gain matrix of the PFC should be selected to be positive definite. That is, $C_a B_a = C_f B_f > \mathbf{0}$

3. SLIDING MODE CONTROL

Let $\boldsymbol{\sigma}(t) = \mathbf{0}$ in (11) be the switching hyperplanes. Then we consider the following sliding mode control law.

$$\boldsymbol{v}(t) = -(C_a B_a)^{-1}\Big(C_a A_a \boldsymbol{x}_a(t) + K^T(t)\frac{\boldsymbol{\sigma}(t)}{\|\boldsymbol{\sigma}(t)\| + \delta}\Big) \tag{12}$$

where $K(t) \in \boldsymbol{R}^{m\times m}$, $K(t) > 0$ denotes the adaptive switching input gain matrix, constant $\delta > 0$ is introduced to avoid the chattering phenomenon and $\|\cdot\|$ indicates the vector norm. Here $K(t)$ is tuned based on the following adaptive tuning rule:

$$\dot{K}(t) = \frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\Gamma\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t), \tag{13}$$

where $\Gamma \in \boldsymbol{R}^{m\times m}$, $\Gamma = \Gamma^T > 0$. Sliding will occur along the hyperplanes $\boldsymbol{\sigma}(t) = [\sigma_1(t), \cdots, \sigma_m(t)]^T = \mathbf{0}$ as long as the necessary sliding condition $\sigma_i\dot{\sigma_i} \le 0$ holds in the neighborhood of the given hyperplanes. When sliding occurs on all m switching hyperplanes simultaneously, the state slides on the subspace, the intersection of the m hyperplanes. Stability of the system is discussed as follows.

[Theorem 1] Assume that Assumption 1 and 2 hold. Then the control law (12) and adaptive rule (13) attain $\lim_{t\to\infty}\boldsymbol{\sigma}(t) = \mathbf{0}$. Further it guarantees the boundness of the tracking error $\boldsymbol{e}(t)$. Especially if there exists no disturbance, it is guaranteed that tracking error $\boldsymbol{e}(t)$ converges to zero asymptotically.

[Proof] The proof of the stability is divided into two stages.

The first stage: Consider the following candidate of the Lyapunov function.

$$V(t) = V_1(t) + V_2(t) \tag{14}$$

$$V_1(t) = \frac{1}{2}\boldsymbol{\sigma}^T(t)\boldsymbol{\sigma}(t) \tag{15}$$

$$V_2(t) = \frac{1}{2}\mathrm{tr}\left[(K(t) - K^*)^T\Gamma^{-1}(K(t) - K^*)\right] \tag{16}$$

where K^* is an ideal positive definite switching gain matrix and $\mathrm{tr}[X]$ indicates trace of matrix X. The derivative of $V_1(t)$ along the trajectory becomes

$$\begin{aligned}\dot{V}_1(t) &= \boldsymbol{\sigma}^T(t)\dot{\boldsymbol{\sigma}}(t)\\ &= \boldsymbol{\sigma}^T(t)\left(-K^T(t)\frac{\boldsymbol{\sigma}(t)}{\|\boldsymbol{\sigma}(t)\| + \delta} + C_a\boldsymbol{g}_a(t)\right) .\end{aligned}$$

Since

$$\begin{aligned}C_a\boldsymbol{g}_a(t) &= \begin{bmatrix} 0 & I_m & 0 & \cdots & 0 & C_f\end{bmatrix}\begin{bmatrix} B_1\bar{d}(t)\\ 0\\ \vdots\\ 0\end{bmatrix}\\ &= 0 ,\end{aligned} \tag{17}$$

then

$$\begin{aligned}\dot{V}_1(t) &= -\boldsymbol{\sigma}^T(t)K^T(t)\frac{\boldsymbol{\sigma}(t)}{\|\boldsymbol{\sigma}(t)\| + \delta}\\ &= -\frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\mathrm{tr}\left[K^T(t)\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t)\right] .\end{aligned} \tag{18}$$

Next the derivative of (16) along the trajectory becomes

$$\begin{aligned}\dot{V}_2(t) &= \frac{1}{2}\mathrm{tr}\left[\dot{K}^T(t)\Gamma^{-1}(K(t) - K^*)\right]\\ &\quad + \frac{1}{2}\mathrm{tr}\left[(K(t) - K^*)^T\Gamma^{-1}\dot{K}(t)\right]\\ &= \mathrm{tr}\left[K^T(t)\Gamma^{-1}\dot{K}(t)\right] - \mathrm{tr}\left[K^*\Gamma^{-1}\dot{K}(t)\right] .\end{aligned}$$

Taking into account (13), the above equation becomes

$$\begin{aligned}\dot{V}_2(t) &= \frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\mathrm{tr}\left[K^T(t)\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t)\right]\\ &\quad - \frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\mathrm{tr}\left[{K^*}^T\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t)\right] .\end{aligned} \tag{19}$$

From (18)(19), the derivative of $V(t)$ becomes

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t)\\ &= -\frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\mathrm{tr}\left[{K^*}^T\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t)\right] < 0 ,\end{aligned}$$

thus, $\lim_{t\to\infty}\boldsymbol{\sigma}(t) = \mathbf{0}$ holds.

The second stage: In the sliding mode, switching hyperplane and its derivative satisfy $\boldsymbol{\sigma}(t) = \mathbf{0}$,$\dot{\boldsymbol{\sigma}}(t) = \mathbf{0}$. Hence the equation governing the system dynamics may be obtained by substituting an equivalent control(Edwards and Spurgeon, 1998)

$$\boldsymbol{v}_{eq}(t) = -(C_a B_a)^{-1}C_a A_a\boldsymbol{x}_a(t) \tag{20}$$

into the original controlled plant (11) as $\boldsymbol{v}(t) = \boldsymbol{v}_{eq}(t)$. Then we have

$$\begin{cases}\dot{\boldsymbol{x}}_a(t) = A_{eq}\boldsymbol{x}_a(t) + \boldsymbol{g}_a(t)\\ \boldsymbol{\sigma}(t) = C_a\boldsymbol{x}_a(t)\end{cases} \tag{21}$$

where

$$A_{eq} = A_a - B_a(C_a B_a)^{-1}C_a A_a . \tag{22}$$

Now let us introduce the following similarity transformation

$$\bar{\boldsymbol{x}}_a(t) = T_a \boldsymbol{x}_a(t)\ ,\ T_a = \begin{bmatrix} C_a \\ P \end{bmatrix}\ . \tag{23}$$

In (23), P is a matrix chosen to ensure that T_a is nonsingular. By this similarity transformation (23), (22) can rewritten in the following:

$$\begin{cases} \dot{\bar{\boldsymbol{x}}}_a(t) = \begin{bmatrix} \mathbf{0}_m & \mathbf{0} \\ PA_{eq}C_a^g & PA_{eq}P^g \end{bmatrix} \bar{\boldsymbol{x}}_a(t) + T_a \boldsymbol{g}_a(t) \\ \boldsymbol{\sigma}(t) = [\boldsymbol{I}_m\ \mathbf{0}]\, \bar{\boldsymbol{x}}_a(t). \end{cases} \tag{24}$$

where C_a^g and P^g are the generalized inverses of C_a and P, respectively.

As to the disturbance term in (24), we can obtain the following relation

$$T_a \boldsymbol{g}_a(t) = \begin{bmatrix} C_a \\ P \end{bmatrix} \boldsymbol{g}_a(t) = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{h}(t) \end{bmatrix}$$

where $\boldsymbol{h}(t) = P\boldsymbol{g}_a(t)$ is $n + \rho m + n_f - m$-th order bounded disturbance. By dividing state vector $\bar{\boldsymbol{x}}_a(t)$ into two partial state vector, $\bar{\boldsymbol{x}}_{a_1}(t) \in \boldsymbol{R}^m$, $\bar{\boldsymbol{x}}_{a_2}(t) \in \boldsymbol{R}^{n+\rho m+n_f-m}$, the following equation is obtained.

$$\begin{cases} \dot{\bar{\boldsymbol{x}}}_{a_1}(t) = \mathbf{0} \\ \dot{\bar{\boldsymbol{x}}}_{a_2}(t) = PA_{eq}P^g \bar{\boldsymbol{x}}_{a_2}(t) + \boldsymbol{h}(t)\ . \end{cases} \tag{25}$$

Taking into account that $\boldsymbol{\sigma}(t) = \mathbf{0}$, $\dot{\boldsymbol{\sigma}} = \mathbf{0}$ holds, $\bar{\boldsymbol{x}}_{a_1}(t) = \mathbf{0}$. Further eigenvalues of A_{eq} consist of m zeros and equivalent to $n+\rho m+n_f-m$ system zeros. According to Assumption 2, system under consideration satisfies ASPR condition. It follows that the eigenvalues of A_{eq} coincides with that of stable system zeros in (11). From the boundness of the disturbance $\boldsymbol{h}(t)$ in (25), we can obtain the conclusion in Theorem 1. **[Q.E.D.]**

Expression $C_a A_a \boldsymbol{x}_a(t)$ in (12) can be rewritten as

$$\begin{aligned} C_a A_a \boldsymbol{x}_a(t) &= \begin{bmatrix} \bar{C}\ C_f \end{bmatrix} \begin{bmatrix} \bar{A} & 0 \\ 0 & A_f \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{x}}(t) \\ \boldsymbol{x}_f(t) \end{bmatrix} \\ &= \dot{\boldsymbol{e}}(t) + C_f A_f \boldsymbol{x}_f(t)\ . \end{aligned} \tag{26}$$

From $C_a B_a = C_f B_f$ and (26), (12) can be written as follows:

$$\begin{aligned} \boldsymbol{v}(t) = -\left(C_f B_f\right)^{-1} \Big(&\dot{\boldsymbol{e}}(t) + C_f A_f \boldsymbol{x}_f(t) \\ &+K(t)\frac{\boldsymbol{\sigma}(t)}{\|\ \boldsymbol{\sigma}(t)\| + \delta}\Big)\ . \end{aligned} \tag{27}$$

It shows that $\boldsymbol{v}(t)$ can be constructed by using output, its derivative $\dot{\boldsymbol{e}}(t)$ and the state vector $\boldsymbol{x}_f(t)$ of PFC.

From (13), it is clear that the integral adaptive gains would diverge whenever perfect following is not possible due to internal or external disturbances. This problem can be easily alleviated by modifying the integral adaptive law by using small positive constant σ,

$$\dot{K}(t) = \frac{1}{\|\boldsymbol{\sigma}(t)\| + \delta}\Gamma\boldsymbol{\sigma}(t)\boldsymbol{\sigma}^T(t) - \sigma K(t), \tag{28}$$

maintains robustness of the adaptive control system in the presence of any bounded input and output disturbances.

4. DESIGN OF PFC

In order to realize the above stated control system, we have to introduce PFC. As to the design of PFC, many methods have been proposed(Kaufman *et al.*, 1998; Iwai and Mizumoto, 1994; Iwai *et al.*, 1994). Most design methods require some restrictive conditions. For example, the so-called ladder network structure PFC has relatively simple structure so that it is often used in practical case. However it requires that the plant is minimum phase and the relative degree of the plant should be given before hand. In order to alleviate the restrictive conditions, Iwai et al. proposed new PFC construction method(Iwai *et al.*, 2006). It is sometimes called model based PFC(MBPFC). Its specific feature is to use an approximated plant model for PFC design.

Let $G_p^*(s)$ and $\bar{G}_p^*(s)$ be an approximated model transfer function matrix of (1) and (9), respectively. Then $\bar{G}_p^*(s)$ is defined as

$$\bar{G}_p^*(s) = \frac{1}{D(s)} G_p^*(s)\ .$$

Further let $G_{ASPR}(s)$ be an ASPR transfer function matrix which can be indicated by designers. Then we construct PFC (10) as follows:

$$G_f(s) = G_{ASPR}(s) - \bar{G}_p^*(s)\ . \tag{29}$$

It follows that

$$\begin{aligned} G_a(s) &= \bar{G}_p(s) + G_f(s) \\ &= G_{ASPR}(s) + \bar{G}_p(s) - \bar{G}_p^*(s) \\ &= G_{ASPR}(s)\left(I_m + \Delta(s)\right), \end{aligned} \tag{30}$$

where

$$\Delta(s) = G_{ASPR}^{-1}(s)\left(\bar{G}_p(s) - \bar{G}_p^*(s)\right)\ . \tag{31}$$

Then the following Theorem 2 and Lemma 1 hold (Iwai *et al.*, 2006).

[Theorem 2] Extended system (11) becomes ASPR if the following conditions are satisfied.

(1) $G_{ASPR}(s)$ is ASPR.
(2) $\Delta(s) \in RH_{\infty}^{m\times m}$.
(3) $\|\Delta(s)\|_{\infty} < 1$ (where $\|\cdot\|_{\infty}$ indicates the H_{∞} norm of the transfer function matrix.)

[Lemma 1] The condition (2) of Theorem 2 is satisfied if $G(s)$ and $G^*(s)$ are stable transfer function matrices and D(s) is a stable polynomial.

From Lemma 1, $D(s)$ should be stable polynomial when we use the above mentioned MBFPC. Thus from (3), step input is not included in this case. To improve the situation, we assume

$$D(s)[\boldsymbol{r}(t)] = \bar{\boldsymbol{r}}(t) \neq \boldsymbol{0} \ . \tag{32}$$

Then, $\boldsymbol{g}(t)$ in (9) and (17) becomes

$$\boldsymbol{g}(t) = \left[\, B_1\bar{d}(t), 0, \cdots, 0, -\bar{\boldsymbol{r}}(t) \,\right] \ , \tag{33}$$

$$C_a\boldsymbol{g}_a(t) = \left[\, 0 \ I_m \ 0 \ \cdots \ 0 \ C_f \,\right] \begin{bmatrix} B_1\bar{d}(t) \\ 0 \\ \vdots \\ 0 \\ -\bar{\boldsymbol{r}}(t) \\ 0 \end{bmatrix} = 0 \ , \tag{34}$$

respectively. From this, it can be seen that the condition (2) of Theorem 2 does not affect the final result of Theorem 1.

5. SIMULATION RESULTS

In numerical simulation, consider the following transfer function matrix of liquid level process identified by Prony's method(Iwai *et al.*, 2005).

$$G(s) = \tag{35}$$
$$\begin{bmatrix} \dfrac{0.138s^2 + 9.58\times 10^{-4}s + 7.23\times 10^{-5}}{s^3 + 0.04s^2 + 3.96\times 10^{-4}s + 5.92\times 10^{-7}} & \dfrac{6.360\times 10^{-6}}{s^3 + 0.043s^2 + 1.85\times 10^{-4}s + 2.77\times 10^{-7}} \\ \dfrac{1.087680\times 10^{-5}}{s^3 + 0.163s^2 + 3.98\times 10^{-4}s + 4.13\times 10^{-7}} & \dfrac{0.02s + 2.01\times 10^{-4}}{s^3 + 0.19s^2 + 1.62\times 10^{-3}s + 2.91\times 10^{-6}} \end{bmatrix} .$$

Based on (35), $G^*(s)$ is chosen as follows:

$$G^*(s) = \tag{36}$$
$$\begin{bmatrix} \dfrac{7.20\times 10^{-5}}{s^3 + 0.04s^2 + 3.96\times 10^{-4}s + 5.92\times 10^{-7}} & \dfrac{6.0\times 10^{-6}}{s^3 + 0.043s^2 + 1.85\times 10^{-4}s + 2.77\times 10^{-7}} \\ \dfrac{1.10\times 10^{-5}}{s^3 + 0.163s^2 + 3.98\times 10^{-4}s + 4.13\times 10^{-7}} & \dfrac{2.0\times 10^{-4}}{s^3 + 0.19s^2 + 1.62\times 10^{-3}s + 2.91\times 10^{-6}} \end{bmatrix} .$$

ASPR model $G_{ASPR}(s)$ and other design parameters are set as follows:

$$G_{ASPR}(s) = \begin{bmatrix} \dfrac{122}{600s + 1.0} & \dfrac{21.7}{400s + 1.0} \\ \dfrac{26.7}{400s + 1.0} & \dfrac{68.8}{450s + 1.0} \end{bmatrix} ,$$
$$D(s) = \mathrm{diag}\,[1.0, \ 1.0] \ , \Gamma = \mathrm{diag}\,[0.01, 0.01] \, ,$$
$$\delta = 10.0, \ \sigma = 0.01 \ . \tag{37}$$

In all numerical simulations, white noise is added as output disturbance.
Reference input :$\boldsymbol{r}(t) = [40.0, \ 30.0]^T$
Input disturbance : $B_1 = B$,

$$d(t) = \begin{cases} [0.08, 0.08]^T & , \ t \geq 3000 \text{ [sec]} \\ [0.0, 0.0]^T & , \text{ otherwise} \end{cases} .$$

Control input limitation: $-8.0 \sim 4.0$ (dimensionless expression)
To confirm the effectiveness of proposed method, we compare our method with fixed SMC and fixed PID in the following. The MIMO PID parameter matrices are chosen by partial model matching method(Eguchi *et al.*, 2005). Fixed SMC gain matrix and PID gain matrices are chosen as follows:

$$K = \begin{bmatrix} 0.6 & 0.01 \\ 0.01 & 0.4 \end{bmatrix} ,$$
$$K_p = \begin{bmatrix} 1.38\times 10^{-2} & -2.46\times 10^{-3} \\ -4.22\times 10^{-3} & 1.94\times 10^{-2} \end{bmatrix} ,$$
$$K_i = \begin{bmatrix} 2.77\times 10^{-5} & -1.07\times 10^{-5} \\ -1.13\times 10^{-5} & 4.19\times 10^{-5} \end{bmatrix} ,$$
$$K_d = \begin{bmatrix} 2.01 & 1.04 \\ 3.98\times 10^{-2} & -5.62\times 10^{-4} \end{bmatrix} .$$

Results are shown in Fig. 1−4. The upper row of Fig. 1 shows $y_1(t)$, the lower raw shows $y_2(t)$. The upper, middle and lower row of Fig. 2 and Fig. 3 shows control inputs of the proposed method, fixed SMC and fixed PID, respectively.

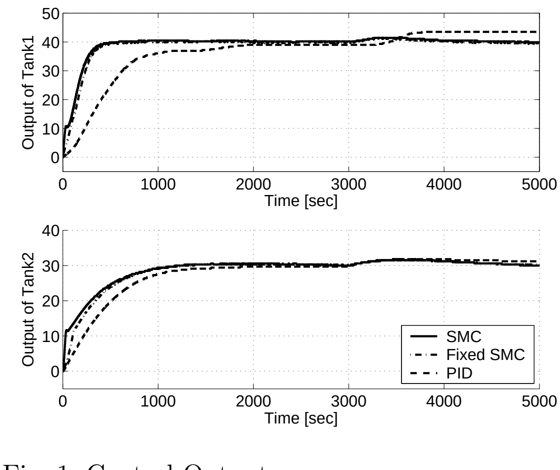

Fig. 1. Control Output

From these results, it can be concluded that, although the both of proposed method and fixed

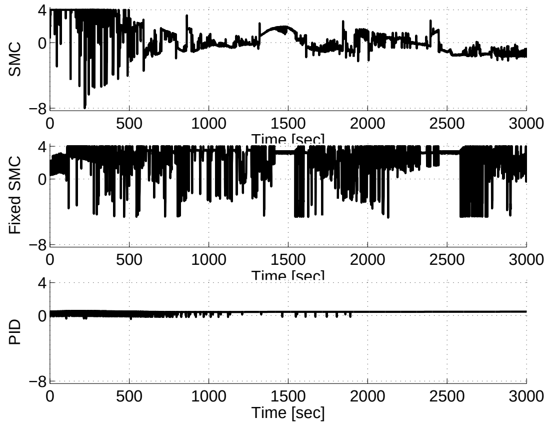

Fig. 2. Control Input $u_1(t)$

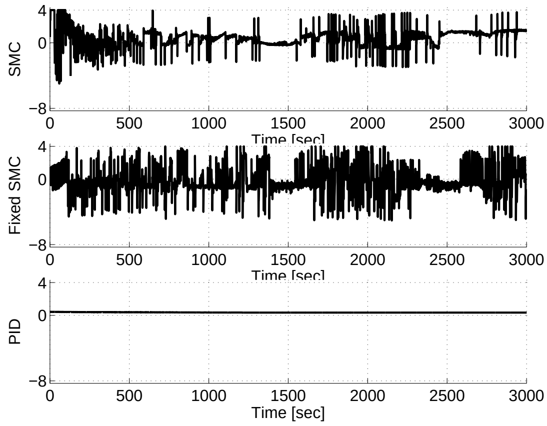

Fig. 3. Control Input $u_2(t)$

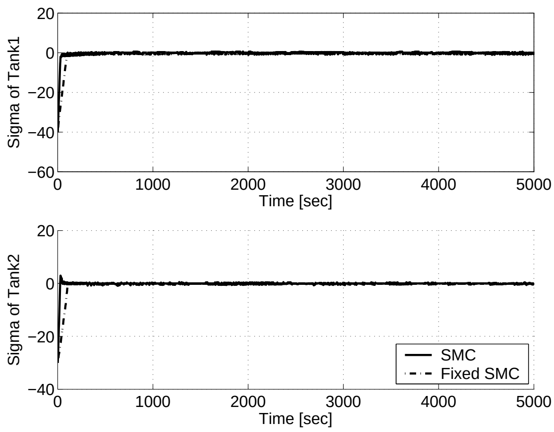

Fig. 4. $\boldsymbol{\sigma}(t)$

SMC attain good control performance, the chattering phenomenon in steady state is suppressed by adaptive SMC. Moreover, in this case, SMC is more effective than MIMO PID.

6. CONCLUSION

We have proposed a design scheme of MIMO adaptive output feedback sliding mode tracking controller by using MBPFC. Also stability analysis of the control system is derived. The effectiveness of the proposed method was confirmed through numerical simulations on liquid-level control process.

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