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TRANSIENT IN 2-DOF NONLINEAR SYSTEMS

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Abstract
In this paper a transient in some nonlinear systems is constructed by using the multiple-scale method. In particular, a system containing a linear oscillator, linearly coupled to an essentially nonlinear attachment with a comparatively small mass, is considered. A damping is taken into account. A transfer of energy from the initially perturbed linear subsystem to the nonlinear one can be observed. A similar construction is made to describe a transient in a system which contains a linear oscillator and a vibro-impact attachment with a comparatively small mass. A transient in such system under the external periodical excitation is considered too. Besides, a transient in a system which describes an interaction of some rotating subsystem and the elastic one is constructed.

Key words
Transient, nonlinear system, multiple-scale method

1 Introduction

An investigation of transient is important in engineering, in particular, in problem of absorption. Over the past years different new devices have been used for the vibration absorption and for the reduction of the transient response of structures [Shaw, Shaw and Haddow, 1989; Frolov K.V., 1995; Manevitch L. and al., 2003]. It seems interesting to study nonlinear passive absorbers for this reduction.

In presented paper the transient in a system containing a linear oscillator, linearly coupled to an essentially nonlinear attachment with a comparatively small mass, is considered. A damping is taken into account. It is assumed that some initial excitation implies vibrations of the linear oscillator.

The multiple scales method [Nayfeh, 1973] is used to construct a process of transient in the system under consideration. A transfer of energy from the initially perturbed linear subsystem to the nonlinear absorber can be observed. A similar construction is made to describe the transient in a system which contains a linear oscillator and a vibro-impact absorber. Both an exact integration with regards to impact conditions, and the multiple scales method are used for this construction. The transient in the vibro-impact system under the external periodical excitation was considered too. Numerical simulation confirms an efficiency of the analytical construction in these systems.

2 Transient in a system containing an essentially nonlinear oscillator as absorber

Let us consider a system with two connected oscillators, namely one linear and one nonlinear with a comparatively small mass (Fig.1).

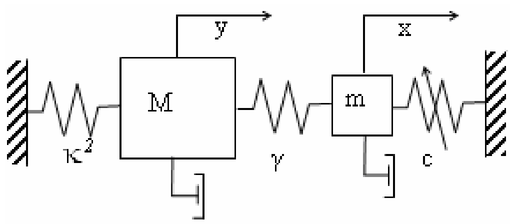

Fig.1. The system under consideration

This system is described passing by the following differential equations:

$$\begin{cases}\varepsilon m\ddot{x}+\varepsilon cx^3+\varepsilon^2\delta\dot{x}+\varepsilon\gamma(x-y)=0,\\ M\ddot{y}+\kappa^2 y+\varepsilon^2\delta\dot{y}+\varepsilon\gamma(y-x)=0,\end{cases} \qquad (1)$$

where δ is the dissipation coefficient, ε is a formal small parameter. In the first equation the small parameter characterizes the smallness of the absorber principal characteristics with respect to ones in the linear subsystem.

The solution of the system (1) will be found by the multiple-scale method. One has

$$x=x_0(t_0,t_1,t_2,\ldots)+\varepsilon x_1(t_0,t_1,t_2,\ldots)+\varepsilon^2 x_2(t_0,t_1,t_2,\ldots)+\ldots,$$
$$y=y_0(t_0,t_1,t_2,\ldots)+\varepsilon y_1(t_0,t_1,t_2,\ldots)+\varepsilon^2 y_2(t_0,t_1,t_2,\ldots)+\ldots, \quad (2)$$

where

$t_0=t, t_1=\varepsilon t, t_2=\varepsilon^2 t,\ldots,t_n=\varepsilon^n t,\ldots,$

$$\frac{d}{dt}=\frac{\partial}{\partial t_0}\frac{dt_0}{dt}+\frac{\partial}{\partial t_1}\frac{dt_1}{dt}+\frac{\partial}{\partial t_2}\frac{dt_2}{dt}+\ldots=$$
$$=\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}+\varepsilon^3\frac{\partial}{\partial t_3}+\ldots=D_0+\varepsilon D_1+\varepsilon^2 D_2+\varepsilon^3 D_3+\ldots$$

etc.

One obtains in zero approximation by the small parameter the next equation:

$\varepsilon^0: MD_0^2 y_0+\kappa^2 y_0=0.$

The solution of this equation is

$$y_0=A_1(t_1,t_2,\ldots)\cos\psi_0,$$

where $\psi_0=\Omega t_0+\varphi_0(t_1,t_2,\ldots)$, $\Omega^2=\dfrac{\kappa^2}{M}$.

One has in the next approximation by the small parameter the following ODE system:

$$\varepsilon^1:\begin{cases}mD_0^2 x_0+cx_0^3+\gamma(x_0-y_0)=0,\\ MD_0^2 y_1+2MD_0D_1y_0+k^2y_1+\gamma(y_0-x_0)=0.\end{cases}$$

Let us use the presentation of the x_0 in the form

$$x_0=B_1(t_1,t_2,\ldots)\cos\psi_0+B_2(t_1,t_2,\ldots)\cos\psi_1,$$

where $\psi_1=\Omega_1(t_1,t_2,\ldots)t_0+\varphi_1(t_1,t_2,\ldots)$. Equating cosine coefficients in the first equation and eliminating secular terms in the second one we get nonlinear algebraic equations:

$$\begin{cases}-mB_1\Omega^2+c\left(\frac{3}{4}B_1^3+\frac{3}{2}B_1B_2^2\right)+\gamma B_1=\gamma A_1\\ \gamma-m{\Omega_1}^2+\frac{3}{4}cB_2^2+\frac{3}{2}cB_1^2=0\end{cases},$$

$$\begin{cases}2MA_1\Omega\dfrac{\partial\varphi_0}{\partial t_1}+\gamma B_1-\gamma A_1=0\\ \dfrac{\partial A_1}{\partial t_1}=0\end{cases}$$

Thus $A_1=A_1(t_2,t_3,\ldots)$, $\dfrac{\partial\varphi_0}{\partial t_1}=\dfrac{\gamma(A_1-B_1)}{2MA_1\Omega}$.

Escaping calculations of the next approximations in the multiple scale method we give expressions for the amplitudes, frequencies and phases of zero-approximation x_0, y_0 of (2):

$$B_2=\bar{c}(t_2,t_3,\ldots)e^{-\frac{\delta}{2m}t_1},$$
$$B_1=c_0(t_2,t_3,\ldots)+c_2(t_2,t_3,\ldots)e^{-\frac{\delta}{m}t_1},$$
$$A_1=\frac{\gamma-m\Omega^2}{\gamma}c_0+\frac{3}{4\gamma}cc_0^3,$$
$${\Omega_1}^2=\frac{1}{m}\left(\gamma+\tfrac{3}{4}cB_2^2+\tfrac{3}{2}cB_1^2\right)=\text{[after time-averaging]}=\frac{1}{m}\left(\gamma+\tfrac{3}{2}cc_0^2\right),$$
$$\varphi_0=\frac{\gamma}{2M\Omega}t_1-\frac{\gamma}{2M\Omega A_1}\left(c_0t_1-c_2\frac{m}{\delta}e^{-\frac{\delta}{m}t_1}\right)+c_2^*,$$

where $c_2=\dfrac{\frac{3}{2}c\bar{c}^2c_0}{m\Omega^2-\gamma-\frac{9}{4}cc_0^2}$.

In such a way we have got a zero-approximation of sought solution containing four constants with respect to time t_0, namely $c_1^*=c_1^*(t_3,t_4,\ldots), c_2^*=c_2^*(t_2,t_3,\ldots), c_3^*=c_3^*(t_2,t_3,\ldots), \bar{c}=\bar{c}(t_2,t_3,\ldots)$ They could be found numerically by Newton method from the next initial conditions, which model the instant impact to the linear subsystem: $x(0)=\dot{x}(0)=0$, $y(0)=0, \dot{y}(0)=V$.

Figs. 2 and 3 present results of comparing the analytical solution (zero-approximation) with the numerical simulation obtained by using the Runge-Kutta procedure for different initial values and parameters, namely

$\varepsilon=0.01$, $m=M=\sqrt{10}$, $\kappa=0.9/\sqrt{10}$,

$c=0.18\sqrt{10}$, V=0.127;

besides, $\delta=5$, $\gamma=2/\sqrt{10}$ in Fig.2;

$\delta=1$, $\gamma=3/\sqrt{10}$ in Fig.3.

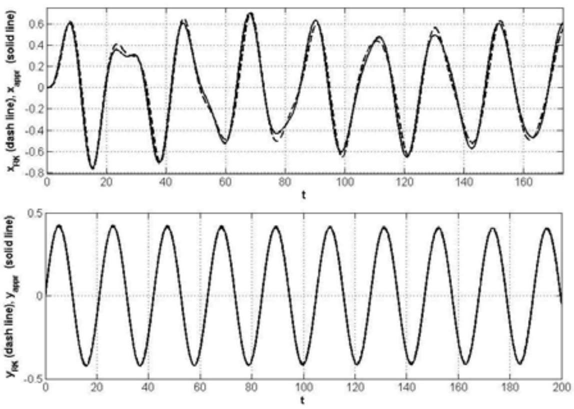

Fig.2. Transient simulation ($\delta=5$, $\gamma=2/\sqrt{10}$)

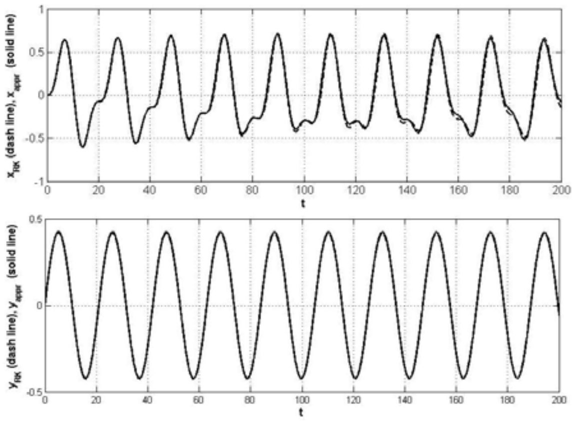

Fig.3. Transient simulation ($\delta=1$, $\gamma=3/\sqrt{10}$)

3 Transient in the vibro-impact system

One considers the 2-DOF vibro-impact system with the one-sided catch (Fig.4). This system contains the linear oscillator and the absorber with a comparatively small mass. It is presupposed to obtain analytical description of transient, both for free and forced oscillations, by using the multiple-scale method.

Equations of motion for the system under consideration in a case of the free vibrations are the following:

$$\begin{cases} \varepsilon m\ddot{x}+\varepsilon\gamma(x-y)+\varepsilon^2\delta\dot{x}=0; \\ M\ddot{y}+\kappa^2 y+\varepsilon\gamma(y-x)+\varepsilon^2\delta\dot{y}=0, \end{cases} \tag{3}$$

here M is a mass of the main linear subsystem, m is a mass of the absorber, δ characterizes a linear dissipation force, γ and κ^2 characterize elastic springs. The formal small parameter is introduced to select a smallness of the absorber mass, the connection between oscillators and the dissipation.

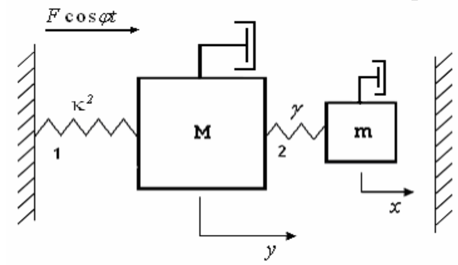

Fig.4. The vibro-impact system under consideration

It is presupposed that an impact here is instantaneous. The restoration coefficient ($0\le e\le 1$) characterizes a lost of velocity in the instant of impact. One has the following conditions of the impact:

$$x(t_k{}^+)=x(t_k{}^-)=x_{\max}, \qquad \dot{x}(t_k{}^+)=-e\dot{x}(t_k{}^-),$$

$$y(t_k{}^+)=y(t_k{}^-),\ \dot{y}(t_k{}^+)=\dot{y}(t_k{}^-) \tag{4}$$

Here t_k is the impact instant (k is a number of the impact, t_k^- is an instant before impact, t_k^+ is one after impact), $x_{\max}$ is a distance between the equilibrium state and the catch.

3.1 Free oscillations in the vibro-impact system

To construct an analytical solution by using the multiple scale method, the expansions (2) are used. In zero approximation by small parameter the next solution can be obtained:

$$y_0=A_0(t_1,t_2,t_3,...)\cos\Omega_0 t_0+B_0(t_1,t_2,t_3,...)\sin\Omega_0 t_0,$$

where $\Omega_0^2=\kappa^2/M$;

$$\begin{aligned} x_0 &= \beta(A_0(t_1,...)\cos\Omega_0 t_0+B_0(t_1,...)\sin\Omega_0 t_0)+ \\ &+A_1(t_1,...)\cos\sqrt{\gamma/m}\,t_0+B_1(t_1,...)\sin\sqrt{\gamma/m}\,t_0, \end{aligned} \tag{5}$$

where $\beta=\dfrac{\gamma}{m(\gamma/m-\Omega_0^2)}$. Conditions of elimination of secular terms in the next approximation by the small parameter give us the following expressions for amplitudes of the zero approximation:

$$A_0=-C_1\sin\Omega_1 t_1+C_2\cos\Omega_1 t_1,$$

$$B_0=C_1\cos\Omega_1 t_1+C_2\sin\Omega_1 t_1,$$

where $\Omega_1=\dfrac{\gamma(\beta-1)}{2M\Omega_0}$.

Taking onto account the next approximation, one has the approximate solution of the form:

$$x=\beta\,(\cos\Omega_2 t\cdot(-R_1C_1+R_2C_2)+\sin\Omega_2 t\cdot(R_2C_1+R_1C_2))+ e^{\alpha\varepsilon t}\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\},$$

$$y=C_1\sin\Omega_2 t+C_2\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\},$$

here

$$R_1=\frac{\varepsilon\delta\Omega}{m\left(\gamma/m-\Omega^2\right)},\ R_2=1-\frac{2\varepsilon\Omega\Omega_1}{\gamma/m-\Omega^2},$$

$$\beta_3=\sqrt{\gamma/m}-\beta_2\varepsilon,\quad \Omega_2=\Omega-\varepsilon\Omega_1.$$

Impact conditions (4) gives the next relations connecting coefficients C_i before (C_i^k) and after impact (C_i^{k+1}):

$$\begin{aligned} &\beta(\cos\Omega_2 t\cdot(-R_1C_1^{k+1}+R_2C_2^{k+1})+\sin\Omega_2 t\cdot(R_2C_1^{k+1}+R_1C_2^{k+1}))+ \\ &+e^{\alpha\varepsilon t}\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\}=\beta(\cos\Omega_2 t\cdot(-R_1C_1^k+R_2C_2^k)+ \\ &+\sin\Omega_2 t\cdot(R_2C_1^k+R_1C_2^k))+e^{\alpha\varepsilon t}\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\}. \end{aligned}$$

$$\begin{aligned} &\Omega_2\beta\left(-\sin\Omega_2 t\cdot(-R_1C_1^{k+1}+R_2C_2^{k+1})+\cos\Omega_2 t\cdot(R_2C_1^{k+1}+R_1C_2^{k+1})\right)+ \\ &+e^{\alpha\varepsilon t}\left(\alpha\varepsilon\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\}+\beta_3\{C_3^{k+1}\cos\beta_3 t-C_4^{k+1}\sin\beta_3 t\}\right)= \\ &=-\mathrm{e}\Omega_2\beta\left(-\sin\Omega_2 t\cdot(-R_1C_1^k+R_2C_2^k)+\cos\Omega_2 t\cdot(R_2C_1^k+R_1C_2^k)\right)+ \\ &+e^{\alpha\varepsilon t}\left(\alpha\varepsilon\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\}+\beta_3\{C_3^k\cos\beta_3 t-C_4^k\sin\beta_3 t\}\right) \end{aligned}$$

$$\begin{aligned} &C_1^{k+1}\sin\Omega_2 t+C_2^{k+1}\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\}= \\ &=C_1^k\sin\Omega_2 t+C_2^k\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\} \end{aligned}$$

$$\begin{aligned} &\Omega_2\left(C_1^{k+1}\cos\Omega_2 t-C_2^{k+1}\sin\Omega_2 t\right)+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{\alpha\varepsilon\left(C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\right)+\right. \\ &\left.+\beta_3\left(C_3^{k+1}\cos\beta_3 t-C_4^{k+1}\sin\beta_3 t\right)\right\}= \\ &=\Omega_2\left(C_1^k\cos\Omega_2 t-C_2^k\sin\Omega_2 t\right)+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{\alpha\varepsilon\left(C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\right)+\right. \\ &\left.+\beta_3\left(C_3^k\cos\beta_3 t-C_4^k\sin\beta_3 t\right)\right\}. \end{aligned}$$

The numeric simulation was realized for the next values of parameters: M=1, m=0.01, $\varepsilon=1$, $\delta=0.001$, e=0.9, $x_{\max}=1.4$, $\gamma=1.5$, $\kappa=1$. Initial values model the instant impact to the linear subsystem: $x(0)=0, \dot{x}(0)=0,\ y(0)=0, \dot{y}(0)=\dot{V}_0=1$. Comparison of the analytical solution and numerical simulation shows a good exactness of the analytical approximation (Fig.5). A number of the integration steps is shown on the horizontal axis.

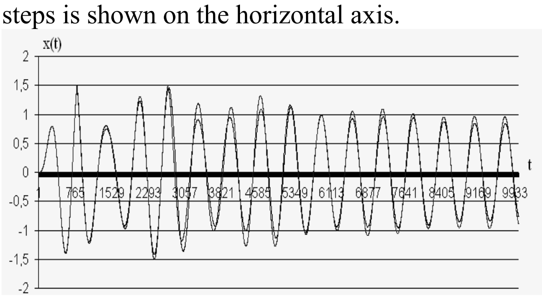

Fig.5. Transient in a case of free oscillations in the vibro-impact system

3.2 Transient in a case of forced oscillations

One considers the same 2-DOF vibro-impact system in a case when an external periodic force acts to linear subsystem. The multiple scales method can be successfully used in this case too. In contrast with the solution, obtained in the sub-section 3.1, the part, corresponding to the external excitation, have to be added. One has

$$x=\beta(\cos\Omega_2 t\cdot(-R_1C_1+R_2C_2)+\sin\Omega_2 t\cdot(R_2C_1+R_1C_2))+$$
$$+e^{\alpha\varepsilon t}\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\}+(F_2+\varepsilon F_5)\cos\varphi t+\varepsilon F_6\sin\varphi t,$$
$$y=C_1\sin\Omega_2 t+C_2\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\}+$$
$$+(F_1+\varepsilon F_3)\cos\varphi t+\varepsilon F_4\sin\varphi t,$$

$$\text{where}\ F_1=\frac{F}{(\Omega^2-\varphi^2)},\quad F_2=\frac{\gamma F_1}{m\left(\gamma/m-\varphi^2\right)},$$

$$F_3=\frac{-\gamma(F_1-F_2)}{M(\Omega^2-\varphi^2)},\quad F_4=\frac{2\varphi F_1}{\Omega^2-\varphi^2},$$

$$F_5=\frac{\gamma F_3}{m\left(\gamma/m-\varphi^2\right)},\quad F_6=\frac{\frac{\gamma}{m}F_4+\left(2+\frac{\delta}{m}\right)F_2\varphi}{\gamma/m-\varphi^2}. \qquad (6)$$

Impact conditions (4) gives some relations connecting coefficients C_i before (C_i^k) and after impact (C_i^{k+1}). These relations are not presented here.

Numerical simulation was made for the same parameters and initial values, as in the preceding sub-section. Comparison of the analytical solution and numerical simulation (Fig.6) shows a good exactness of obtained analytical approximation. A number of the integration steps is shown on the horizontal axis.

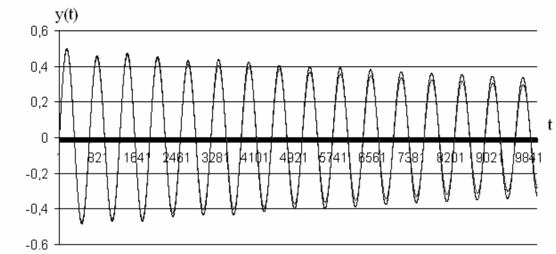

Fig.6. Transient in a case of forced vibrations in the vibro-impact system.

4 Transient in 2-DOF nonlinear system with limited power supply

One considers a transient in a system which describes an interaction of some rotating subsystem and the linear elastic one. A model of this system is presented in Fig.7. Equations of motion of this system are the following:

$$\begin{cases} m\ddot{x}+\beta\dot{x}+cx=c_1 r\sin(\varphi) \\ I\ddot{\varphi}=L(\dot{\varphi})-H(\dot{\varphi})+c_1 r(x-r\sin(\varphi))\cos(\varphi) \end{cases} \qquad (7)$$

Here the function $L(\dot{\varphi})$ is a controlled torque of the unbalanced rotor of DC motor; $H(\dot{\varphi})$ is a resistance torque of the rotor. The system under consideration is known as non-ideal system [Kononenko, 1969; Balthazar et al., 2003]. It means that the excitation is influenced by the response of the supporting elastic structure and that the energy source has a limited power supply (non-ideal excitation).

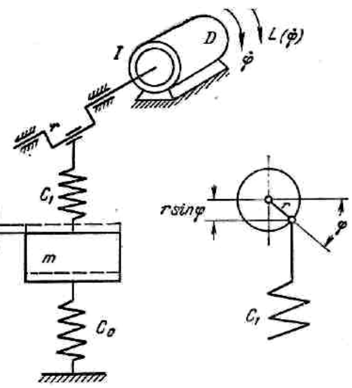

Fig.7. The nonlinear system with limited power supply.

The equations (7) may be simplified when we accept the torques as linear. One has in this case:

$$L(\dot{\varphi})-H(\dot{\varphi})=A-B\dot{\varphi}$$

Introducing the new dimensionless variables, $y=x/r,\ \tau=\omega t$, and the next parameters, $M=A/(I\omega^2), N=B/(I\omega^2), \varepsilon q=c_1r^2/(I\omega^2)$, $\varepsilon K=c_1/(m\omega^2),\ \varepsilon h=\beta/(m\omega)$,
(here ε is a formal small parameter) one can rewrite the equations (7) as

$$\begin{aligned} &y''+\varepsilon h y'+\omega^2 y=\varepsilon K\sin\varphi \\ &I\varphi''=M-N\omega+\varepsilon q(y\cos\varphi-0.5\sin 2\varphi) \end{aligned} \qquad (8)$$

Here prime denotes a derivation by τ. A procedure of multiple-scale method which is similar to (2) can be used here.

One has the next equations of the zero approximation by the small parameter, and the corresponding solution in the form:

$$\varepsilon^0: \begin{cases} D_0^2 y_0 + y_0 = 0 \\ D_0^2\varphi_0 = M - N\omega D_0\varphi_0 \end{cases} \qquad (9)$$

$$y_0 = A_0(T_1,T_2)\sin(T_0 + \Psi_0(T_1,T_2))$$
$$\varphi_0 = \Phi_0(T_1,T_2) + \frac{M}{N\omega}T_0 + F_0(T_1,T_2)\frac{1}{N\omega}e^{-N\omega T_0}$$

(The operator D_0 was introduced in the Section 2)

To simplify a construction of solution in the next approximations we will consider the transient after some instant when the exponent in (9) can be negligible. Note that in concrete systems this instant is very small.

In this case to eliminate secular terms in the next approximation by the small parameter ε, it must satisfy the next equations:

$$-2\frac{\partial A_0}{\partial T_1} - A_0 = 0 \;\rightarrow\; A_0 = A_1(T_2)e^{-hT_1/2}$$
$$-A_0\frac{\partial \Psi_0}{\partial T_1} = 0 \;\rightarrow\; \Psi_0 = \Psi_1(T_2) \qquad (10)$$
$$-N\omega\frac{\partial \Phi_0}{\partial T_1} = 0 \;\rightarrow\; \Phi_0 = \Phi_1(T_2)$$

Then we can write equations of the first approximation by the small parameter, namely:

$$D_0^2 y_1 + y_1 = K\sin(\Phi_0 + MT_0/(N\omega)$$
$$D_0^2\varphi_1 + N\omega D_0\varphi_1 = qA_0\sin(T_0 + \Psi_0) \qquad (11)$$
$$-0.5q\sin(2(\Phi_0 + MT_0/(N\omega)$$

After some transformation the transient can be presented as

$$y(\tau) = A_1(T_2)e^{-hT_1/2}\sin(T_0 + \Psi_1(T_2)) + \varepsilon K(1-\Omega^2)^{-1}\sin(\Phi_1(T_2) + \Omega T_0);$$

$$\varphi(\tau) = \Phi_1(T_2) + \Omega T_0 + F_1(T_2)e^{-N\omega T_0}/(N\omega) +$$
$$\varepsilon(-\frac{q}{2}A_1(T_2)e^{-hT_1/2}((N\omega)^2 + (1-\Omega^2)^2)^{-0.5}/|1-\Omega|\times$$
$$\sin((1-\Omega)T_0 + \Psi_1 - \Phi_1 + arctg(N\omega/(1-\Omega)))) +$$
$$\varepsilon(-\frac{q}{2}A_1(T_2)e^{-hT_1/2}((N\omega)^2 + (1+\Omega^2)^2)^{-0.5}/|1+\Omega|\times$$
$$\sin((1+\Omega)T_0 + \Psi_1 - \Phi_1 + arctg(N\omega/(1+\Omega)))) +$$
$$\varepsilon(\frac{q}{2}((N\omega)^2 + (2\Omega)^2)^{-0.5}/|2\Omega|\times \qquad (12)$$
$$\sin(2\Omega T_0 + 2\Phi_1 + arctg(N\omega/(2\Omega))))$$

Here $\Omega = M/(N\omega)$. The solution (12) describes a transfer to a non-resonance stationary solution having frequencies Ω and 2Ω. Vibration amplitudes of this non-resonance regime are not large. The numerical checking calculation shows a very good exactness of the transfer analytical presentation in a region of the non-resonance stationary regime stability. But if this stationary regime is unstable, on has a transfer to the 1:1 resonance stationary regime with large amplitudes.

5 Conclusion

The obtained results show an affectivity of the multiple-scale method to describe a transient in different kinds of nonlinear systems, including essentially nonlinear systems. It is important that an exactness of the analytical presentation is good for sufficiently large time interval.

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