OPTIMAL CONTROL OF PERIODIC MOTIONS OF VIBRATION-DRIVEN SYSTEMS

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Abstract: An optimal control problem is solved for a rigid body that moves along a straight line on a rough horizontal plane due to the motion of two internal masses. One of the masses moves horizontally parallel to the line of motion of the system's main body and the other mass moves vertically. Such a mechanical system models a vibration-driven robot able to move in a resistive medium without special propelling devices (wheels, legs or caterpillars). A periodic motion of the internal masses is constructed to ensure a velocity-periodic motion of the main body with a maximum average velocity, provided that the period is fixed and the accelerations of the internal masses relative to the main body lie within prescribed limits. This statement does not constrain the amplitude of vibrations of the internal masses. Based on the solution of the problem, a suboptimal control that takes this constraint into account is constructed.

Keywords: Vibration-driven Systems, Optimal Control, Robotics

1. INTRODUCTION

A rigid body with internal masses that perform periodic motions can move progressively in a resistive medium with nonzero average velocity. This phenomenon can be used as a basis for the design of new-type mobile systems able to move without special propelling devices (wheels, legs, caterpillars or screws) due to direct interaction of the body with the environment. Such systems have a number of advantages over systems based on the conventional principles of motion. They are simple in design, do not require gear trains to transmit motion from the motor to the propellers, and their body can be made hermetic and smooth, without any protruding components. The said features make this principle of motion prospective for being used in capsule-type microrobots designed for motion in strongly restricted space (e.g., inside narrow tubes) and in vulnerable media, for example, inside a human body for delivering a drug or a diagnostic sensor to an affected organ. Automatic transport systems moving due to periodic motion of internal masses are sometimes referred to as vibration-driven robots. Issues of control and optimization of motion of systems with internal movable masses have been studied by Chernousko (2002, 2005, 2006) and Figurina (2007). The dynamics and design of vibration-driven robots have been considered by Gradetsky *et al.* (2003), Li *et al.* (2005), Chernousko *et al.* (2005), Bolotnik *et al.* (2006), and Vartholomeos and Papadopoulos (2006).

In the present paper, a vibration-driven system consisting of a main body and two internal masses

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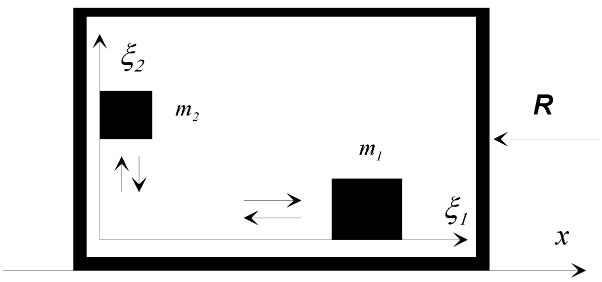

Fig. 1. Schematic of the system

is considered. The main body is based on a rough horizontal plane and can move along a straight line over the plane. There is dry (Coulomb's) friction acting between the body and the plane. One of the internal masses moves horizontally along a straight line parallel to the line of motion of the body, while the other mass moves vertically. The motion of the internal masses is controlled by forces acting between the masses and the body. Therefore, the control of the horizontal motion of the internal mass enables one to control the magnitude and direction of the friction force applied to the body, which provides the progressive motion of the entire system. The control of the vertical motion of the internal mass ensures an additional possibility of control of the dry friction magnitude due to the change of the normal pressure force exerted on the body by the supporting surface.

Periodic modes of motion of the internal masses are constructed to provide a velocity-periodic progressive motion of the main body with a maximum average velocity. The average velocity of the steady-state motion of the body is a basic operating characteristic of vibration-driven robots, and the maximization of this velocity is an important task for planning motions of such systems.

2. DESCRIPTION OF THE MODEL

Consider a rigid body of mass m_0 that is able to move along a straight line on a rigid rough plane. Inside this body, there are two movable internal point masses m_1 and m_2. Mass m_1 moves horizontally along a line parallel to the line of motion of the body and mass m_2 moves vertically. The system described is shown in Fig. 1.

The system is controlled by moving the internal masses relative to the body due to internal forces acting between the masses and the body. Let x be the displacement of the body relative to a fixed (inertial) reference frame, ξ_1 the horizontal displacement of mass m_1 relative to the body, ξ_2 the vertical displacement of mass m_2 relative to the body, and R the friction force exerted on the body by the supporting plane. Let the friction be dry friction modeled by Coulomb's law. We assume that the x and ξ_1 axes are co-directed and that the ξ_2 axis points vertically upward. Then the motion of the body is governed by the relations

$$M\ddot{x} + m_1\ddot{\xi}_1 = R, \quad M = m_0 + m_1 + m_2, \qquad (1)$$

$$R = \begin{cases} -kN\operatorname{sign}\dot{x}, \text{ if } \dot{x} \neq 0, \\ m_1\ddot{\xi}_1, \text{ if } \dot{x} = 0 \text{ and } |m_1\ddot{\xi}_1| \leq kN, \\ kN\operatorname{sign}(m_1\ddot{\xi}_1), \\ \text{if } \dot{x} = 0 \text{ and } |m_1\ddot{\xi}_1| > kN, \end{cases} \qquad (2)$$

$$N = Mg + m_2\ddot{\xi}_2, \qquad (3)$$

where k is the coefficient of friction between the supporting plane and the body, g is the acceleration due to gravity, and N is the normal pressure force applied to the body by the plane. Since the plane resists the penetration of the body but does not resist the separation, the quantity N must be nonnegative. Therefore, in accordance with (3), the contact of the body with the plane implies the inequality

$$Mg + m_2\ddot{\xi}_2 \geq 0. \qquad (4)$$

3. STATEMENT OF THE OPTIMAL CONTROL PROBLEM

Periodic motions of the internal masses will be constructed to provide a velocity-periodic motion of the body with a maximum average velocity for a prescribed period T. Due to the periodicity, it suffices to construct the desired motion on the interval $0 \leq t \leq T$. Assume without loss of generality that

$$\xi_1(0) = 0, \quad \xi_2(0) = 0, \quad x(0) = 0. \qquad (5)$$

These initial conditions are ensured by an appropriate choice of origin for the respective coordinates. The periodicity of the functions $\xi_1(t)$, $\xi_2(t)$, and $\dot{x}(t)$ implies the relations

$$\begin{gathered} \dot{x}(0) = \dot{x}(T) \\ \xi_i(0) = \xi_i(T) = 0, \quad \dot{\xi}_i(0) = \dot{\xi}_i(T), \quad i = 1,\ 2. \end{gathered} \qquad (6)$$

The accelerations of the internal masses relative to the body will be taken as the control variables subject to the constraints

$$|\ddot{\xi}_1(t)| \leq U_1, \quad -U_2^- \leq \ddot{\xi}_2(t) \leq U_2, \qquad (7)$$

$$\int_0^T \ddot{\xi}_i(t)dt = 0, \quad i = 1,\ 2, \qquad (8)$$

where

$$U_2^- = \min\left\{U_2, \frac{Mg}{m_2}\right\}. \qquad (9)$$

The parameters U_1 and U_2 are prescribed positive quantities characterizing the possibilities of the drives that control the motion of the internal masses. The inequalities of (7) take into account the characteristics of the drives and the condition of (4). The integral relations of (8) are equivalent to the conditions of (6) that relate $\dot{\xi}_i(0)$ and $\dot{\xi}_i(T)$. For the system's body to be able to be moved from a state of rest, it is assumed that

$$U_1 > k\left(\frac{Mg}{m_1} - \frac{m_2}{m_1}U_2^-\right). \tag{10}$$

For a given function $\ddot{\xi}_i(t)$ satisfying the condition of (8), there exist a unique function $\xi_i(t)$ satisfying the condition $\xi_i(0) = \xi_i(T) = 0$. This function is expressed by

$$\xi_i(t) = \frac{t}{T}\int_0^T \tau\ddot{\xi}_i(\tau)d\tau + \int_0^t (t-\tau)\ddot{\xi}_i(\tau)d\tau. \tag{11}$$

It is proved for the velocity-periodic motion of the body on a rough plane that the variable $\dot{x}$ vanishes at some instant on the interval of length T. By taking this instant as zero for measuring time, one can let $\dot{x}(0) = 0$ without loss of generality.

With reference to the aforesaid observations, the following optimal control problem is stated: *for the system of (1)–(3) considered on a fixed time interval* $0 \le t \le T$, *subject to the initial conditions* $x(0) = 0$ *and* $\dot{x}(0) = 0$, *find time histories for the relative accelerations of the internal masses,* $\ddot{\xi}_1(t)$ *and* $\ddot{\xi}_2(t)$, *that satisfy the constraints of (7) and (8) and maximize the average velocity* $V = x(T)/T$, *provided that* $\dot{x}(T) = 0$.

Introduce the dimensionless variables

$$\begin{aligned} &x' = \frac{x}{L}, \quad t' = \sqrt{\frac{g}{L}}t, \\ &T' = \sqrt{\frac{g}{L}}T, \quad r = \frac{R}{Mg}, \\ &\xi_i' = \frac{\xi_1}{L}, \quad u_i = \frac{m_i\ddot{\xi}_i}{Mg}, \\ &U_i' = \frac{m_iU_i}{Mg}, \quad i = 1,\ 2. \end{aligned} \tag{12}$$

where L is an arbitrary parameter having the dimension of length. In the dimensionless variables (the primes are omitted) the optimal control problem is stated as follows: *for the system*

$$\ddot{x} + u_1 = r, \tag{13}$$

$$r = \begin{cases} -k(1+u_2)\mathrm{sign}\dot{x}, \text{ if } \dot{x} \ne 0, \\ u_1, \text{ if } \dot{x} = 0 \text{ and } |u_1| \le k(1+u_2), \\ k(1+u_2)\mathrm{sign}(u_1), \\ \ \text{if } \dot{x} = 0 \text{ and } |u_1| > k(1+u_2), \end{cases} \tag{14}$$

considered on a fixed time interval $0 \le t \le T$, *subject to the boundary conditions*

$$x(0) = 0, \quad \dot{x}(0) = \dot{x}(T) = 0, \tag{15}$$

find the controls $u_1(t)$ *and* $u_2(t)$ *that satisfy the constraints*

$$|u_1(t)| \le U_1, \quad -U_2^- \le u_2(t) \le U_2, \tag{16}$$

$$\int_0^T u_i(\tau)d\tau = 0, \quad i = 1,\ 2, \tag{17}$$

where

$$U_2^- = \min\{U_2,\ 1\}, \quad U_1 > k(1 - U_2^-). \tag{18}$$

and maximize the quantity $V = x(T)/T$.

The controls $u_1(t)$ and $u_2(t)$ that satisfy the constraints of (16) and (17) will be referred to as admissible controls.

4. SOLUTION OF THE PROBLEM

The construction of an optimal control is based on the three propositions stated next.

Proposition 1. In an optimal motion, the inequality $\dot{x} \ge 0$ holds on the time interval $0 \le t \le T$, i.e., the body never moves backward.

Proposition 2. Optimal controls can be sought among the functions $u_1(t)$ and $u_2(t)$ that generate the motion possessing the following property:

$$\begin{aligned} &\dot{x} > 0 \text{ almost everywhere for } 0 \le t \le \delta, \\ &\dot{x} \equiv 0 \text{ for } \delta \le t \le T, \end{aligned} \tag{19}$$

where δ is some time instant.

Proposition 3. On the interval $[0, \delta]$, optimal controls providing the property of (19) have the form

$$u_1(t) = \begin{cases} -U_1, \text{ if } t \in [0, \tau_1), \\ U_1 \text{ if } t \in [\tau_1, \delta], \end{cases} \tag{20}$$

$$u_2(t) = \begin{cases} -U_2^-, \text{ if } t \in [0, \tau_2), \\ U_2 \text{ if } t \in [\tau_2, \delta], \end{cases} \tag{21}$$

where τ_1 and τ_2 are yet unknown switching instants.

The proof of these propositions is omitted in the present paper.

According to (19), (13), and (14), the motion of the body on the interval $[0, \delta]$ is governed by the equation

$$\ddot{x} + u_1 = -k(1 + u_2). \tag{22}$$

Since $\dot{x}(0) = \dot{x}(\delta) = 0$, integration of both sides of Eq. (22) on the interval $[0, \delta]$ leads to the relation

$$\int_0^\delta u_1(\tau)d\tau + k\delta + k\int_0^\delta u_2(\tau)d\tau = 0. \tag{23}$$

Substitute the expressions of (20) and (21) for u_1 and u_2 into (23) to obtain the relationship between the parameters τ_1, τ_2, and δ

$$\tau_1 = \frac{\delta[U_1 + k(1+U_2)] - 2k\widetilde{U}_2\tau_2}{2U_1}, \tag{24}$$

where

$$\widetilde{U}_2 = \frac{U_2^- + U_2}{2}. \tag{25}$$

The points τ_1 and τ_2 must lie on the interval $[0, \delta]$. This requirement, with reference to (24), is equivalent to the set of inequalities

$$\max\left\{0,\ \frac{\delta[k(1+U_2) - U_1]}{2k\widetilde{U}_2}\right\} \le \tau_2 \le \delta. \tag{26}$$

Since the structure of the optimal controls on the interval $[0, \delta]$ has been established, the solution of the problem is reduced to the determination of the length of the interval δ and the switching times τ_1 and τ_2 that maximize the quantity $V = x(\delta)/T$, provided that the controls $u_1(t)$ and $u_2(t)$ of (20) and (21) can be continued to the interval $[\delta, T]$ in such a way that the constraints of (16) and (17) hold and, in addition,

$$|u_1(t)| \le k(1 + u_2(t)),\ t \in [\delta, T]. \tag{27}$$

Since $\dot{x}(\delta) = 0$, the last condition ensures that the body stays at rest for $t \in [\delta, T]$.

If functions $u_1(t)$ and $u_2(t)$ satisfy the inequalities of (16) and (27) on the interval $[\delta, T]$, then these inequalities are valid for the mean values of the control functions on this interval. Therefore, one can replace the control functions by their mean values. Use (17), with reference to (20), (21), and (24), to obtain

$$\int_\delta^T u_1(t)dt = k[\delta(1+U_2) - 2\widetilde{U}_2\tau_2], \tag{28}$$

$$\int_\delta^T u_2(t)dt = 2\widetilde{U}_2\tau_2 - U_2\delta. \tag{29}$$

Hence, the mean values of the control functions on the interval $[\delta, T]$ are given by

$$\begin{aligned} \bar{u}_1^{[\delta,T]} &= \frac{k[\delta(1+U_2) - 2\widetilde{U}_2\tau_2]}{T-\delta}, \\ \bar{u}_2^{[\delta,T]} &= \frac{2\widetilde{U}_2\tau_2 - U_2\delta}{T-\delta}. \end{aligned} \tag{30}$$

Thus, the control functions for the entire interval $[0, T]$ can be defined as follows:

$$u_1(t) = \begin{cases} -U_1, \text{ if } t \in [0, \tau_1), \\ U_1, \text{ if } t \in [\tau_1, \delta), \\ \bar{u}_1^{[\delta,T]}, \text{ if } t \in [\delta, T], \end{cases} \tag{31}$$

$$u_2(t) = \begin{cases} -U_2^-, \text{ if } t \in [0, \tau_2), \\ U_2, \text{ if } t \in [\tau_2, \delta), \\ \bar{u}_2^{[\delta,T]}, \text{ if } t \in [\delta, T]. \end{cases} \tag{32}$$

By construction, these functions satisfy the integral constraints of (17) and the inequality constraints of (16) on the interval $[0, \delta]$. To provide the inequality constraints of (16) and (27) on the interval $[\delta, T]$, one should require these constraints to be satisfied for the mean values $\bar{u}_1^{[\delta,T]}$ and $\bar{u}_2^{[\delta,T]}$, i.e.,

$$\begin{gathered} |\bar{u}_1^{[\delta,T]}| \le U_1, \quad -U_2^- \le \bar{u}_2^{[\delta,T]} \le U_2, \\ |\bar{u}_1^{[\delta,T]}| \le k(1 + \bar{u}_2^{[\delta,T]}). \end{gathered} \tag{33}$$

Substituting the expressions of (30) into the relations of (33) yields inequalities for the desired parameters δ and τ_2. These inequalities should be added by those of (26) to obtain a complete set of inequalities constraining the parameters δ and τ_2. This set of inequalities can be reduced to the form

$$\delta[U_1 + k(1+U_2)] \le U_1T + 2k\widetilde{U}_2\tau_2, \tag{34}$$

$$2\widetilde{U}_2\tau_2 \le U_2T, \tag{35}$$

$$2\widetilde{U}_2\delta \le U_2^-T + 2\widetilde{U}_2\tau_2, \tag{36}$$

$$(1+U_2)\delta \le \frac{T}{2} + 2\widetilde{U}_2\tau_2, \tag{37}$$

$$\max\left\{0,\ \frac{\delta[k(1+U_2) - U_1]}{2k\widetilde{U}_2}\right\} \le \tau_2 \le \delta. \tag{38}$$

To determine the expression for the average velocity of the body to be maximized with respect to τ_2 and δ, substitute the controls of (31) and (32) into Eq. (3.1), solve the resulting equation, subject to the initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$, on the interval $0 \le t \le \delta$ to obtain the expression for $x(\delta)$, and then divide this expression by T. This yields

$$\begin{aligned} &V(\tau_2, \delta) = \frac{x(\delta)}{T} = \frac{[U_1^2 - k^2(1+U_2)^2]}{4U_1T}\delta^2 + \\ &\frac{k[U_1 + k(1+U_2)]\widetilde{U}_2}{U_1T}\tau_2\delta - \frac{k(U_1 + k\widetilde{U}_2)\widetilde{U}_2}{U_1T}\tau_2^2. \end{aligned} \tag{39}$$

Note that $x(\delta) = x(T)$, since the body stays at rest on the interval $\delta \le t \le T$.

By straightforward differentiation one can show that

$$\frac{\partial V}{\partial \delta} > 0, \text{ if } \tau_2 > \frac{\delta[k(1+U_2) - U_1]}{2k\widetilde{U}_2}. \tag{40}$$

From (38) it follows that the second inequality of (40) is valid in the interior of the admissible set for τ_2 and δ. Hence, the maximum of the function $V(\tau_2, \delta)$ lies on the boundary of the set of (34)–(38).

5. SPECIAL CASES. THE ESTIMATE OF THE EFFECT DUE TO THE VERTICALLY MOVING INTERNAL MASS

Consider two limiting cases of the vertical motion of the internal mass m_2, corresponding to $U_2 = 0$ and $U_2 \to \infty$. In the former case, mass m_2 does not move and the system behaves as a system with one internal mass moving horizontally. In the latter case, no constraint is imposed on the vertical motion of the internal mass m_2, apart from the condition of (4), ensuring permanent contact of the body with the plane. The solution of the optimal control problem in this case yields an upper limit for the average velocity of the body that theoretically can be attained due to a coordinated optimal control of motion of both internal masses under the constraint imposed on the magnitude of the relative acceleration of the internal mass m_1 moving horizontally. A comparison of the maximum velocities of the body in these two cases for the same constraint imposed on the acceleration of the internal mass moving horizontally enables one to evaluate the effect due to the introduction of the internal mass moving vertically.

Case 1. $U_2 = 0$. This case has been studied in detail by Figurina (2007). The relation $U_2 = 0$ implies $U_2^- = 0$, in accordance with (18), and, hence, $\widetilde{U}_2 = 0$, in accordance with (25). Substitute $U_2 = 0$ into the expression of (39), the inequalities of (34)–(37), and that of (38) multiplied by $2k\widetilde{U}_2$ to obtain

$$V = \frac{U_1^2 - k^2}{4U_1 T}\delta^2, \tag{41}$$

$$\delta \le \frac{U_1}{U_1 + k}T, \quad \delta \le \frac{T}{2}. \tag{42}$$

Minimization of the function V of (41) under the constraints of (42), with reference to the relation $U_1 > k$, following from (18) for $U_2^- = 0$, yields

$$V = \frac{U_1 T}{16}\left(1 - \frac{k^2}{U_1^2}\right), \quad \delta = \frac{T}{2}. \tag{43}$$

Substitute $\delta = T/2$, $U_2 = 0$, and $U_2^- = 0$ into (24) and (30) to find the optimal switching time of the control u_1 and the value of this control on the interval $\tau_1 \le t \le T$,

$$\tau_1 = \frac{T}{4}\left(1 + \frac{k}{U_1}\right), \quad \bar{u}_1^{[\delta,T]} = k. \tag{44}$$

Substitute the relations of (44) into (31) to obtain the expression for the optimal control $u_1(t)$. Return to the original dimensional variables to represent the solution of the problem in the form

$$\tau_1 = \frac{T}{4}(1+\rho), \quad \delta = \frac{T}{2}, \quad \rho = \frac{kMg}{m_1 U_1}, \tag{45}$$

$$\ddot{\xi}_1(t) = U_1 \begin{cases} -1, \text{ if } t \in [0,\ \tau_1), \\ 1, \text{ if } t \in [\tau_1,\ T/2), \\ \rho, \text{ if } t \in [T/2,\ T], \end{cases} \tag{46}$$

$$V = V_{\max}^{(1)} = \frac{m_1 U_1}{16M} T(1-\rho^2). \tag{47}$$

Case 2. $U_2 \to \infty$. We will present the final result, omitting the details of the solution. The dimensional values of τ_1, τ_2, and δ are defined by

$$\tau_1 = \frac{T}{2}, \quad \tau_2 = \delta = T, \tag{48}$$

the relative accelerations of the internal masses (optimal controls) have the form

$$\begin{aligned} \ddot{\xi}_1(t) &= \frac{Mg}{m_1}\operatorname{sign}\left(t - \frac{T}{2}\right), \\ \ddot{\xi}_2(t) &= -\frac{Mg}{m_2}\left(1 - T\delta(t-T)\right), \quad t \in [0,\ T], \end{aligned} \tag{49}$$

where $\delta(t-T)$ is Dirac's delta-function, and the maximum average velocity of the body is expressed as follows:

$$V = V_{\max}^{(2)} = \frac{m_1 U_1}{4M} T. \tag{50}$$

In the case under consideration, the acceleration of the internal mass m_2, moving vertically, involves an impulse component, which is expressed by the delta-function concentrated at the time instant T in (49). This implies that at the end of the period the velocity of mass m_2 undergoes a jump discontinuity $\Delta\dot{\xi}_2 = MgT/m_2$, i.e., an impact occurs. At the impact instant, mass m_2 finds itself at the extreme lower position.

Divide the expression of (50) by that of (47) to obtain the ratio of the maximum average velocities of the body in cases 1 and 2

$$\frac{V_{\max}^{(2)}}{V_{\max}^{(1)}} = \frac{4}{1-\rho^2}. \tag{51}$$

Hence, the introduction of the vertically moving mass to the system enables one, in principle, to obtain at least a 4-fold increase in the average velocity of the body, as compared with the maximum average velocity that can be attained in the system with one internal mass, moving horizontally, for the same period T and the total mass of the system M.

6. CONTROL WITH CONSTRAINTS IMPOSED ON THE DISPLACEMENT OF THE INTERNAL MASSES

The optimal control problem solved in the previous section does not involve constraints on the overall displacements of the internal masses. However, when designing control modes for realistic vibration-driven systems one has to impose such constraints because the systems's body, inside which the internal masses can move, has certain finite dimensions. Using the solution obtained, one can construct a control that takes into account the inequalities

$$\Xi_i \le L_i, \quad \Xi_i = \max_{t\in[0,T)} \xi_i(t) - \min_{t\in[0,T)} \xi_i(t), \quad i = 1,\ 2, \tag{52}$$

where L_i is the maximum overall displacement allowed for mass m_i. We will construct such a control for the case of $U_2 \to \infty$. The respective relations for the case of $U_2 = 0$ are presented in (Figurina, 2007).

Substitute the expressions of (49) into (11) to find $\xi_i(t)$. Use the expression obtained to calculate the overall displacement of both masses in accordance with (52). This yields

$$\Xi_1 = \frac{U_1 T^2}{16}, \quad \Xi_2 = \frac{MgT^2}{8m_2}. \tag{53}$$

The maximum average velocity for a fixed period is defined by (50) and increases as T increases. Therefore, we choose the maximum T for which the constraints of (52), in which Ξ_i are defined in accordance with (53), hold. As a result, we obtain

$$\begin{aligned} T &= 4\sqrt{\frac{L_1}{U_1}}\min\{1,\ \sigma\}, \\ V_{\max}^{(2)} &= \frac{m_1}{M}\sqrt{U_1 L_1}\min\{1,\ \sigma\}, \\ \sigma &= \sqrt{\frac{m_2 L_2 U_1}{2MgL_1}}. \end{aligned} \tag{54}$$

From Eq. (54) it follows that the average velocity of the body increases without limit as U_1 increases.

7. CONCLUSIONS

Optimal periodic motions for the internal masses of a vibration-driven system have been constructed to provide a velocity-periodic motion of the main body along a rough horizontal plane with a maximum average velocity. A system with two internal masses, one of which moves horizontally and the other moves vertically, has been considered. With the mass moving horizontally, one can control the direction and the magnitude of dry friction between the body and the supporting plane but cannot control the normal pressure acting on the body. The control of the normal pressure of the plane on the body due to the vertical motion of an internal mass enables one to obtain a substantial (theoretically greater than 4-fold) increase in the average velocity of motion of the system, as compared with the case, where motion of an internal mass along the vertical does not occur. The results obtained indicate that involving vertically moving masses in the design of vibration-driven systems and the joint optimization of the modes of motion of the internal masses are advisable.

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