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SIMPLIFIED DYNAMICS MODEL FOR SUBCRITICAL REACTOR CONTROLLED BY LINEAR ACCELERATOR

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Abstract

Subcritical reactor controlled by linear proton accelerator is considered. Problems of such systems dynamics modeling, taking into account the temperature feedbacks on coolant and fuel are investigated. The obtained dynamics equations describe physical processes with characteristic times differing by orders of magnitude. Due to this feature some physical approximations were introduced to simplify the equations of dynamics based on the point kinetics model. This simplification allows to use standard methods for numerical integration of ODE.

Key words

Subcritical reactor, dynamics with feedbacks, ADS control, linear accelerator.

1 Introduction

Accelerator driven systems (ADS) is a new type of reactor which produces power even though it remains sub-critical throughout its life [Carminati and et al, 1993]. The main elements of ADS are subcritical reactor, neutron producing target and charged particle accelerator. Nowadays ADS is considered to be one of the most prospective ways to solve the main problems of nuclear power engineering such as transmutation of long-lived radioactive waste, safe energy production and breeding new fissionable elements [Gerasimov and Kiselev, 2001], [Golovkina et al., 2016a].

In contrast to traditional critical reactors, where the control on reactor power rate is carried out by the neutron absorbing rods, subcritical reactor is controlled by charged particle accelerator in ADS. Reactivity coefficient changes in time due to feedbacks on temperature effects (fuel and coolant) and simultaneous fuel burning and fission products accumulation. So problem of ADS power-level maintenance with accelerator as well as reactor dynamics investigation is arised.

In [Golovkina, 2017] was shown that the considered physical processes progress in time with speeds different by orders of magnitude. In order to correctly take this speciality into account in numerical calculations simplified dynamics model based on point kinetics is used in [Golovkina, 2017]. In this paper four successive initial dynamics equations simplification were proposed and their correctiveness on each step was shown. As a result the physical limits of applicability of these simplifications are formulated. It is shown that characteristic times for physical processes in ADS reactor meet these conditions. Using the finally obtained equations physically appropriate results were calculated.

2 Subcritical Reactor Control with Accelerator

Dynamics of the ADS subcritical reactor depends on internal and external feedbacks. Internal feedbacks are defined by physical properties of the reactor core, whereas external feedbacks reflect reactor connection with power plant (coolant flow, coolant temperature at the reactor inlet).

For stable ADS operation at the constant power-level the reactor core must have a negative feedback on the fuel and coolant temperature as well as the average negative reactivity coefficient, which ensures reactor self-regulation and maintenance of the average temperature.

Thermal power-level of the reactor core is determined by the following expression:

$$N_T = \frac{E_f Q_f}{\nu}, \tag{1}$$

where E_f — energy released per one fuel nucleus fission, ν — average number of neutrons coming out in fission event, Q_f — intensity of fission neutrons generation, which in first approximation can be calculated

by the following formula

$$Q_f = S\frac{1-k_{\text{eff}}}{k_{\text{eff}}}. \quad (2)$$

Here S — external neutron source intensity.

As can be seen from equations (1) and (2), the reactor power depends on the intensity of the electronuclear neutron source and the value of effective multiplication factor k_{eff}, which is chosen to provide nuclear safety and nowadays for ADS it is admitted not to exceed value 0.98.

3 Dynamics of Subcritical Reactor Based on Point Kinetics Model

Here and later instead of efficient multiplication factor k_{eff} we will consider reactivity of the reactor $\rho = \frac{k_{\text{eff}}-1}{k_{\text{eff}}}$ which is a dimensionless quantity used to characterize reactor deviation from the critical state [Keepin, 1965].

Internal feedbacks cause the dependence of reactivity on the fuel elements and coolant temperature. The effect of the reactor temperature on its reactivity is called the fuel temperature effect, and the influence of the coolant temperature — coolant temperature effect. Temperature effects are characterized by the respective temperature coefficients of reactivity α_T and α_{TH}. Usually the dependence of reactivity on temperature is represented by a linear function [Beckman, 2005]

$$\rho = \alpha_T\left(T_T - T_T^{\text{av}}\right) + \alpha_{TH}\left(T_{TH} - T_{TH}^{\text{av}}\right), \quad (3)$$

where T_T and T_{TH} — current fuel and coolant temperature correspondently, T_T^{av} T_{TH}^{av} — temperature of fuel and coolant in the operating point.

The reactivity temperature effect is determined by two components: dependence of the core materials density on temperature and the Doppler effect [Usynin and Kusmartsev, 1985].

Taking into account the remarks made above, reactor core dynamics with thermal feedbacks is described by the following system of equations:

$$\begin{aligned}
&\frac{d\varphi(t)}{dt} = \frac{(\rho(t)-\beta_{\text{eff}})\,\varphi(t)}{l} + \sum_{i=1}^{N}\lambda_i C_{\text{eff}}^i(t) + q_{\text{eff}}(t),\\
&\frac{dC_{\text{eff}}^i(t)}{dt} = \frac{\beta_{\text{eff}}^i\varphi(t)}{l} - \lambda_i C_{\text{eff}}^i(t), \quad (4)\\
&\rho(t) = \rho_{\text{av}} + \alpha_T\left(\widehat{T}_T(t) - T_T^{\text{av}}\right) +\\
&\alpha_{TH}\left(T_{TH}(t) - T_{TH}^{\text{av}}\right),\\
&M_{TH}C_{TH}\frac{dT_{TH}(t)}{dt} = 2GC_{TH}(t)\left(T_{\text{in}} - T_{TH}(t)\right) +\\
&+hS\left(T_{\text{w}}(t) - T_{TH}(t)\right), \quad (5)\\
&\rho_T(T_T,r)C_T(T_T,r)\frac{\partial T_T(r,t)}{\partial t} = \quad (6)\\
&= \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_T(T_T,r)\frac{\partial T_T(r,t)}{\partial r}\right) + q_v(r,t),\\
&t>0, \quad 0<r<R,\\
&\varphi(0) = \varphi^{\text{ini}},\ C_{\text{eff}}(0) = C_{\text{eff}}^{\text{ini}},\\
&\rho(0) = \rho^{\text{ini}},\ T_{TH}(0) = T_{TH}^{\text{ini}},\ T_T(r,0) = T_T^{\text{ini}}(r).
\end{aligned}$$

Here t — time, φ — neutron flux amplitude, β_{eff}^i — effective delayed neutron fraction, $\beta_{\text{eff}} = \sum_{i=1}^{N}$, N — number of delayed neutrons groups, l — mean prompt generation time, C_{eff}^i — effective concentration of delayed neutron precursors, q_{eff} — effective external neutron source, λ — r — fuel element radius coordinate, M_{TH} — mass of coolant, $T_T(r,t)$ — fuel element temperature distribution, $\widehat{T}_T(t)$ — volume average temperature of the fuel element, T_{w} — temperature of the fuel element wall, T_{TH} — coolant temperature, G — coolant mass flow, S — area of heat delivery surface of the fuel elements in the reactor core, α_T — fuel temperature coefficient, α_{TH} — coolant temperature coefficient, h — coolant heat-transfer coefficient, λ_T — heat conductivity coefficient, C_T — specific heat capacity of the fuel element, ρ_T — density of the fuel element.

Taking into account equation (1) and assumption of time and spatial variables separation, made during point kinetics equations derivation [Usachev, 1955], change in time of energy release spatial distribution in the core is determined by the expression

$$N_0(\mathbf{r},t) = \frac{\varphi(t)E_f\int dE\int d\mathbf{\Omega}\mathbf{M_f}\tilde{F}(\mathbf{r},\mathbf{\Omega},E)}{\nu},$$

and change in time of the integral energy release:

$$N(t) = \frac{\varphi(t)E_f\left\langle\mathbf{M_f}\tilde{F}(\mathbf{r},\mathbf{\Omega},E)\right\rangle}{\nu}, \quad (7)$$

then distribution of specific (by volume) energy release in equation (6) is defined as $q_v(r,t) = \frac{N(r,t)}{V_0}$,

where V_0 — volume of the reactor core, $\tilde{F}(\mathbf{r}, \mathbf{\Omega}, E)$ — neutron flux spatial–angle–energy distribution, $\varphi(t)$ — shape factor of the neutron flux, $\mathbf{M_f}$ — linear fission operator, $\langle \cdot, \cdot \rangle$ — scalar product in l_2.

Simultaneous integration of equations (4), (5) and (6) with given initial and boundary conditions is rather difficult problem, since the physical processes described by them are characterized by time constant differing in orders of magnitude [Strakhovskaya and Fedorenko, 1998].

4 Physical Simplifications

Four physical components with different characteristic time, can be separated [Golovkina, 2017]:

1. *Prompt neutrons.* Average prompt neutrons lifetime l in the reactor depends on the neutron energy spectrum and changes from $5 \cdot 10^{-7}$ s (for fast reactors) to $5 \cdot 10^{-4}$ s (for thermal reactors).
2. *Delayed neutrons.* Average delayed neutrons lifetime $t_{\text{del}} = 0.1 - 10$ s.
3. *Accelerator driver current.* Micro impulses period in the linear accelerator is $T = 5 \cdot 10^{-9}$ s, and macro impulses period is $T = 5 \cdot 10^{-3}$ s.
4. *Thermal feedbacks.* The time constant, characterizing the rate of the fuel elements temperature change with energy release change in time is not less than 0.01 s. The time constant characterizing the rate of coolant temperature change is determined by the time of its passage through the reactor core and is a few seconds.

Thus, the system of nonstationary equations (4)–(6) is characterized by significant variety of time constants defining the dynamics of simulated physical processes. So numerical solution of this system by standard methods [Hairer et al., 1993] requires the use of integration step, corresponding to the physical process with a minimum characteristic time (about 10^{-7} s). This approach is not appropriate, since the reactor dynamics should be determined within a long period of time. In this regard, the influence of each of these physical components on the dynamics of subcritical reactor controlled by linear accelerator was analyzed and approximate models, which make it possible to use traditional methods for ODE numerical solution are obtained.

4.1 Prompt-jump Approximation

As noted above, the time constant characterizing the rate of fuel elements temperature change in time, and the lifetime of delayed neutrons are approximately 0.01 s, that is several orders of magnitude greater than the prompt neutrons average lifetime in the reactor. In this regard, the dynamics of subcritical reactor taking into account the feedbacks can be described by quasistatic prompt-jump approximation [Hetrick, 1971], because it predicts a sudden change in the reactor power following a sudden change in reactivity. This approximation is based on the assumption, that the mean prompt neutrons generation time is extremely small and can be equated to zero.

Prompt-jump approximation is valid only in case when the relative rate of change of reactor power in a mean prompt neutrons generation time is sufficiently small so that holds [Akcasu et al., 1971]:

$$\left| \frac{l}{\beta_{\text{eff}}} \frac{d\varphi(t)/dt}{\varphi(t)} \right| \ll \left| 1 - \frac{\rho(t)}{\beta_{\text{eff}}} \right|.$$

In this case the term $\dfrac{l}{\beta_{\text{eff}}} \dfrac{d\varphi(t)}{dt}$ in (4) can be neglected and resulting approximate equations read

$$\begin{aligned} \varphi(t) &= \frac{\left(\sum\limits_{i=1}^{N} \lambda_i C^i_{\text{eff}}(t) + q_{\text{eff}}(t) \right) l}{\beta_{\text{eff}} - \rho(t)}, \\ \frac{dC^i_{\text{eff}}(t)}{dt} &= \frac{\beta^i_{\text{eff}} \varphi(t)}{l} - \lambda_i C^i_{\text{eff}}(t), \\ \varphi(0) &= \varphi^{\text{ini}},\ C_{\text{eff}}(0) = C^{\text{ini}}_{\text{eff}}. \end{aligned} \tag{8}$$

In the Figure 1 the relative ADS power rate change in time with external pulsed neutron source is shown. The duration and the period of source pulses correspond to the linear accelerator current macro pulses (fig. 2). Calculations were carried out for the fast reactor core ($l = 3 \cdot 10^{-6}$ s) using models (4)–(6) and (8), (5)–(6). It is clear from the Figure 1, that these results almost coincide, so that demonstrates the possibility of prompt neutrons approximation usage in calculations.

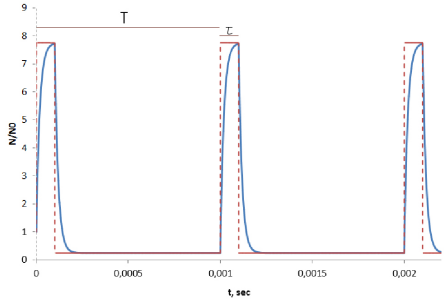

Figure 1. Dependence on time of the relative power rate for ADS with fast reactor (general point kinetic approximation and prompt-jump approximation)

Integration step for point kinetics equations (4) shouldn't exceed the average neutron life time in the reactor [Golovkina et al., 2016b]. For the fast reactors this value is $\sim 10^{-8} : 10^{-7}$ s, so the solving of equation (4) for the long lasting period of time becomes impossible. On the other hand, the usage of prompt neutrons approximation allows to increase the integration

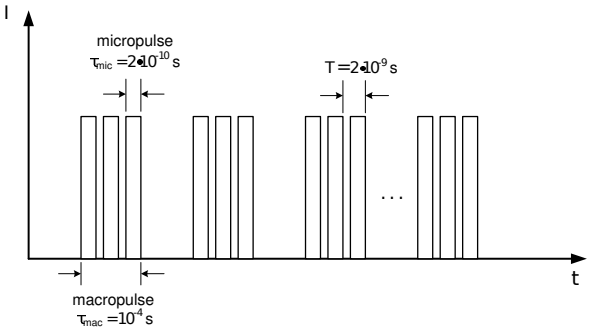

Figure 2. Charged particles current change in time in a linear accelerator

step in several orders of magnitude. Also prompt neutrons approximation reduces the number of differential equations by one.

4.2 Accelerator current continuity approximation

Period of current micropulses in the linear accelerator ($T = 5 \cdot 10^{-9}$ s) is rather smaller than the mean prompt neutrons generation time, so the intensity of the additional neutron source in ADS with a linear accelerator can be treated as a sequence of square pulses with period and length corresponding to the average current value in the macropulse.

Period of current macropulse in the linear accelerator, proposed to be used in ADS, is $T = 5 \cdot 10^{-3}$. Let us analyze the influence of this value on a fuel element temperature state, which defines the longterm ADS dynamics. For this reason let us consider the heat-balance equation for a cylindrical fuel element:

$$M_T C_T \frac{d\widehat{T}_T(t)}{dt} = N - \frac{2}{R} h(\widehat{T}_T(t) - T_{TH}), \qquad (9)$$

$$\widehat{T}_T(0) = T_{TH}.$$

Here M_T — mass of fuel, R — radius of fuel element, temperature of coolant T_{TH} and power of internal energy-release N are supposed to be constant, also temperature change on the radius of fuel element is neglected.

Let us estimate the fuel element time constant (time for which its temperature changes in e times) using equation (9). Turn to the dimensionless variables $u = \dfrac{\Delta\widehat{T}_T}{\Delta T_T^0} = \dfrac{\widehat{T}_T(t) - T_{TH}}{T_T - T_{TH}}$ and $\tau = \dfrac{tN}{\rho_T C_T \Delta T_T^0}$:

$$\frac{du(\tau)}{d\tau} = 1 - \frac{2h\Delta T_T^0}{RN} u(\tau), \;\; u(0) = 0. \qquad (10)$$

The solution of equation (10) is an exponential function like $u(\tau) = 1/a - \exp(-a\tau)/a$, where $a = \dfrac{2h\Delta T_T^0}{RN}$. Then $u(t) = 1/a - \exp(-t/t)/a$ and time constant of the fuel element will be calculated using the formula

$$t = \frac{R\rho_T C_T}{2h}. \qquad (11)$$

In case $t_{\text{f.e.}} \gg t_{\text{mac}}$, then the temperature of the fuel element won't change in the time between power pulses, so the temperature oscillation can be neglected. Due to this fact external neutron source can be treated as constant in time with intensity equal to the average time value of $q(t)$:

$$q_{\text{av}} = \frac{Q^{\max}\tau}{T}. \qquad (12)$$

Here $Q^{\max}$ — amplitude of neutron source power macropulse, τ — macropulse length, T — macropulse period.

$t_{\text{f.e.}}$ depends on physical characteristics of fuel element material and lies in the range 0.01–1 s, that is in order of magnitude greater than current macropulse period in linear accelerator.

To illustrate the formulated criteria let's consider the problem of fuel element heating-up during the ADS reactor start from the cold state with pulsed neutron source with period $t_{\text{f.e.}} \gg t_{\text{mac}}$ and $t_{\text{f.e.}} < t_{\text{mac}}$. This process is described by nonstationary heat conductivity equation (6) with initial

$$T_T(r, 0) = T_{TH}$$

and boundary condition on the external surface of the fuel element:

$$-\lambda_T \frac{\partial T_T(r,t)}{\partial r}\bigg|_{r=R} = h(T_T(r,t) - T_{TH}). \qquad (13)$$

Here h — heat-transfer coefficient.

In figs. 3 and 4 the dependence of ADS fuel element temperature with pulsed neutron source for which in the first case condition $t_{\text{f.e.}} \gg t_{\text{mac}}$ is realized, and in the second case — $t_{\text{f.e.}} < t_{\text{mac}}$. When $t_{\text{f.e.}} \gg t_{\text{mac}}$ external neutron source pulses doesn't influence on the temperature change mode, so in this case the approximation (12) can be successfully used for the reactor longterm dynamics analysis.

4.3 Point Approximation of a Fuel Element

As was shown above, in the general case fuel element temperature distribution is described by nonstationary heat conductivity equation (6) with given initial and boundary conditions.

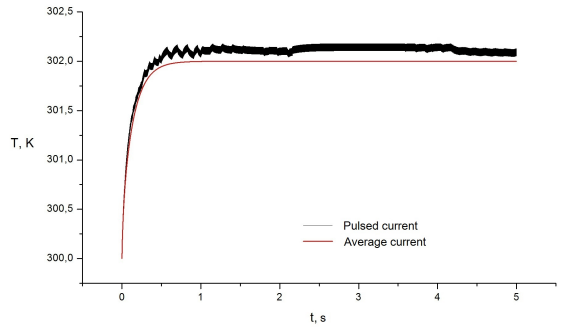

Figure 3. Temperature of the fuel element change in time ($t_{\text{fel}} \gg t_{\text{mac}}$). Red – average current, black pulsed current.

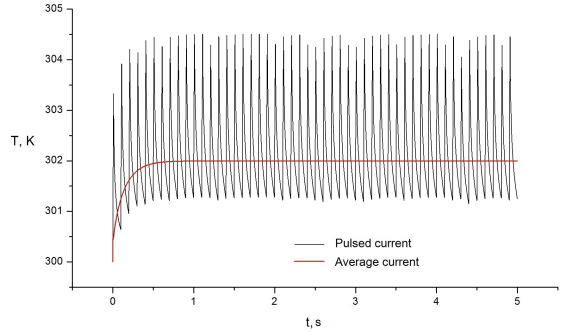

Figure 4. Temperature of the fuel element change in time ($t_{\text{fel}} < t_{\text{mac}}$). Red – average current, black pulsed current.

The main criteria defining the solution of nonstationary heat conductivity equation are Bi (Bio) Fo (Fourier) [Lykov, 1967]:

$$Bi = \frac{h}{\lambda} R, \tag{14}$$

$$Fo = \frac{\lambda_T t}{\rho_T C_T R^2}. \tag{15}$$

The process of fuel element heating or cooling can be divided into three stages. Initial temperature distribution plays a greater role on the first stage. It means that any irregularity in initial temperature distribution is reflected on temperature distribution in the next instants of time. The second stage is called regular mode: temperature distribution inside the fuel element doesn't depend on initial distribution. The third stage corresponds to the steady state, when the temperature in every point of fuel element equals to the temperature of surrounding environment.

The dependence $Fo^{\text{st}}(Bi)$, defining the regular thermal conditions coming for cylinder is presented in Figure 5 [Lykov, 1967], $\lim\limits_{Bi \to \infty} Fo^{\text{st}} = 0.58$.

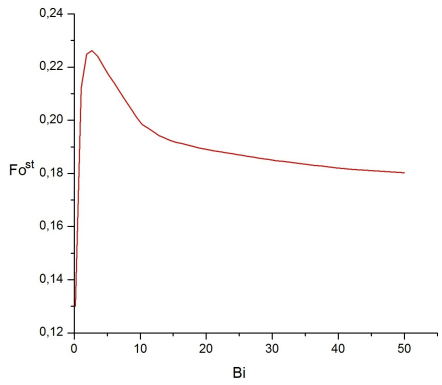

Figure 5. Dependence of Fo value on Bi value, defining the regular thermal conditions beginning.

Bi value is uniquely defined for the fuel element of known radius and given heat conductivity and heat transfer coefficients (14). The correspondent value Fo^{st} is found using the plot in Figure 5. With this value the transient period of regular thermal conditions can be defined:

$$t_{\text{reg}} = \frac{Fo^{\text{st}}(Bi)\rho_T C_T R^2}{\lambda_T}. \tag{16}$$

In case, when the characteristic time of delayed neutrons t_{del} exceeds the time of regular thermal conditions setting up, then instead of solving of nonstationary heat conduction equation (6) point model, based on the thermal balance in the fuel element, can be used:

$$M_T C_T \frac{d\widehat{T}_T(t)}{dt} = N(t) - hS\left(T_{\text{w}}(t) - T_{TH}(t)\right). \tag{17}$$

Here $\widehat{T}_T(t)$ — change in time of the average volume temperature of the fuel element, $N(t)$ — change in time of thermal power released inside the fuel element (7), $hS\left(T_{\text{w}}(t) - T_{TH}(t)\right)$ — quantity of heat, removed by coolant from the fuel element surface area S, $T_{\text{st}}(t)$ — temperature of the fuel element surface, M_T — mass of the fuel element, C_T — specific heat capacity of the fuel element material.

Temperature of the fuel element heat delivery surface is presented in the right side of equation (17). Let's find its connection with the average temperature. The dependence of temperature of cylindrical fuel element on its radius in stationary heat conductivity mode is described by the following function [Isachenko et al., 1975]:

$$T(r) = T_{TH}^{\text{st}} + \frac{N^{\text{st}} R}{2h} + \frac{N^{\text{st}}}{4\lambda_T}\left(R^2 - r^2\right), \tag{18}$$

where R — radius of the fuel element.

Let's find the coupling coefficient between the average and surface temperature of the fuel element using the expression (18):

$$\mu = \frac{T_{\text{w}}^{\text{st}}}{T_{\text{av}}^{\text{st}}} = \frac{T_{TH}^{\text{st}} + N^{\text{st}}R/2h}{T_{TH}^{\text{st}} + N^{\text{st}}R/2h + N^{\text{st}}R^2/8\lambda_T},$$

then the equation (17) takes on form

$$M_T C_T \frac{d\widehat{T}_T(t)}{dt} = N(t) - hS\left(\mu\widehat{T}_T(t) - T_{TH}(t)\right). \tag{19}$$

Let us consider two systems, in which coefficients of heat conductivity and heat transfer are chosen in way that condition $Bi \ll 1$ is achieved for the first system, and $Bi \gg 1$ — for another one. In figs. 6, 7 dependence of volume average temperature on time calculated using (6) and dependence of point temperature calculated using (19) are presented. As can be seen from the figure, these plots almost coincide when $Bi \ll 1$, so in this case point fuel element model (19) is sufficient to use for average temperature of the fuel element.

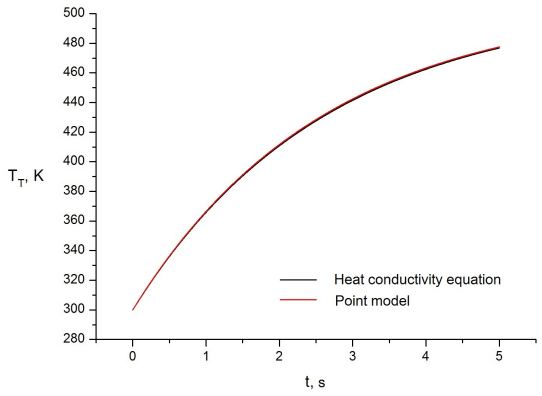

Figure 6. Fuel element average temperature change in time. Black line — heat conductivity equation (6), red line — point approximation model (19) ($Bi \ll 1$).

It should also be noted, that time constant of the coolant is defined by the speed of its circulation in the reactor core and is approximately several seconds. Due to this fact the coolant feedback can be excluded from the equation (3).

4.4 Final Simplified Model for Long-term ADS Dynamics Calculation

Under these physical assumptions: prompt-jump approximation [Keepin, 1965], accelerator current continuity approximation and point approximation of the

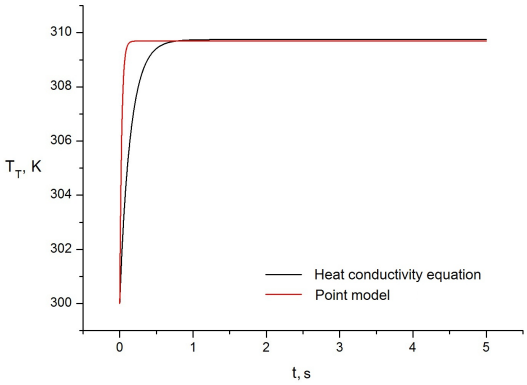

Figure 7. Fuel element average temperature change in time. Black line — heat conductivity equation (6), red line — point approximation model (19) ($Bi \gg 1$).

fuel element, the resulting system of differential–algebraic equations for longtime subcritical reactor dynamics calculation are obtained:

$$\begin{aligned}
&\varphi = \frac{\left(\sum\limits_{i=1}^{N} \lambda_i C_{\text{eff}}^i(t) + q_{\text{eff}}^{\text{av}}\right) l}{\beta_{\text{eff}} - \rho(t)},\\
&\frac{dC_{\text{eff}}^i(t)}{dt} = \frac{\beta_{\text{eff}}^i \varphi(t)}{l} - \lambda_i C_{\text{eff}}^i(t),\\
&\rho(t) = \rho_{\text{av}} + \alpha_T\left(\widehat{T}_T(t) - T_T^{\text{av}}\right),\\
&M_{TH}C_{TH}\frac{dT_{TH}(t)}{dt} = 2GC_{TH}(t)\left(T_{\text{in}} - T_{TH}(t)\right) +\\
&hS\left(\mu\widehat{T}_T(t) - T_{TH}(t)\right),\\
&M_T C_T \frac{dT_T(t)}{dt} = N(t) - hS\left(\mu\widehat{T}_T(t) - T_{TH}(t)\right).\\
&C_{\text{eff}}(0) = C_{\text{eff}}^{\text{ini}},\ \rho(0) = \rho^{\text{ini}},\ T_{TH}(0) = T_{TH}^{\text{ini}},\\
&\widehat{T}_T(0) = T_{TH}^{\text{ini}}.
\end{aligned} \tag{20}$$

System of equations (20) can successfully be used for longterm ADS dynamics analysis.

5 Calculation Results

Let us consider ADS with fast subcritical reactor and external pulsed neutron source after start-up from a cold state and coming up to the given power level. In Figure 8 and 9 as an example, calculation results of reactivity and power rate change are presented [Golovkina et al., 2014]. As can be seen from Figure 8 in the initial moments there is a power excursion, which is suppressed by fuel temperature feedback (the Doppler effect). It also should be noted that fuel temperature remains constant after reactor start up due to fuel elements thermal inertia [Golovkina et al., 2016b].

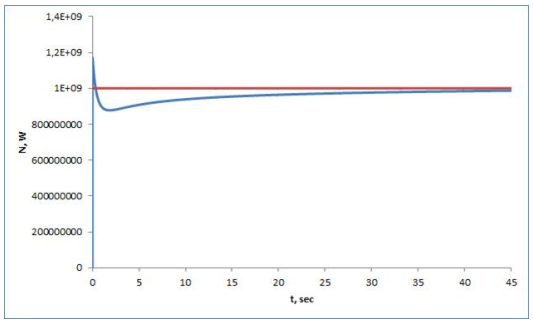

Figure 8. The ADS reactor power level change in time

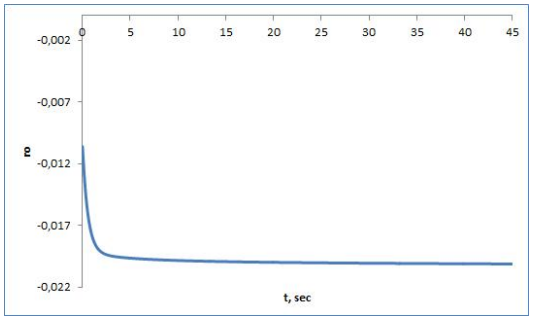

Figure 9. The reactivity coefficient change in time

6 Conclusion

Questions of dynamics modeling in subcritical reactor, taking into account the temperature feedbacks on the basis of point kinetics model are considered. Influence of the modeling processes characteristic time on the longtime reactor core dynamics is investigated. After comprehensive analysis the initial dynamics equation were simplified in order to use for their numerical integration standard methods, particularly Runge-Kutta method of 4th order. As an example, subcritical reactor dynamics during start-up was calculated. As a result a short-time power surge higher the power rating level can be observed, wheres the fuel temperature doesn't exceed its rated value.

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