QUALITATIVE BEHAVIOR PREDICTION OF TRANSITION STATES IN A CONVECTION LOOP

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***Abstract*— In this work a procedure to qualitative long-term prediction of limit sets of a convection-loop system is presented. The procedure employs estimation of system parameters of a previously defined process structure with time-varying coefficients and an interpolator to track in advance the path of the parameter vector on a basin of stationary limit sets. Numeric simulations with a convection-loop system illustrate the features of the approach.**

I. INTRODUCTION

The dynamics of a thermal convection loop heated from below and cooled from above is investigated widely for control purposes in the literature (see for instance [1]). This is represented by three equations similar to the celebrated Lorenz equations employed as a general model to local weather predictions [2]. Roughly speaking, when the Rayleigh number increases, *i.e.*, the heat from below increases, the loop dynamics exhibits a diversity of transitions from a no-motion state to uniform convection and finally to chaos. The last transition occurs through a subcritical Hopf bifurcation.

Behavior prediction is an important task related to monitoring, supervision and diagnostics for control purposes. Predictions of the system behavior from time sequences can be generally possible when certain periodicity conditions are fulfilled. When transitions occur from one state to another one, a more complex time-varying dynamics takes place. In this situation predictions are more difficult due to permanent change in the system parameters and/or structure. Assuming a fixed model structure, the prediction is based on certain estimations of the physical parameters at any time point.

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Due to the presence of chaotic states in the transition, one is generally more interested to define what kind of qualitative state is being travelled by the dynamics path in the parameter space and if from this point in advance, a prediction is possible to next qualitative states, quantifying also how long they should remain in time. In this way, slow state transition can be interpreted as a successive change of stationary limit sets, from which a qualitative prediction of the behavior is possible.

This paper is concerned with the identification of qualitative long-term tendencies of the system behavior drawn out from tracking of estimated parameter trajectories over a dimensional bifurcation diagram of the system, which is constructed a-priori using Ljapunov spectra. To this end a time-varying model of the convection loop is proposed and a strategy for prediction is developed.

The interaction among temperature gradients points situated on and at both sides of the torus, points situated up and down, and the flow rate of a viscous fluid inside, is emulated by a modified Lorenz-like equation set. The problem that arrives in the identification when signals are not sufficiently informative, is dealt in the context of the bifurcation diagram and classes of limit sets on a basin. To this end, on-line parameter estimation can provide advantages beside other techniques with statistical background, mainly if physical structural models are available. Numerical simulations will illustrate the features of our approach.

II. THERMAL CONVECTION LOOP

The thermal convection loop consists of a circular pipe standing in a vertical plane and containing viscous fluid. The fluid particles in motion can be tracked by means of a bead as shown in an implementation depicted in Fig. 1, which has low inertia and a similar density to the one of the fluid at every temperature. One particularity of the system is that the heat rate Q over the entire loop is zero.

Regarding the information in Fig. 1, let the state vector be $\mathbf{x} = (x, y, z)^T$ with $x = u$, $y = T_w(0) - T_w(\pi)$ and $z = T_w(\pi/2) - T_w(\pi 3/2)$, where $T_w(\theta)$ is the wall temperature at a angle position θ. Thus the proposed dynamics equations are

$$\begin{aligned} \dot{x} &= \sigma(t)(-x+y) \\ \dot{y} &= \beta(t)x - \alpha_y(t)y - \gamma_v(t)xz \\ \dot{z} &= -\alpha_z(t)z + \gamma_h(t)xy - r(t), \end{aligned} \quad (1)$$

where σ is the Prandtl number and r the Rayleigh number.

For the stationary system with σ positive and $0 < r \leq 1$ the system has one globally attracting equilibrium $(0, 0, -r)^T$ that corresponds to the no-motion state of the thermal convection. For $r > 1$ two equilibria appear that represent the states of the steady convection. They are respectively: $x^* = y^* = \pm\sqrt{r-1}$, $z^* = -1$. At $r = \sigma(\sigma+4)/(\sigma-2)$ the equilibria loose their stability and appear a Hopf bifurcation [3], and for greater values of r the system has no more equilibrium points [4]. Finally, the coefficient β defines the linear interaction of the flow over the temperature gradient horizontally. In many approaches the gains are considered $\gamma_v = \gamma_h = \alpha_y = 1$ while r, β and α_z are more changeable within large ranges. It is worth noticing that model (1) has a Lorenz-similar structure with time-varying parameters.

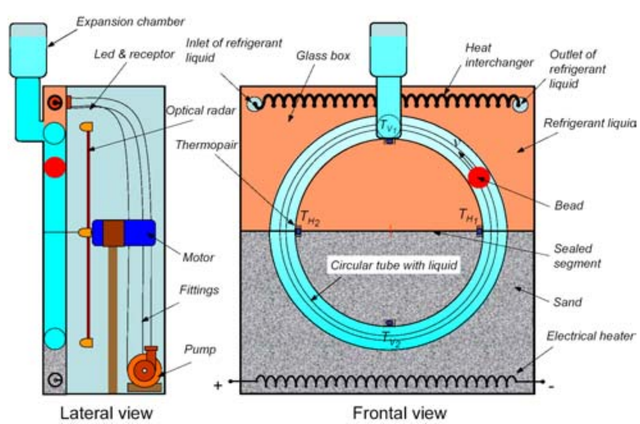

Figure 1-Thermal convection loop

III. Predictor

Our basic approach is illustrated in Fig. 2. The prediction problem is posed basically as a parameter identification problem of a time-variant system. It consists in the on-line estimation of a set of physical parameters of the model (1) with data being provided from the behavior during a state transition. In this form the estimated parameter trajectory can be superposed in a basin of such parameters.

It is then aimed that predictions of the path for the immediate time in advance can be plotted in the same basin. In this way, path crossings of the contours of limit sets help to detect qualitative states of the system. The key idea is to use a model for the time-dependency of each parameter in such a way that a prediction of the future evolution of the path be possible. So qualitative behavior changes can be detected and assigned in a true time scale with certain precision. Furthermore, it is supposed that the parameter change sufficiently slow so as to endorse the hypothesis of stationarity of the basin and that the domains of attractions for the basin are sufficiently extensive so as to guarantee that trajectories enter them when travelling over the diverse basin regions.

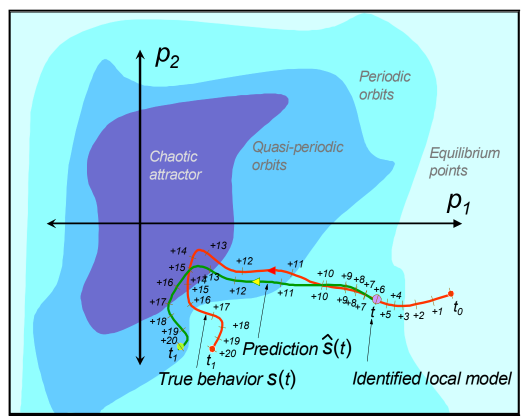

Figure 2 - Prediction of parameter path in a basin

Let us suppose a time-varying processes with coefficients that change slowly in the time $t \in [t_0, t_1]$ so that its temporal behavior can be capture in a immediate time by

$$\hat{p}_i(t) = \hat{\theta}_{i_0} + \hat{\theta}_{i_1}\, t + ... + \hat{\theta}_{i_n} t^n, \quad (2)$$

with $\hat{p}_i \in \{\sigma, \beta, \alpha_y, \gamma_v, \gamma_h, \alpha_z, r\}$ and $\hat{\theta}_{i_j}$ will be assumed constant within $[t_0, t_1]$ and the $\boldsymbol{\phi}_i$'s are the regressors constructed with the signals and time-dependent parameter laws. A normalization is applied for maintained the regressors Lebesgue-∞

measurable by means of $\varphi_i(t) = \frac{1}{\sqrt{1+\phi_i^T\phi_i}}\phi_i(t)$. Also special filters are applied in order to eliminate the need to measure derivatives [5]. Thus the normalized estimation error results in

$$\varepsilon_i(t) = \frac{-\breve{\boldsymbol{\theta}}_i\boldsymbol{\phi}_i}{\sqrt{1+\boldsymbol{\phi}_i^T\boldsymbol{\phi}_i}} + \frac{\eta_i}{\sqrt{1+\boldsymbol{\phi}_i^T\boldsymbol{\phi}_i}} \tag{3}$$

with $\breve{\boldsymbol{\theta}}_i$ the parameter error vector and η_i a bounded vector. Finally, the parameter estimation is implemented as a gradient-based adaptive law by the ODE

$$\dot{\hat{\boldsymbol{\theta}}}_i(t) = \Gamma_i \frac{\varepsilon_i(t)}{(1+\boldsymbol{\phi}_i^T\boldsymbol{\phi}_i)}\boldsymbol{\phi}_i(t), \tag{4}$$

where $\Gamma_i = \Gamma_i^T > 0$ is a gain matrix influencing the rate of convergence of $\hat{\boldsymbol{\theta}}$. The trajectories of the estimated vectors $\hat{\boldsymbol{\theta}}_i$ are proved to converge inside a residual set, which measure is of the order of magnitude of the disturbance $\eta_i/\sqrt{1+\boldsymbol{\phi}_i^T\boldsymbol{\phi}_i}$. For accuracy reasons, the algorithm need to be reset after each prognosis using the present path point as initial condition for the next estimation.

As the system has no input, it is stated in the paper mathematically that the system is able to generate by self conditions of Persistency of Excitation (PE) provided some requirements are satisfied. These are for any regressor

$$\sigma_1 I \geq \int_t^{t+T} \boldsymbol{\phi}_i \,\mathrm{diag}(\frac{1}{1+\phi_{i_1}^2},\ldots,\frac{1}{1+\phi_{i_n}^2})\boldsymbol{\phi}_i^T \, d\tau \geq \sigma_2 I, \tag{5}$$

for $t \geq 0$ and $T > 0$, $\sigma_1, \sigma_2 > 0$, where σ_1 is known as the level of persistency of $\boldsymbol{\phi}_i$.

It is proved in the paper that the only way that any relation in (5) be not satisfied by solutions of the convection-loop system is that the system dynamics stays at an equilibrium point. This is the only condition under which the parameter identification does not converge, i.e. equivalently, under which the dynamics does not generate conditions PE.

So the convection-loop dynamics evolves in a proper behavior that can range within a wide spectrum of oscillations from transient to a limit set, for instance periodic orbits like PN-, PPN-, PPNN or chaotic states given by strange attractors.

IV. Basins of Ljapunov Spectra

Now we will describe one analytical method for characterizing qualitative behaviors, that concerns the Ljapunov spectrum. The exponents of the Ljapunov spectrum are defined as

$$\lambda_i = \lim_{t\to\infty}\frac{1}{t}\log_2\frac{s_i(t)}{s_i(0)} \tag{6}$$

where $i = 1, \ldots, n$, with n being the order of the dynamic system and s_i the length of one of the n principal lengths of an ellipsoid at t in the n-dimensional phase space of the system, which has started as an infinitesimal n-sphere of radius $s_i(0)$ and changed its form due to the locally deforming nature of the flow, *i.e.*, in some main directions it contracts, in others it suffers no change and in the rest it expands. In [6] an algorithm for detecting main directions and ortho-normalization of them via the Grandt-Schmidt method is presented.

The signs of the Ljapunov exponents provide a quantitative scenario of the system dynamics and will be used in further analysis. In the context of extended Lorenz-like dynamic systems, a three-dimensional continuous dissipative system like this, is described by the Ljapunov spectrum composed by the following possible combination of exponent signs [6].

Lyapunov exponent signs (λ_1, λ_2, λ_3)	Type of limit set
(+, 0, -)	Strange attractor
(0, 0, -)	Two-torus
(0, -, -)	Limit cycle
(-, -, -)	Fixed point

Table I – Identification of limit sets

Besides the Ljapunov spectrum, there is another related indicator that quantifies chaotic behavior on the basis of information theoretic terms. These also are very relevant for our approach to behavioral prediction. This is a measure of the rate at which system processes create or destroy information that is useful for behavior prediction. So, the Ljapunov exponents can be expressed in bits of information per second or bits per orbit. Accordingly, for a chaotic Lorenz attractor with coefficients $\sigma = 16.0$,

$\beta = 45.92$ and $\alpha_z = 4.0$, and $r = 0$, the maximal Ljapunov exponent results 2.16, which means 2.16 (bits/sec). The interpretation is as following: if the initial behavior was established with an accuracy, for instance, of about one part per million, i.e., 1/220 (bits); so the next evolution could be not predicted with so a precision after 20 (bits) / 2.16 (bits/s)=9 (s). After this time, there is an inability to predict the behavior except to say that the orbit stays somewhere on the strange attractor.

In Figs. 3 up to 8, different subdivisions of the space are made. Additionally, while Fig. 3 shows the region conformed by the Prandtl coefficient σ versus the Rayleigh number r, the rest depicts the control space parameter conformed by the gain β for bead velocity and the Rayleigh number r. Clearly, the richness in behavioral diversity seen in the region β vs. r is quite superior to that obtained in the subspace σ vs. r. Moreover, in local vicinities of β vs. r, several limit cycles are obtained, included PPN and NNP orbits.

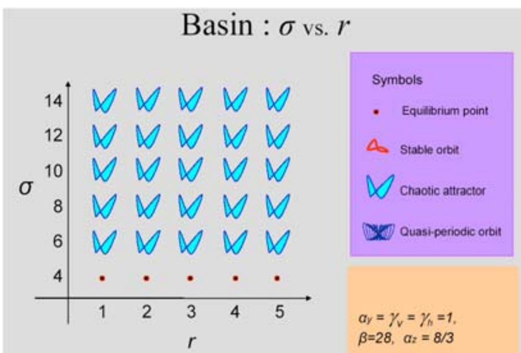

Figure 3 - Basin of the Prandtl coefficient σ and the Rayleigh number r

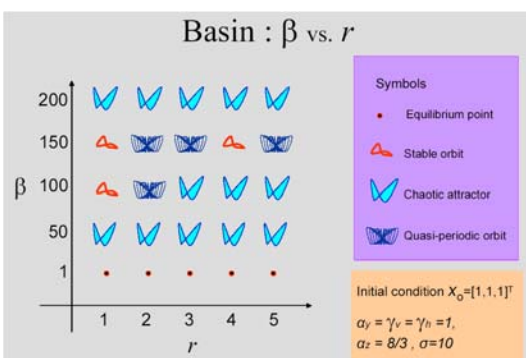

Figure 4 - Basin of the gain β and the Rayleigh number r

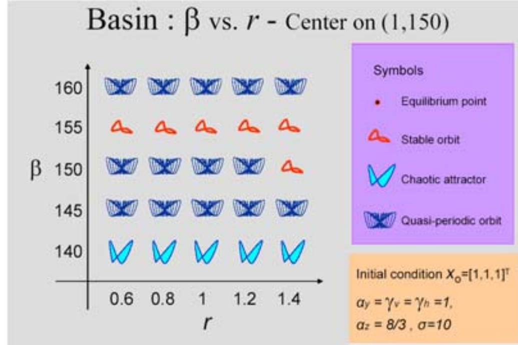

Figure 5 - Basin of the gain β and the Rayleigh number r

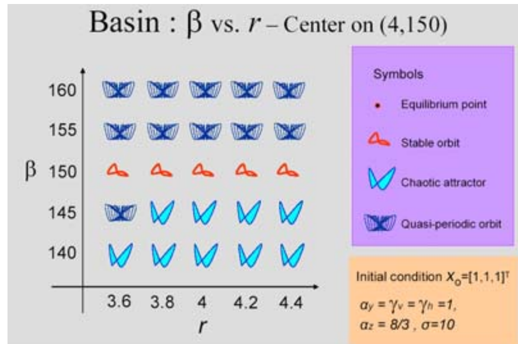

Figure 6 - Basin of the gain β and the Rayleigh number r

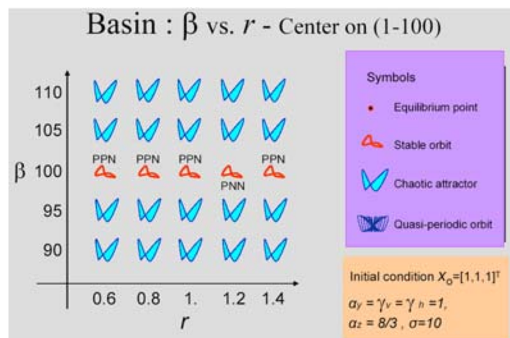

Figure 7 - Basin of the gain β and the Rayleigh number r

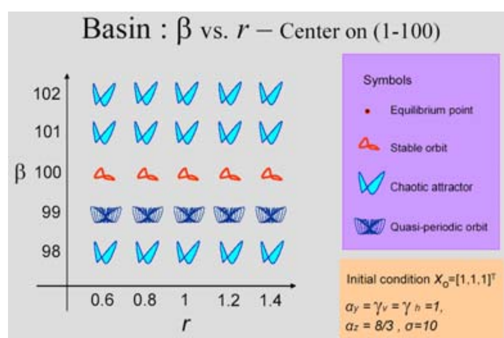

Figure 8 - Basin of the gain β and the Rayleigh number r

In Fig. 9 the maximal Lyapunov exponent is plotted. Basically, this indicator is employed as building-brick to construct the basin of our approach. Clearly, the diversity is detected with the sign of this indicator, so that the domain β vs. r is divided into sub-regions, in where the system maintains its features of limit set. It is noticing that the limit sets corresponding to limit cycles are confined to a segment without area that is characterized by the Lyapunov exponent equal to zero for $\beta = 4.5$ approximately.

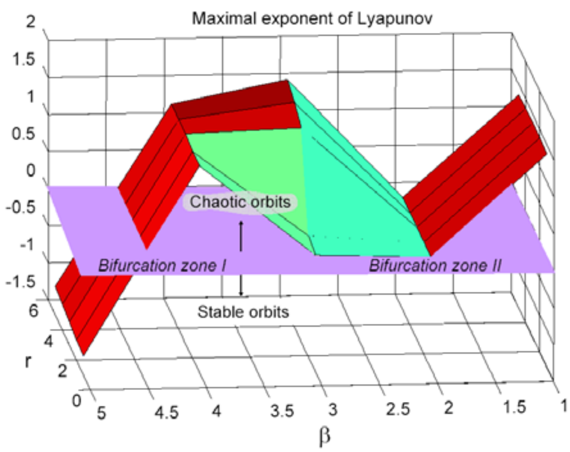

Figure 9 - Plot of the maximal Lyapunov exponent in the region (r, β)

V. Numerical simulations

The approach is illustrated with numerical simulations. A time-varying convection loop with coefficients $\sigma = 10$, $\alpha_y = 1$, $\gamma_v = 1$, $\alpha_z = 8/3$, $\gamma_h = 1$, $r = 0$ and $\beta(t) = 1 + \cos\left(\frac{\pi}{200}t\right)$, and with initial conditions: $x(0) = 14$, $y(0) = 27$ and $z(0) = 78$ is considered. In Figs. 10 and 11 the evolution and the estimated evolution are depicted. It can be shown that the system dynamics starts from an equilibrium point at $\beta(0) = 1$ and transits a series of equilibria with a short transitory till $\beta(12.68) = 24.74$ where a Hopf bifurcations and consequently a strange attractor is formed for the asymptotic behavior. This chaotic pattern remains in time induced by a series of strange attractors up to the arrival in time intervals where stable orbits with period doubling are inserted from time to time, for instance for $\beta = 99.65$ a PPNPPN-orbit and for $\beta = 100.5$ a PPN-orbit. Up to $t = 100$[hours] the coefficient $\beta(t)$ begins to decrease along the time, passing through the same limit sets and showing the same qualitative behaviors. At $t = 187.32$[hours] the asymptotic dynamics enters the path through a series of equilibrium points up to the end of the simulation.

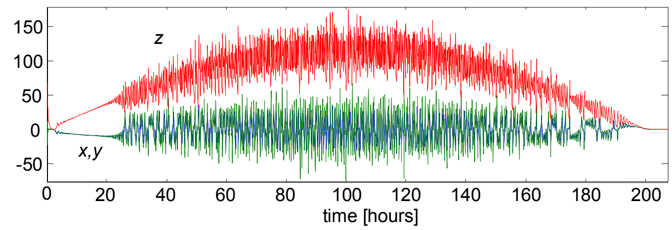

Figure 10 - Evolution of the system

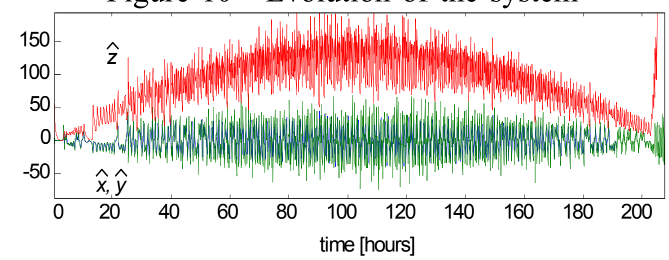

Figure 11 - Etimated evolution of the system

In the estimation it is assumed that no coefficient is known beforehand. The only time-varying parameter is β and this is assumed to have the law $\hat{\beta}(t) = \beta_0 + \beta_1 t + \beta_2 t^2$. Thus the parameter vectors are built up as: $\hat{\boldsymbol{\theta}}_x = [\hat{\sigma}]^T$, $\hat{\boldsymbol{\theta}}_y = \left[\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\alpha}_y, \hat{\gamma}_v\right]^T$ and $\hat{\boldsymbol{\theta}}_z = \left[\hat{\alpha}_z, \hat{\gamma}_h, \hat{r}\right]^T$. The method of the static normalization is applied with gains $\Gamma_x = 10$, $\Gamma_y =$diag$(10^3, 10^2, 10.14, 10^3, 10^2)$ and $\Gamma_z =$diag$(10, 500, 600)$. .

It is clearly seen that the estimation of the state variables preserves qualitatively the patterns of the paths during chaotic behaviors. Moreover, as predicted by the theory, the presence of stable equilibrium points cause lack of persistency of excitation in the intervals $[0, 12.68]$[hours] and $[187.32, 220]$[hours], i.e., when the dynamics transits through a series of limiting sets composed by equilibrium points the convergence fails. Specially on this last interval, the divergence of the parameters is more marked.

Also it is remarking that far away from $t = 200$[hours], the prediction of β begins to fail since a quadratic evolution does not fit a cosinus-shaped outside one semi-period of it. This would require at least a third-order approximation of $\beta(t)$ or the reset of the estimation as remarked previously.

VI. Conclusions

In this work a procedure to qualitative long-term prediction of limit sets of a convection-loop system is presented. The procedure employs estimation of system parameters of a previously defined process structure with time-varying coefficients and an interpolator to track in advance the path of the parameter vector on a basin of stationary limit sets. The estimation algorithm is based on a gradient law with normalizations in order to provide Lebesgue-∞ measurable regressors. A necessary condition for parameter path tracking is the meeting of good persistent excitation of the system. Accordingly, the convergence analysis shows that the stability of the estimation algorithm is ensured in any path of the dynamics that includes either chaotic or periodic orbits, but excludes any permanent transit over equilibria points. Numeric simulations with a convection-loop system have shown that the qualitative prediction of the future evolution can precisely be made with relatively larger anticipation in time than in the case done from the analysis of realizations. Future research is focused on experimental data on the convection loop.

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