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DYNAMICS OF FINITE ELEMENT MECHANICAL MODELS WITH UNILATERAL CONTACTS AND FRICTION

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Abstract
A numerical method is developed, which is suitable for determining dynamic response of finite element models of mechanical systems involving unilateral contact and friction. In classical structural dynamics approaches, such constraints are usually modeled by special contact elements. The characteristics of these elements must be selected in a delicate way, but even so the success of these methods can not be guaranteed. The present method is based on a proper combination of recent results from two classes of analytical and numerical methodologies. The first one includes the standard methods that determine dynamic response of models resulting by employing the finite element method to systems with smooth nonlinearities. The second class of methods includes specialized methodologies that simulate the response of simple dynamical systems with unilateral constraints. The validity of the method is illustrated with numerical results.

Key words
Unilateral contacts, friction, finite element models.

1 Introduction

Fast and accurate simulation of mechanical structures with complex geometry requires frequently application of the finite element method. These structures usually have supports and connecting elements, which involve a suitable combination of discrete springs and dampers. Typically, the finite elements that are used to model the action of the structural components have linear properties, while the action of the supports and the connecting elements is characterized by nonlinear properties. This category of systems has already been studied intensively in the past and a series of reliable numerical solution methods is available, when the nonlinearities involved are smooth [Bathe, 1982; Hughes, 1987].

On the other hand, some important phenomena may arise in typical mechanical structures, such as the establishment or loss of contact and the sticking or sliding that may take place between two contacting surfaces of a composite structure. Such models can be handled appropriately by employing set-valued force law theory [Brogliato, 1999; Glocker 2001; Leine and Nijmeijer, 2004]. However, with a few exceptions (e.g., [Simo and Laursen, 1992; Chetouane, Dubois, Vinches and Bohatier, 2005]), the results presented up to now in this scientific area, refer to relatively simple systems, with rigid components and a rather small number of degrees of freedom.

The main objective of the present study is to develop an appropriate direct integration scheme for investigating the dynamics of mechanical systems possessing a relatively large number of degrees of freedom and involving unilateral constraints. For such structures, the classical numerical integration methods do not work properly or fail to work at all. Currently, the main methods used to simulate the response of small scale non-smooth dynamical systems are based on either event-driven [Natsiavas, 1993; Pfeiffer and Glocker, 1996] or time-stepping [Moreau and Panagiotopoulos 1988; Jean, 1999] approaches. In general, the former are not efficient when the number of unilateral constraints is large [Jean, 1999]. For such cases, the most frequently applied methods are those employing time-stepping schemes. Among them, the midpoint rule proposed by Moreau is the most commonly employed scheme [Moreau and Panagiotopoulos, 1988]. However, when the number of the degrees of freedom is relatively large, more efficient numerical integration techniques are needed. In the present study, a new method is developed for studying dynamics of the class of mechanical models examined. This is achieved by combining a time-stepping integration scheme with more classical schemes, which are applicable to systems with smooth nonlinearities.

The organization of this paper is as follows. The class of mechanical systems examined and the analysis applied are briefly presented in the following section. Then, typical numerical results are presented for three characteristic finite element models involving contacts and friction. These results illustrate the performance of the method developed as well as the effect of some important mechanical properties. The final section includes a synopsis of the conclusions and the highlights of the study.

2 Method of Analysis

The equations of motion of the class of dynamical systems examined in the present study are represented by the following set of equations

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K\underline{q} - \underline{h}(\underline{q},\dot{\underline{q}}) - W\underline{\lambda} = \underline{f}(t) \,. \quad (1)$$

The set of the generalized coordinates $\underline{q} \in \Re^n$ is selected as minimal, so that the forces developed at all the bilateral constraints drop out from (1). Moreover, the terms M , C and K represent the $n \times n$ mass, damping and stiffness matrix of the system, respectively. Likewise, the term $\underline{h}(\underline{q},\dot{\underline{q}})$ includes the nonlinear smooth forces, while the term $W(t,\underline{q})\underline{\lambda}(t)$ involves the non-smooth forces developed at the points where the unilateral constraints are imposed. Finally, the term $\underline{f}(t)$ includes the externally imposed forces. In general, the set of the equations of motion is accompanied by the initial conditions

$$\underline{q}(0) = \underline{q}_0 \quad \text{and} \quad \dot{\underline{q}}(0) = \underline{u}_0 \,. \quad (2)$$

Adopting the notation and the approach style introduced in references [Glocker, 2001] and [Leine and Nijmeijer, 2004], the vector of generalized velocities is defined according to

$$\underline{u} = \dot{\underline{q}} \,. \quad (3)$$

This definition holds for almost all time t, since the set of the time intervals where the discontinuities occur has zero measure. Next, the equations of motion are complemented by constitutive laws describing the action along the normal and tangential direction at the contact points. More specifically, in order to treat the unilateral forces developed due to contact and friction within the same theoretical framework, the set-valued Signorini and Coulomb friction laws and the related normal cone formulations are selected, respectively, as explained briefly next.

First, if $\underline{g}_N(t,\underline{q})$ is a vector containing the normal relative distances at the $i = 1,\ldots,n_C$ potential contact points of the system, while the vector $\underline{\lambda}_N(t)$ includes the corresponding normal forces, the condition for no interpenetration in the normal direction at the contact points can originally be expressed by the following complementarity relations

$$\underline{g}_N \geq \underline{0}\,, \quad \underline{\lambda}_N \geq \underline{0}\,, \quad \underline{g}_N^T \underline{\lambda}_N = 0\,,$$

whose validity is meant to hold for each component separately. Alternatively, the above conditions, known as Signorini's normal contact law, can be cast in the form of non-smooth potential functions and eventually as

$$-\underline{g}_N \in N_{C_N}(\underline{\lambda}_N)\,, \quad (4)$$

where $N_{C_N}(\underline{\lambda}_N)$ represents the normal cone of the convex set

$$C_N = \{\underline{\lambda}_N \in \Re^{n_C} \mid \underline{\lambda}_N \geq \underline{0}\}$$

at point $\underline{\lambda}_N$. Finally, when some continuity conditions are satisfied, these laws can also be expressed in the velocity level [Leine and Nijmeijer, 2004; Jean, 1999].

Likewise, the law employed in the tangential direction of a contact point is the set-valued Coulomb friction law, which can eventually be put in the form

$$-\underline{\gamma}_T \in N_{C_T}(\underline{\lambda}_T)\,, \quad (5)$$

where the vector $\underline{\gamma}_T$ includes the relative velocity at the contact points along the corresponding tangent plane, while $N_{C_T}(\underline{\lambda}_T)$ is the normal cone of the convex set

$$C_T = \{\underline{\lambda}_T \in \Re^{n_C} \mid \left|\lambda_{T_i}\right| \leq \mu_i \lambda_{N_i}\,; \quad i = 1,\ldots,n_C\}$$

at point $\underline{\lambda}_T$.

The vectors including the normal and tangential velocities at the contact points are obtained in the form

$$\underline{\gamma}_N(t,\underline{q},\underline{u}) = W_N^T(t,\underline{q})\underline{u} + \underline{\tilde{w}}_N(t,\underline{q})$$

and

$$\underline{\gamma}_T(t,\underline{q},\underline{u}) = W_T^T(t,\underline{q})\underline{u} + \underline{\tilde{w}}_T(t,\underline{q})\,,$$

respectively, where by considering the system kinematics it turns out that

$$W_N^T(t,\underline{q}) = \frac{\partial \underline{g}_N}{\partial \underline{q}}(t,\underline{q})\,,$$

$$W_T^T(t,\underline{q}) = \frac{\partial \underline{\gamma}_T}{\partial \underline{u}}(t,\underline{q},\underline{u})$$

and

$$\underline{\tilde{w}}_N(t,\underline{q}) = \frac{\partial \underline{g}_N}{\partial t}(t,\underline{q})\,.$$

At present, the solution of the mathematical problem posed by expressions (1)-(5) is obtained numerically by applying an event-driven or a time-stepping method [Brogliato, 1999; Glocker 2001; Leine and Nijmeijer, 2004]. In general, the response of systems involving many unilateral constraints is most frequently determined by employing a time-stepping scheme. Traditionally, the midpoint rule proposed by Moreau is the most commonly employed time-stepping scheme [Moreau and Panagiotopoulos 1988; Jean, 1999]. This method is able to capture events like contact-detachment and sticking-sliding. However, when the number of the degrees of freedom is relatively large, faster and more reliable numerical techniques are needed. The finite difference or the Newmark integration schemes are two common examples of such techniques [Bathe, 1982; Hughes, 1987]. These methods are adequate to handle nonlinear smooth forces but they are not appropriate for systems involving set-valued forces. On the other hand, Moreau's technique can not handle systems with nonlinear elements effectively, since direct application of this method requires frequently a relatively small time step.

In the sequel, it is assumed that the systems examined belong to the class of dynamical systems involving planar friction only. Moreover, a Linear Complementarity Problem (LCP) formulation is preferred over an Augmented Lagrangian approach. Then, the equations of motion (1) are first recast in the following equality of measures

$$M\,d\underline{u} + C\,d\underline{q} + K\underline{q}dt - \underline{h}dt - Wd\underline{\Lambda}\,dt = \underline{f}dt.$$

Moreover, it is convenient to split the term of the constraint forces in the normal and tangential direction in the following form

$$W\underline{\lambda} = W_N\underline{\lambda}_N + W_T\underline{\lambda}_T\,.$$

Furthermore, in order to decompose the two-corner tangential contact law into two separate unilateral primitives, the following quantities

$$\underline{\gamma}_R = \tfrac{1}{2}(|\,\underline{\gamma}_T\,| + \underline{\gamma}_T),\ \underline{\gamma}_L = \tfrac{1}{2}(|\,\underline{\gamma}_T\,| - \underline{\gamma}_T),$$

represent the vectors including the right and the left sliding velocities at the contact points, respectively [Leine and Nijmeijer, 2004]. Combination of the last two relations implies that

$$\underline{\gamma}_T = \underline{\gamma}_R - \underline{\gamma}_L\,.$$

In a similar fashion, a decomposition of the impulsive friction saturations is performed, so that

$$\underline{\Lambda}_R = \overline{\mu}\underline{\Lambda}_N + \underline{\Lambda}_T \text{ and } \underline{\Lambda}_L = \overline{\mu}\,\underline{\Lambda}_N - \underline{\Lambda}_T,$$

with

$$\overline{\mu} = diag\{\mu_i\} \quad \text{and} \quad \underline{\Lambda} = \int_{\Delta t}\underline{\lambda}\,dt\,.$$

Then, combining a version of the Newmark-β method with Moreau's time-stepping technique and excluding spatial friction, the integration process developed requires the solution of a linear complementarity problem formulation at the end of each time step. In particular, this LCP formulation exhibits the following structure

$$\underline{y} = A\underline{x} + \underline{b}\,, \tag{6}$$

where the matrix A involves quantities related to the system dynamics and kinematics as well as characteristics of the numerical discretization scheme employed. Moreover, the vector quantities in (6) satisfy the following definitions and properties

$$\underline{x} = (\underline{\Lambda}_N^T \quad \underline{\Lambda}_L^T \quad \underline{\gamma}_R^T)^T\,,$$

$$\underline{y} = (\underline{\gamma}_N^T \quad \underline{\gamma}_L^T \quad \underline{\Lambda}_R^T)^T\,,$$

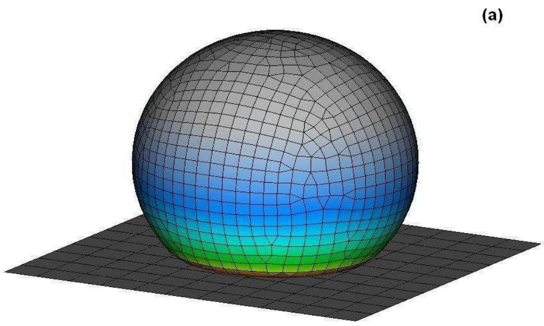
$$\underline{0} \le \underline{y} \perp \underline{x} \ge \underline{0}\,.$$

Numerical solution of the last set of equations, yields the impulsive forces, needed for the subsequent determination of the response of the class of the mechanical systems examined.

3 Numerical Results

The validity, accuracy and effectiveness of the method developed were illustrated by a large variety of examples. A selected set of such examples is presented next.

The first example model is shown in Fig. 1a. It is an elastic shell, with a radius of 50mm and a thickness of 1mm, which is dropped from a height and bounces on a rigid ground. The geometry is descretized by shell finite elements, leading to a model with 7710 degrees of freedom. Originally, the study focused on the effect of material properties. In particular, the results of Figs. 1b and 1c were obtained by selecting the elasticity modulus and density to correspond to a rubber or a steel shell, respectively. The continuous and dashed lines correspond to the vertical displacement of the lowest point and a neighboring point of the shell. Finally, Fig. 1d compares results obtained for a shell possessing material properties with intermediate values. The results indicate the convergence obtained for three different sizes of the time integration steps Δt. Also, apart from the convergence achieved, qualitatively similar results were obtained in all cases examined, while no sign of numerical instability was detected.



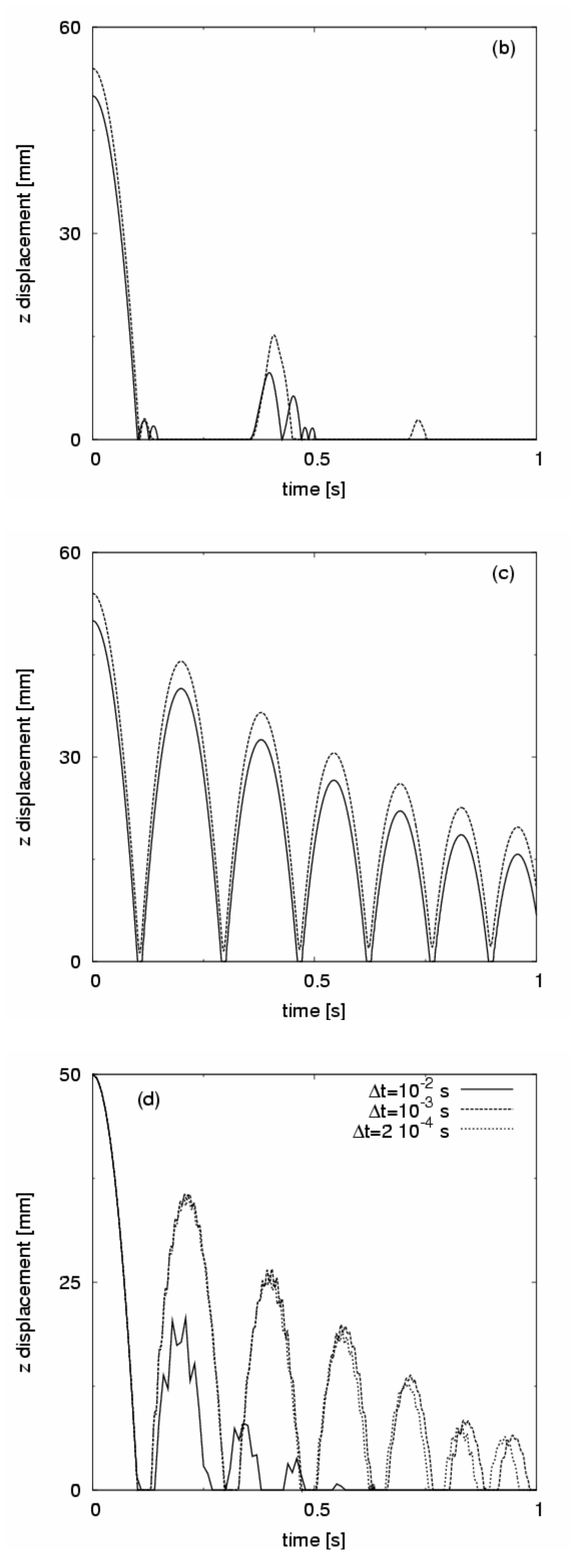
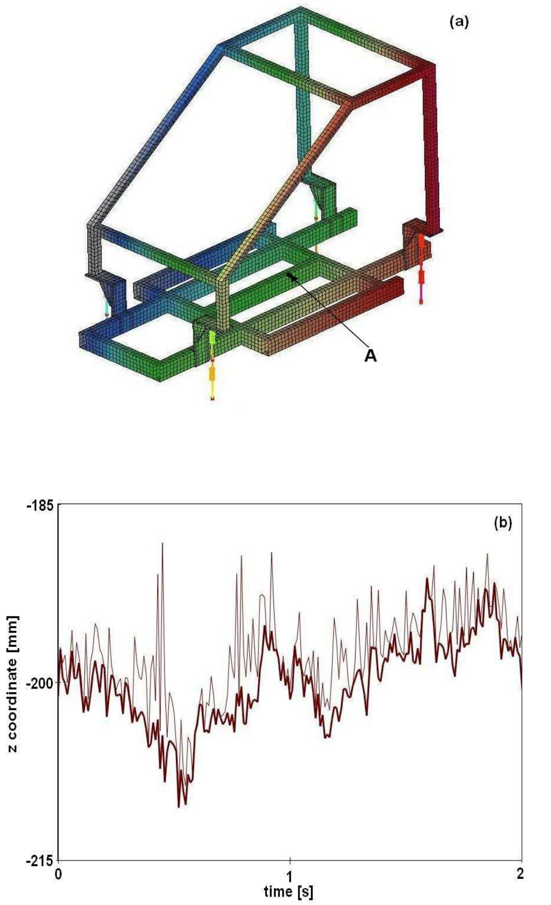


Figure 1.Elastic shell bouncing on rigid ground: (a) deformed geometry; (b) vertical displacement of a rubber shell; (c) vertical displacement of a steel shell and (d) effect of time step size on convergence.

The second example model, shown in Fig. 2a, has a much bigger original dimension. It represents a prototype vehicle structure used in earlier experimental studies [Giagopoulos and Natsiavas, 2007]. Basically, it consists of a metallic frame supported on the ground by four suspension subsystems. The frame is modeled by finite elements with linear properties, while the suspension subsystems possess nonlinear stiffness and damping characteristics. Also, the wheels are allowed to separate from the ground. The model possesses a little more than 49,000 degrees of freedom and is subjected to base excitation. For instance, the road profile at the front left wheel is represented by the thick line in Fig. 2b. The thinner line in the same figure represents the vertical displacement of the corresponding wheel. Obviously, there exist extended time intervals where the tire is not in contact with the ground. On the other hand, Figs. 2c and 2d depict the vertical displacement and acceleration histories recorded at point A of the frame of the vehicle (shown in Fig. 2a), during the same time interval. A smoother form of these histories is apparent, which is due to the vibration isolation action of the suspension subsystems.



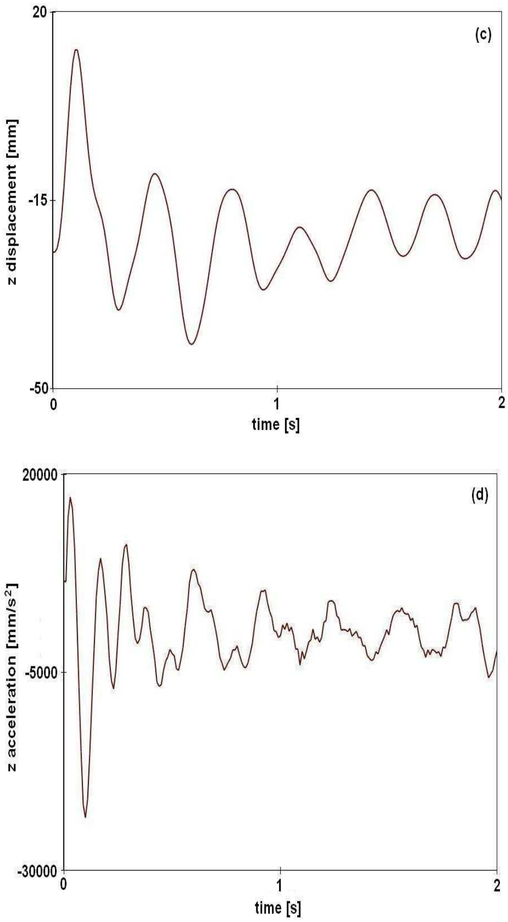


Figure 2. Prototype vehicle structure: (a) deformed geometry; (b) vertical displacement of the front left wheel (thin line) and road profile (thick line); (c) vertical displacement history and (d) vertical acceleration history, at point A on the frame of the vehicle.

The level of complexity is raised further in the last example, which involves point to surface contact combined with friction. Here, an elastic cubic block with an edge of 20mm, shown in Fig. 3a, is dropped from rest at a height and hits a plate with a thickness of 0.5 mm. The block is discretized by a number of solid finite elements, while the thin plate (whose corner points are fixed) is discretized by shell elements, leading to a model with 1861 degrees of freedom. After half a second from start, a horizontal harmonic force is applied on the block, along the x-axis, activating friction phenomena during contact of the block with the ground. First, Fig. 3b displays results obtained for the vertical displacement of a node on the lowest surface of the block that comes in contact with the plate, together with the vertical displacement of a nearby point of the plate. Likewise, Figs. 3c and 3d depict the horizontal displacement and velocity of the same node of the block. The results demonstrate the appearance of stick intervals. In addition, they indicate that the system tends eventually to reach a periodic steady state motion.

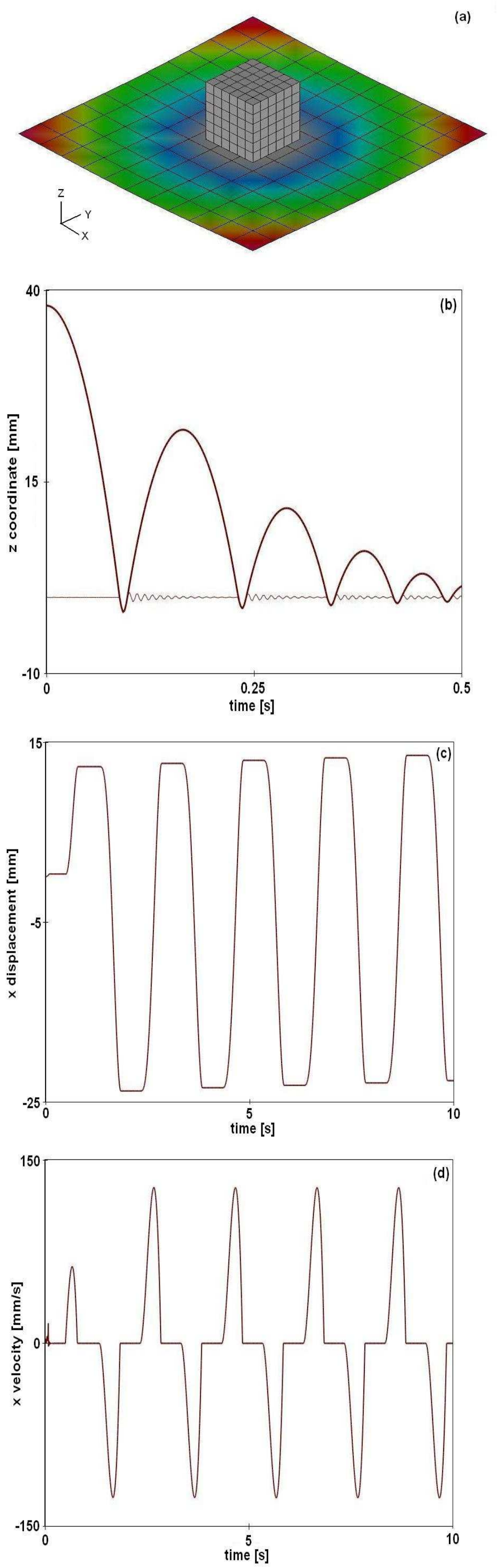


Figure 3.Elastic block hitting a thin flexible plate: (a) deformed geometry; (b) vertical displacement of a node on the lowest surface of the block (thick line) and vertical displacement of a nearby point of the plate (thin line); (c) horizontal displacement and (d) velocity of the same node of the block.

4 Summary

A computationally efficient methodology was developed for determining dynamic response of finite element models, arising in Structural Dynamics applications and involving unilateral contact and friction constraints. The basic idea was to combine classical methodologies employed for the direct integration of mechanical systems involving smooth nonlinearities with recent developments in the area of non-smooth dynamics. The accuracy and effectiveness of the methodology was demonstrated by presenting numerical results for three selected examples. The results presented illustrated that the methodology developed can help efforts directed towards predicting the influence of parameters on the response of large order nonlinear systems with unilateral constraints in a systematic way.

Acknowledgments

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