

ANGULAR GUIDANCE OF NAVIGATION SATELLITES DURING PASSING THE SINGULAR ORBITAL POSES

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Abstract

During the flight of a navigation satellite, the solar-terrestrial reference frame is used, based on the unit vectors of directions to the Sun and Earth measured on board, emanating from the mass center of the satellite (object). The problems on attitude control of a navigation satellite when its passing through singular orbital poses are studied in order to ensure predictability of the influence of solar pressure forces on its movement.

Key words

Navigation satellite, attitude guidance and control, passing the singular orbital poses.

1 Introduction

The problems on guidance, navigation and control are relevant for modern information satellites [Aleksandrov and Tikhonov, 2018; Somov et al., 2021]. Famous Russian *GLOBAL NAVIGATION SATELLITE SYSTEM (GLONASS)* [Perov and Kharisov, 2005; Testoyedov, 2009] contains 24 satellites located in circular orbit at 19100 km altitude with 64.8 deg inclination and 11.25 hour period. This constellation operates in three orbital planes, with eight evenly spaced satellites on each, see the right part in Fig. 1. The angle between the unit vectors from a satellite to the Sun and to the Earth is denoted by SOE (Sun – Object – Earth). The orbital singular poses appear at the following values of the SOE angles: close to 0 deg and to $\pi=180$ deg. We studied the control problems for the Glonass-M navigation satellite (NS) [Somov et al.,

2017] at its passage through singular orbital poses when the satellite performs a preemptive rotational maneuver (PRM). This is due to the surface of the NS has different specular and diffuse reflections of solar radiation, so this effect can lead to unpredictable motion of its mass center. We briefly represent new technological advances that will ensure energy efficiency, reliability, and survivability of the NS motion control systems.

2 Mathematical models and the problem statement

We use the following reference frames:

- geocentric inertial reference frame (IRF) $O_{\oplus}X_e^I Y_e^I Z_e^I$ with unit vectors $\mathbf{i}_i, i = 1, 2, 3 \equiv 1 \div 3$, axis X_e^I is directed to point Υ of the vernal equinox and axis Z_e^I is directed to the world North Pole P_N on the axis of the Earth's rotation with angular velocity vector $\boldsymbol{\omega}_e \equiv \boldsymbol{\omega}_{\oplus}$, Fig. 2;
- inertial solar-ecliptic reference frame $O_s X_s^I Y_s^I Z_s^I$ (SERF) with origin in the Sun center O_s , axis X_s^I is also directed to point Υ , the axes Y_s^I and Z_s^I are obtained by turning the axes Y_e^I and Z_e^I on angle ε_e around axis X_e^I (Fig. 2), the Sun astronomical longitude $\rho_s(t) = \rho_s^o + \omega_s(t - t_0)$ where ω_s is the average angular rate of the Earth's annual revolution around the Sun, and unit vector $\mathbf{e}_s$ is directed to the Earth center $O_{\oplus}$;
- orbital reference frame (ORF) $Ox^o y^o z^o$ with origin in point O and unit vectors $\mathbf{r}^o, \boldsymbol{\tau}^o$ and $\mathbf{n}^o$, Fig. 3;
- the body reference frame (BRF) $Oxyz$, the NS has the solar array panels (SAPs) rotated on angle γ , Fig. 4;
- the SAPs' reference frame (PRF) $Op^p x^p y^p z^p$ with unit vector $\mathbf{p}$ — a normal to the plane $y^p Op^p z^p$, Fig. 4.

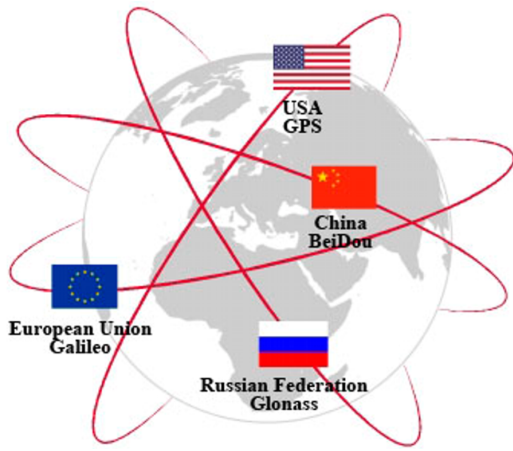


Figure 1. The Russian GLONASS among other global space-based navigation systems

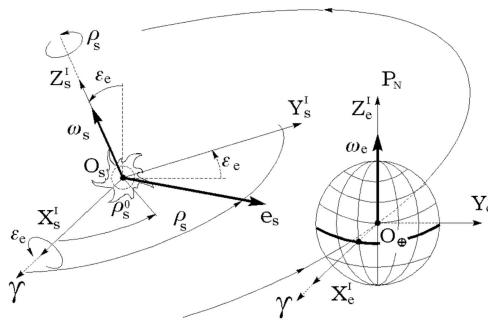


Figure 2. The inertial geocentric and solar-ecliptic reference frames

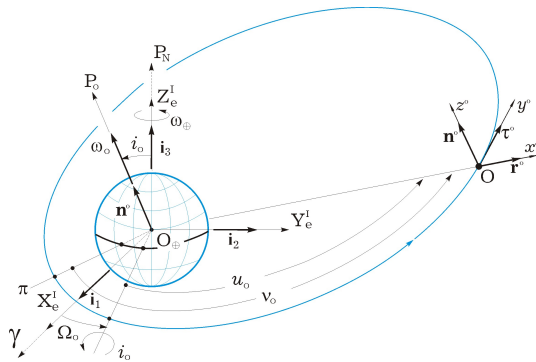
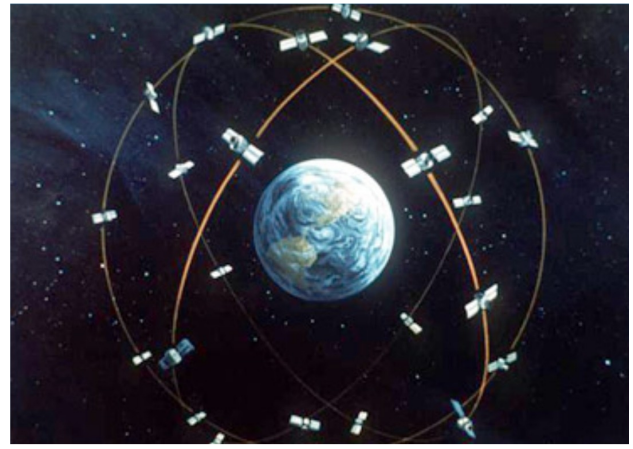


Figure 3. The inertial geocentric and orbital reference frames

The orbit position is determined by longitude Ω_o of the ascending node, inclination i_o and the NS position in the orbit – by vector $\mathbf{r}_o(t)$ with unit vector $\mathbf{r}^o(t)$ and the true anomaly $\nu_o(t)$ connected with the argument of latitude $u_o(t) = \omega_\pi + \nu_o(t)$, Fig. 3.

In what follows, we use conventional notations $\{\cdot\} \equiv \text{col}(\cdot)$, $[\cdot] \equiv \text{line}(\cdot)$ $\langle \cdot, \cdot \rangle$ and $\times$ for vectors, $[\cdot \times]$, $[\cdot]^t$, $[\cdot]_i$ for matrices. We introduce angles by yaw $\phi_1 \equiv \psi$, roll $\phi_2 \equiv \varphi$, pitch $\phi_3 \equiv \theta$ in sequence 132, the attitude matrix $\mathbf{C}^o \equiv \{[c_{ij}^o]\} \equiv [\mathbf{c}_1^o, \mathbf{c}_2^o, \mathbf{c}_3^o] = [\varphi]_2 [\theta]_3 [\psi]_1$ and quaternion $\mathbf{\Lambda}^o = (\lambda_0^o, \lambda^o)$, $\lambda^o = \{\lambda_i^o\}$ to define the BRF orientation in the ORF.

The matrix $\mathbf{C}^o = \mathbf{I}_3 - 2[\lambda^o \times] \mathbf{Q}^t$ with the matrix $\mathbf{Q} = \mathbf{I}_3 \lambda_0^o + [\lambda^o \times]$, and analytic reverse transforma-



tion is carried out by well-known Shepperd's algorithm [Shepperd, 1978; Junkins and Turner, 1986].

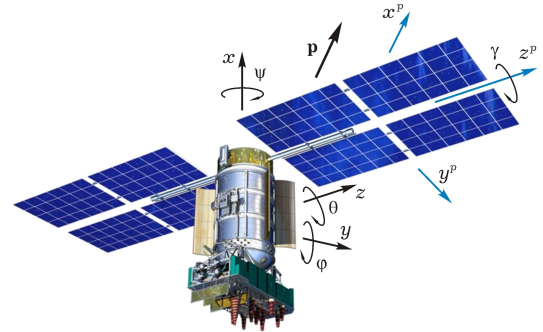


Figure 4. The reference frames of Glonass-M satellite and its SAPs

In IRF the NS quaternion $\mathbf{\Lambda} = (\lambda_0, \lambda)$, $\lambda = \{\lambda_i\}$ is presented in the form $\mathbf{\Lambda} = \mathbf{\Lambda}_o \circ \mathbf{\Lambda}^o$ where quaternion $\mathbf{\Lambda}_o$ corresponds to matrix $\mathbf{C}_o(t) = [u_o(t)]_3 [i_o]_1 [\Omega_o]_3$ of the ORF orientation in the IRF. We introduce also the unit vectors $\mathbf{S} = -\mathbf{e}_s$ and $\mathbf{E} = -\mathbf{r}^o$, see Figs. 2 and 3.

In ORF the unit vectors $\mathbf{e}_s$ and $\mathbf{S}$ have the form

$$\mathbf{e}_s^o = \{e_{sx}^o, e_{sy}^o, e_{sz}^o\} = \mathbf{C}_o[-\varepsilon_e]_1 [-\rho_s(t)]_3 \mathbf{i}_1;$$

$$\mathbf{S}^o = \{s_x^o, s_y^o, s_z^o\} = -\{e_{sx}^o, e_{sy}^o, e_{sz}^o\}.$$

In BRF the unit vectors $\mathbf{S}$ and $\mathbf{E}$ are presented as

$$\mathbf{S} = \{s_x, s_y, s_z\} = \mathbf{C}^o \mathbf{S}^o \text{ and } \mathbf{E} = -\mathbf{C}^o \mathbf{r}^o = -\mathbf{c}_1^o.$$

At last, here the unit vector $\mathbf{p}(t) = [\gamma(t)]_3 \{1, 0, 0\}$ with $\gamma \in [-\pi/2, +\pi/2]$, and angle β between unit vectors $\mathbf{p}$ and $\mathbf{S}$ is defined by the relation $\cos \beta = \langle \mathbf{S}, \mathbf{p} \rangle$.

For a circular orbit its velocity vector is $\boldsymbol{\omega}_o = \omega_o \mathbf{n}^o$ with a constant value $\omega_o = \dot{\nu}_o = \dot{u}_o$, moreover in the ORF this vector is presented by column $\boldsymbol{\omega}_o^o = \{0, 0, \omega_o\}$. In BRF the NS absolute angular velocity vector is defined by column $\boldsymbol{\omega}$ and the kinematic equation for quaternion $\mathbf{\Lambda}^o$ has the form $\dot{\mathbf{\Lambda}}^o = (\mathbf{\Lambda}^o \circ \boldsymbol{\omega} - \boldsymbol{\omega}_o^o \circ \mathbf{\Lambda}^o)/2$.

The NS attitude and orbit control system (AOCS) has the following instrument composition: cluster of four reaction wheels (RWs) by scheme NASA; one-axis drive for the SAP's rotation; device for orientation to the

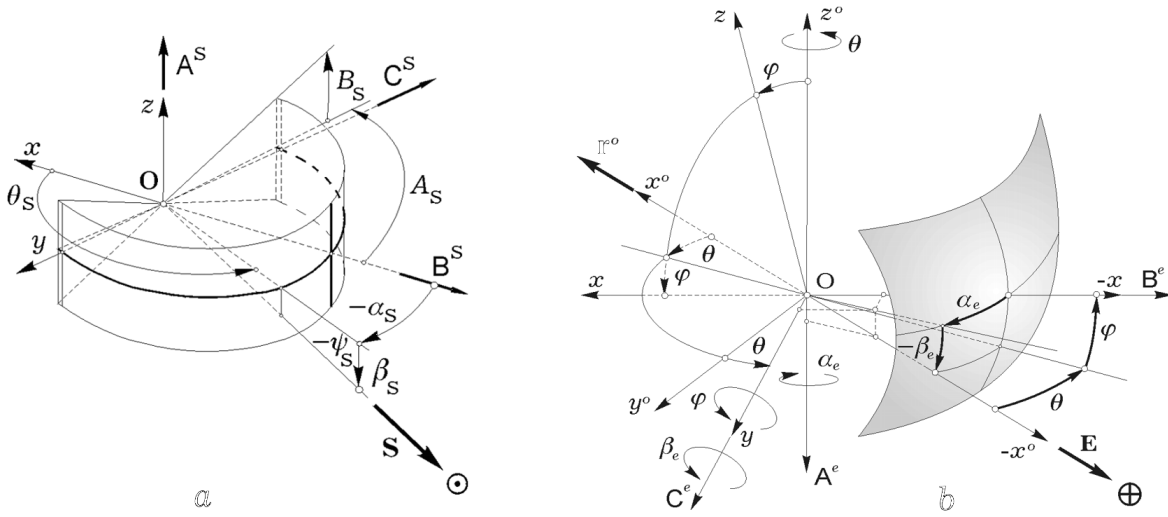


Figure 5. The schemes of onboard devices for the NS attitude determination in ORF: the DOS (a) and the DOE (b).

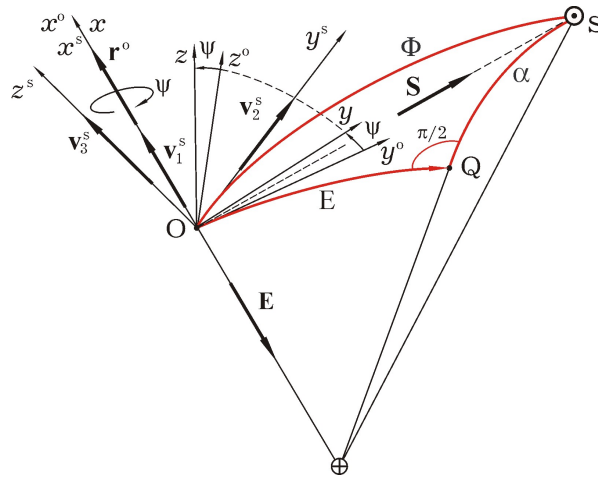


Figure 6. The orbital and solar-terrestrial reference frames

Sun (DOS), Fig. 2a: device for orientation to the Earth (DOE), Fig. 2b; cluster of the gyro angular rate sensors (ARSSs) to measure the angular velocity vector $\omega(t)$ and onboard computer implementing algorithms for discrete filtering and digital control. Models of the DOS and DOE, and also the NS motion with rotated SAPs, were presented in [Somov, 2007] taking into account methods for designing the NS guidance, navigation and control systems [Somov, 2016; Somov et al., 2013].

In the paper we consider problems on an attitude guidance of Glonass-M satellite at its passing through the singular orbit poses while performing a *preemptive rotational maneuver* (PRM). We have previously explored this aspect [Somov et al., 2017] and here we present some refinements and briefly propose the motion control systems for advanced navigation satellites.

3 Algorithms of the NS guidance

We use the STRF $Ox^s y^s z^s$ with unit vectors $\mathbf{v}_1^s, \mathbf{v}_2^s, \mathbf{v}_3^s$ defined on well-known TRIAD algorithm [Shuster, 2006; Shuster and Oh, 1981]:

$$\mathbf{v}_1^s = \mathbf{r}^o = -\mathbf{E}; \quad \mathbf{v}_3^s = (\mathbf{v}_1^s \times \mathbf{S}) / |\mathbf{v}_1^s \times \mathbf{S}|; \quad \mathbf{v}_2^s = \mathbf{v}_3^s \times \mathbf{v}_1^s.$$

Let us α denotes the angle between the NS orbital

plane $x^o y^o$ and the unit vector $\mathbf{S}$, and the angle Φ – between the unit vectors $\mathbf{v}_1^s$ and $\mathbf{S}$ – the angle SOE, moreover $\sin \Phi = |\mathbf{v}_1^s \times \mathbf{S}|$ and it is impossible to calculate an orthogonal basis by the unit vectors $\mathbf{v}_1^s, \mathbf{v}_2^s, \mathbf{v}_3^s$ when $\Phi = 0$ or $\Phi = \pi$. For angle $\alpha = 0$ the angle Φ takes a minimum value ($\Phi = 0$) at the NS passage of areas with the Earth's shadow and the angle Φ is maximum ($\Phi = \pi$) at the NS passing an orbit areas when its arranging between the Earth and the Sun.

In Fig. 6 we applied notation for angle E of the NS current orbital position to the orbit point with minimal (maximum) value of the SOE angle Φ . Assume that the BRF plane xOy is coinciding with the SOE plane $x^s O y^s$ (with the unit vector $\mathbf{S}$) by the BRF rotation on angle ψ . For triangle OSQ in Fig. 6 we have the relations

$$\tan \psi = \tan \alpha / S_E; \quad S_\psi = S_\alpha / S_\Phi; \quad C_\Phi = C_\alpha C_E$$

and the rate $\dot{\psi}$ is computed as [Somov et al., 2017; Fateev et al., 2013; Fateev et al., 2014; Tentilov et al., 2015]

$$\dot{\psi} = \omega_o S_\alpha C_\alpha C_E / (1 - C_\alpha^2 C_E^2), \quad (1)$$

where notations $S_\alpha \equiv \sin \alpha$ and $C_\alpha \equiv \cos \alpha$ are applied. First problem of the SC body attitude guidance is connected with the possible arising of condition $C_\alpha^2 C_E^2 = 1$.

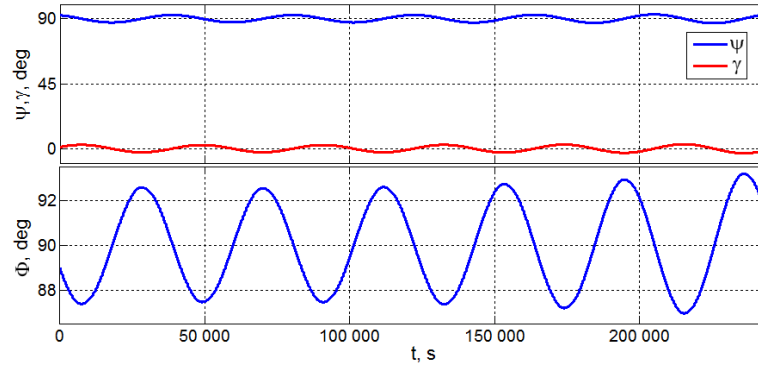


Figure 7. Variations of angles ψ , γ and Φ nearby 5 July when for $t = 0$ the angle $\alpha = 87.35$ deg

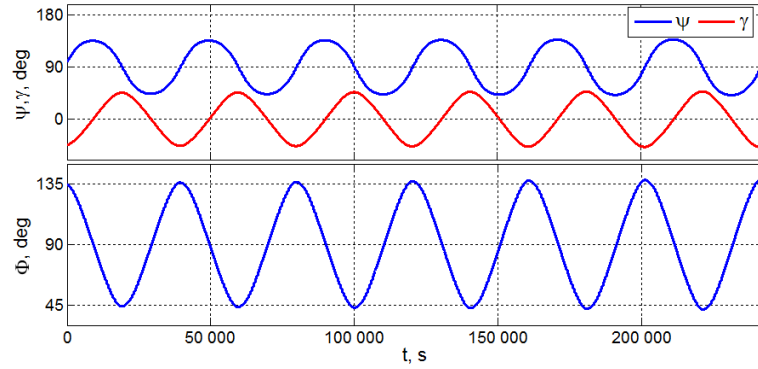


Figure 8. Variations of angles ψ , γ and Φ nearby 21 August when for $t = 0$ the angle $\alpha = 44$ deg

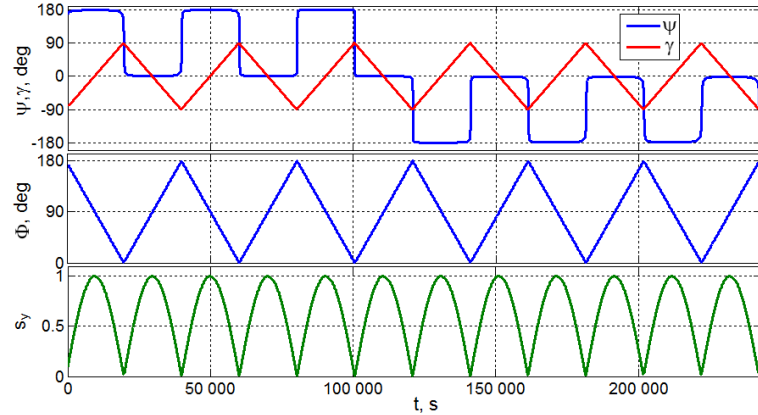


Figure 9. Variations of angles ψ , γ , Φ and component s_y nearby 3 October when for $t = 0$ the angle $\alpha = 1.2$ deg

For the NS body and its SAPs attitude guidance we have developed the laws [Somov et al., 2017; Fateev et al., 2013; Fateev et al., 2014; Tentilov et al., 2015]

$$\begin{aligned}\psi(t) &= \arctan_2(s_z^o(t), s_y^o(t)); \\ \gamma(t) &= -\arctan_2(s_x(t), s_y(t)).\end{aligned}\quad (2)$$

These laws were studied by computer verification, obtained results are presented in Figs. 7, 8 and 9 for different long-term time intervals in July, August and October. Moreover, the NS body surface is always unlit for its side on minus Oy axis, see Fig. 9 with values for component s_y of the unit vector $\mathbf{S}$. In next page Fig. 10 presents the unit vector $\mathbf{S}$ position in the ORF $Ox^o y^o z^o$ during

all year. According to (1) the angular rate $\dot{\psi} \approx 0$ when angle α is close to zero, however for value $E = 0$ the angular rate approaches infinity. Thus, for such situations the guidance laws is impossible without errors on angles ψ and β because of bounded torque of the RWs cluster.

Error $\delta\beta$ is presented in Fig. 11 by the blue line at implementation of the NS and SAPs' guidance laws (2) for value $\alpha = 0$ taking into account the limitation on the control torque vector for the navigation satellite turning.

Principal idea of the preemptive rotational maneuver (PRM) was first published in [Klimarev et al., 1992] to decrease the SAPs tracking error at the communications satellite passing the singular poses in high-elliptic orbit.

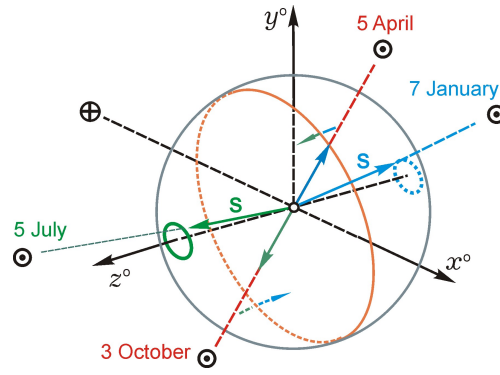


Figure 10. Variations of the unit vector $\mathbf{S}$ position in the ORF during all year

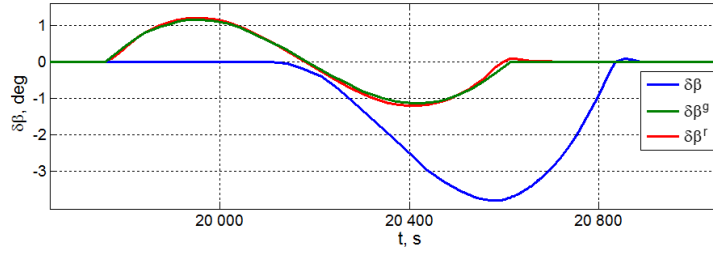


Figure 11. The SAPs angular errors at different implementation of the NS guidance law

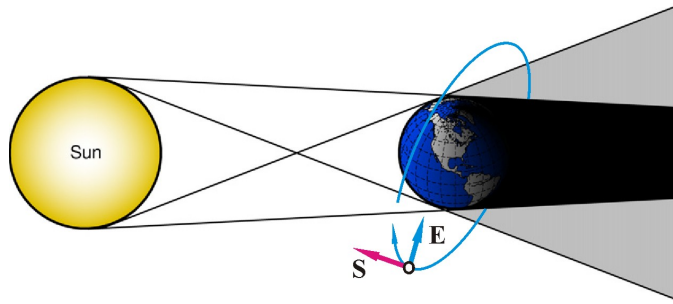


Figure 12. The eclipse season for the Glonass satellite

Then this idea was extended and applied [Fateev et al., 2014] for the navigation satellites of GLONASS system.

At the passage of large (or small) SOE angles with value $|\alpha| \leq 2$ deg when module of the angle between unit vectors $\mathbf{E}$ and $\mathbf{S}$ is less than 5 deg, the PRM on-board calculation is carried out by the following actions:

- the angle E is defined between the NS current position at the time moment t_i and its position at the time moment t_f of passing the maximum (or minimum) SOE angle;
- the required angle ψ^f is computed for the NS rotation around axis Ox to the point of passing the maximum (or minimum) SOE angle in which axis Oy must be perpendicular to the orbit plane $x^\circ Oy^\circ$;
- the turn time $T_1 = t_f - t_i$ is computed for predetermined angular rate of the NS body around axis Ox ;
- the passage time T_2 is computed taking into account the orbit angle from the NS current position to the point of passing the maximum (or minimum) SOE angle;
- the command sign is generated to begin the NS symmetrical PRM if condition $T_1 \geq T_2$ is completed;
- the PRM yawing guidance law $\psi^g(t) \forall t \in [t_i, 2T_1]$ is onboard generated with conditions

$\psi^g(t_i) = 0; \dot{\psi}^g(t_i) = 0$ and $\psi^g(T_1) = 2\psi^f; \dot{\psi}^g(2T_1) = 0$ for the fulfilling of the NS symmetrical PRM.

In Fig. 11 the green line presents the SAPs angular error $\delta\beta^g$ for guidance laws (2) with value $\alpha = 0$ when the NS is carried out symmetrical PRM. When the PRM is fulfilled, the NS body angular motion is stabilized on both attitude and angular velocity vector by the RW cluster using digital information obtained by the ARS cluster. In Fig. 11 the red line presents the SAPs angular error $\delta\beta^r$ taking into account the real (actual) errors and torque restrictions of the AOCs.

Every six months, during the four to eight weeks of the "eclipse season" at the orbital plane, once per revolution each satellite passes through the shadow of the Earth for a short time period. The Earth's shadow is divided into two cone-shaped regions – the umbra and penumbra, Fig. 12; the completely dark part of the shadow cast is umbra. The Glonass-M satellite need up to 54 min for its passage through the umbra [Dilssner et al., 2011]. During this time interval, the DOS signals are *physically* unavailable for the NS control, so here the strapdown inertial navigation system (SINS) is needed.

4 Compensation for antenna phase center errors

The NS attitude control is carried out with respect to its mass center and a mismatch exists of the antenna phase center (APC) in the BRF. The mismatch changes the APC coordinates in the STRF during the PRM. In the GLONASS system, to compensate the APC real position, the calculation is partially performed in consumer equipment. Onboard calculation is carried for the PRM azimuth by the reference model using value of the SOE angle. This reference model is the same onboard and in the consumer equipment. The PRM kinematic parameters depend on the angle α only, except the time data for the PRM beginning and end. So, the information tables are used in the onboard software [Fateev et al., 2013].

5 Advanced technologies for navigation satellites

The original ideas by regular large-size turns of both the NS body and the SAPs to ensure stable exposure to solar pressure forces using inexpensive optoelectronic devices as well as the PRM, were implemented in GLONASS system more than 50 years ago [Dilssner et al., 2011]. Moreover, foreign specialists expressed admiration for the innovative solutions devised by Soviet engineers, who created the AOCS for the Glonass satellites using low-cost components. The use of coatings with tunable reflective properties eliminates the need for such satellites to reorient. The authors have successfully performed a motion simulation of a geostationary satellite featuring these new coatings; however, experimental verification is required before the similar simulation results can be published for the Glonass satellites motion. These new technological advances will ensure energy efficiency, reliability and survivability of the satellite attitude and orbit control systems.

6 Conclusions

We briefly have studied fundamental problems on attitude guidance of the Glonass satellites with focussing on their passing the singular orbit poses with the "eclipse seasons". During these passages the NS is carried out the symmetrical preemptive rotational maneuver for minimizing both the SAPs angular error and influence of the solar pressure forces on orbital motion of the NS mass center. Applied the AOCS logic takes into account the following conditions: the NS body surface is always unlit for its side on minus Oy axis with angle $\psi \in ([-\pi, 0] \cup [0, \pi])$ and the SAPs angle $\gamma \in [\mp\pi/2]$ only. Obtained numerical results were presented for the Glonass-M satellite, including its passing the singular orbit poses at the "eclipse season".

References

Aleksandrov, A. and Tikhonov, A. (2018). Rigid body stabilization under time-varying perturbations with zero mean values. *Cybern. Phys.*, **7**(1), pp. 5–10.

- Dilssner, F., Springer, T., et al. (2011). The Glonass-M satellite yaw-attitude model. *Advances in Space Research.*, **47**(1), pp. 160–171.
- Fateev, A., Emelyanov, D., et al. (2013). Algorithms for definition of a course angle and calculation of the antenna phase center onboard spacecraft Glonass on the sites of preemptive turn maneuver. *Vestnik of Siberian State Aerospace University*, (4(50)), pp. 198–202.
- Fateev, A., Emelyanov, D., et al. (2014). Passage of singular orbit sites by the navigating space vehicle of system GLONASS. *Vestnik of Siberian State Aerospace University*, (4(56)), pp. 126–131.
- Junkins, J. and Turner, J. (1986). *Optimal Spacecraft Rotational Maneuvers*. Elsevier, Amsterdam.
- Klimarev, A., Tentilov, Y., Titov, G., and Bochkarev, A. (1992). Two gyro-wheel attitude control system of a spacecraft in high-elliptic orbit. In *Dynamics and Control of Space Objects*, pp. 76–85. Nauka, Novosibirsk.
- Perov, A. and Kharisov, V., editors (2005). *GLONASS: Principles of Design and Operation*. Radiotekhnika, Moscow, 3 edition. 688 p. In Russian.
- Shepperd, S. (1978). Quaternion from rotation matrix. *Journal of Guidance and Control*, **1**(1), pp. 223–224.
- Shuster, M. (2006). The PTRIAD algorithm as maximum likelihood estimation. *The Journal of the Astronautical Sciences*, **54**(1), pp. 113–123.
- Shuster, M. and Oh, S. (1981). Three-axis attitude determination from vector observations. *Journal of Guidance and Control*, **4**(1), pp. 70–77.
- Somov, S. (2007). Modeling of a flexible satellite motion. *Actual Problems of Aviation and Aerospace Systems*, (3(25)), pp. 85–93.
- Somov, S., Fateev, A., and Vassilyev, A. (2017). Attitude guidance and control of the navigation satellites at a passage of the singular orbital sites. *Mathematics in Engineering, Science and Aerospace*, **8**(1), pp. 43–52.
- Somov, Y. (2016). Guidance, navigation and control of information satellites: Methods for modeling, synthesis and nonlinear analysis. *Mathematics in Engineering, Science and Aerospace*, **7**(2), pp. 223–248.
- Somov, Y., Butyrin, S., and Somov, S. (2021). Dynamics of an autonomous spacecraft control system at initial transition to a tracking mode. *Cybernetics and Physics*, **10**(3), pp. 185–190.
- Somov, Y., Butyrin, S., Somov, S., and Somova, T. (2013). SIRIUS-S software environment for computer-aided designing of attitude control systems for small information satellites. In *Proc. 20th Saint Petersburg International Conference on Integrated Navigation Systems*, pp. 325–328.
- Tentilov, Y., Fateev, A., Emelyanov, D., et al. (2015). *Method for orientation of a navigation satellite. The RF Patent 2569999*.
- Testoyedov, N., editor (2009). *Space Milestones*. Reshetnev ISS Company, Zheleznogorsk. In Russian.