POLE PLACEMENT OF LINEAR MULTI-VARIABLE TIME-VARYING DISCRETE NON-LEXICOGRAPHICALLY-FIXED SYSTEMS

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Abstract

In this paper, a new design method of pole placement for linear time-varying multivariable discrete systems is considered. Using the concept of relative degrees of the system, we propose a simple design procedure to derive the pole placemetn state feedback gain without transforming the system into any canonical form. The method is also applicable to non-lexicigraphically fixed systems.

Key words

Pole Placement, Time-Varying System, Multivariable System, Discrete System

1 Introduction

This paper concerns the pole placement control design method for linear time-varying discrete multivariable systems. The method can be regarded as ”a discrete Ackermann-like algorithm”. The basic problem is to find a time-varying state feedback gain for linear time-varying discrete systems, so that the closed loop system is equivalent to some time-invariant system with desired constant closed loop poles. For this purpose, we focus on the relative degrees of the multi-variable plant. It will be shown that the pole placement controller can be derived simply by finding some particular ”output signal” such that the relative degree from the input to this output is equal to the order of the system. Then, the feedback gain matrix can be calculated directly from the system parameters without transforming the system into any standard form. An other property of the time-varying system is that the reachability (controllability) indices might be variable. Such a system is called a non-lexicographially-fixed system. For continuous systems, N.Olgac et.al. [8] also discussed this problem, and proposed one design method by augmenting the original system. Here, we use this idea for multivariable discrete systems, and propose the new design method of the pole placement state feedback.

2 Preliminaries

Condider the following linear time-varying mutltivariable discrete system.

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1}$$

where $x \in R^n$ and $u \in R^m$ are the state variable and the input. $A(k) \in R^{n\times n}$ and $B(k) \in R^{n\times m}$ are time-varying coefficient matrices. The state transition matrix of the system (1) from $k = j$ to $k = i$, $\Phi(i, j)$, is defined as follows.

$$\Phi(i, j) = A(i-1)A(i-2)\cdots A(j) \quad i > j \tag{2}$$

Definition 1. *System* (1) *is called ”completely reachable in* n *steps” if and only if, for any* $x_1 \in R^n$ *there exists a bounded input* $u(l)$ $(l = k, \cdots, k+n-1)$ *such that* $x(k) = 0$ *and* $x(k+n) = x_1$ *for all k.*

Lemma 1. *System* (1) *is completely reachable in* n *steps if and only if the rank of the reachability matrix defined below is* n *for all k.*

$$U_R(k) = \left[\, B_0(k), B_1(k), \cdots, B_{n-1}(k) \,\right] \tag{3}$$

where,

$$\begin{aligned} B_0(k) &= B(k+n-1) \\ B_1(k) &= \Phi(k+n, k+n-1)B(k+n-2) \\ &\vdots \\ B_{n-1}(k) &= \Phi(k+n, k+1)B(k) \end{aligned} \tag{4}$$

Let $b_i^l(k)$ *be the* i*-th column of* $B_l(k)$*, then, the reachability matrix* $U_R(k)$ *can be written as*

$$U_R(k) = \left[\, b_1^0(k) \cdots b_m^0(k) | \cdots | b_1^{n-1}(k) \cdots b_m^{n-1}(k) \,\right] \tag{5}$$

Note that $b_i^r(k)$ also satisfies (4), i.e.,

$$\begin{aligned} b_i^0(k) &= b_i(k+n-1) \\ b_i^1(k) &= \Phi(k+n,k+n-1)b_i(k+n-2) \\ &\vdots \\ b_i^{n-1} &= \Phi(k+n,k+1)b_i(k) \end{aligned} \tag{6}$$

where $b_i(k)$ is the i-th column of $B(k)$. Suppose that the system (1) is completely reachable in n steps. Then, the reachability indices, $\mu_i (i=1,\cdots,m)$, can be defined such that

$$\sum_{i=1}^{m} \mu_i = n \tag{7}$$

and the $n \times n$ trancated reachability matrix

$$\begin{aligned} R(k) = \big[\, & b_1^0(k),\cdots,b_1^{\mu_1-1}(k)|\cdots| \\ & ,b_m^0(k),\cdots,b_m^{\mu_m-1}\big] \end{aligned} \tag{8}$$

is non-singular. It is assumed that $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_m$ without loss of generallity.

3 Pole Placement

In this section, we consider the pole placement control for linear time-varying multivariable discrete systems. The problem is to design a state feedback for the system (1) so that the resulting closed-loop system becomes equivalent to some linear time-invariant system with arbitrarily stable poles. For this purpose, we first define a new output signal $\eta(k) \in R^m$ by

$$\eta(k) = C(k)x(k) \tag{9}$$

such that the relative degree from $u(k)$ to $\eta(k)$ is equal to the system degree n. Here,

$$\eta(k) = \begin{bmatrix} \eta_1(k) \\ \eta_2(k) \\ \vdots \\ \eta_m(k) \end{bmatrix}, C(k) = \begin{bmatrix} c_1^T(k) \\ c_2^T(k) \\ \vdots \\ c_m^T(k) \end{bmatrix} \tag{10}$$

Then, the problem is to find $C(k) \in R^{m\times n}$ that satisfies this condition.

Using such $\eta(k)$, the pole placement state feedback control can be desined by a simpler procedure than conventional methods.

Let $c_i^T(k)$ be the i-th row of $C(k)$. We define $c_i^{lT}(k)$ by the following.

$$\begin{aligned} c_i^{0T}(k) &= c_i^T(k) \\ c_i^{(l+1)T}(k) &= c_i^{lT}(k+1)A(k) \end{aligned} \tag{11}$$

for $i = 1,\cdots,m$.

Lemma 2. *The relative degree from $u(k)$ to $\eta(k)$ is n if and only if the following equations hold.*

$$\begin{aligned} & c_i^{\mu_i-1T}(k+1)b_i(k) = 1 \\ & c_i^{l\,T}(k+1)b_i(k) = 0, \quad l = 0,1,\cdots,\mu_i-2 \\ & c_r^{l\,T}(k+1)b_j(k) = 0, \quad \begin{cases} l = 0,1,\cdots,\mu_i-1 \\ r = 1,2,\cdots,m \\ (\ r \neq i\ ) \end{cases} \end{aligned} \tag{12}$$

for $i = 1,\cdots,m$. Here, $c_i^{(\mu_i-1)}(k+1)b(k)$ is equal to 1 for simplicity without loss of generality, since its only requirement is to be a non-zero scalar.

(Proof) Using (1), (9) and (11), $\eta_i(k+1)$, $\eta_i(k+2)$, $\cdots$ can be calculated to obtain the following equations.

$$\begin{aligned} \eta_i(k+1) &= c_i^{0T}(k+1)A(k)x(k) \\ &\qquad + c_i^{0T}(k+1)b_i(k)u_i(k) \\ &= c_i^{1T}(k)x(k) \\ \eta_i(k+2) &= c_i^{1T}(k+1)A(k)x(k) \\ &\qquad + c_i^{1T}(k+1)b_i(k)u_i(k) \\ &= c_i^{2T}(k)x(k) \\ &\vdots \\ \eta_i(k+\mu_i) &= c_i^{\mu_i-1T}(k+1)A(k)x(k) \\ &\qquad + c_i^{\mu_i-1T}(k+1)b_i(k)u_i(k) \\ &= c_i^{\mu_i T}(k)x(k) + u_i(k) + \gamma_{i(i+1)}u_{i+1}(k) \\ &\qquad + \gamma_{im}u_m(k) \end{aligned} \tag{13}$$

Here,

$$\gamma_{ij} = c_i^{\mu_i-1T}(k+1)b_j(k) \tag{14}$$

This implies the relative degree from $u_i(k)$ to $\eta_i(k)$ are μ_i. Then, the total relative degree from $u(k)$ to $\eta(k)$ is n and the converse is also clear.

Such $C(k)$ can be obtained by solving (12). Using (5), (6) and (11), (12) can be rewritten as follows.

$$\begin{aligned} & c_i^T(k)b_i^{\mu_i-1}(k-n) = 1 \\ & c_i^T(k)b_i^l(k-n) = 0, \quad l = 0,1,\cdots,\mu_i-2 \\ & c_r^T(k)b_j^l(k-n) = 0, \quad \begin{cases} l = 0,1,\cdots,\mu_i-1 \\ r = 1,2,\cdots,m \\ (\ r \neq i\ ) \end{cases} \end{aligned} \tag{15}$$

These equations contain only $c_j^T(k)$ This makes it possible to derive $C(k)$ directly from (12). Then, we have the following Theorem.

Theorem 1. *If the system is completely reachable in n steps, there exists a new output $\eta(k)$ such that the relative degree from $u(k)$ to $\eta(k)$ is n. And, such $C(k)$ can be calculated by the following equation.*

$$C(k) = WR^{-1}(k-n) \tag{16}$$

where

$$\begin{aligned} W &= diag(w_1, w_2, \cdots, w_m) \\ w_i &= \begin{bmatrix} 0 \cdots 0 \ 1 \end{bmatrix} \in R^{1\times\mu_i} \end{aligned} \tag{17}$$

In the sequel, using this $\eta(k)$, we will obtain the pole placement state feedback gain.

As stated above, if $C(k)$ satisfies (12), we have the following equations.

$$\begin{aligned} \eta_i(k) &= c_i^{0\,T}(k)x(k) \\ \eta_i(k+1) &= c_i^{1\,T}(k)x(k) \\ &\vdots \\ \eta_i(k+\mu_i) &= c_i^{\mu_i\,T}(k)x(k) + u_i(k) \end{aligned} \tag{18}$$

for $i = 1, \cdots, m$. Let $q^i(z)$ be the ideal characteristic polynomial of the closed-loop system from $u_i(k)$ to $\eta_i(k)$, i.e.,

$$q^i(z) = z^{\mu_i} + \alpha^i_{\mu_i-1} z^{\mu_i-1} + \cdots + \alpha^i_1 z + \alpha^i_0 \tag{19}$$

Here, z is the shift operator.

Multiplying $\eta(k+l)$ by $\alpha_l (l = 0, 1, \ldots, \mu_i)$ and then summing them up, the following equation is obtained.

$$q^i(z)\eta_i(k) = D_i^T(k)x(k) + \lambda_i^T(k)u(k) \tag{20}$$

where, $\alpha_{\mu_i} = 1$, and $D_i^T(k)$ and $\lambda_i^T(k)$ are defined as

$$\begin{aligned} D_i^T(k) &= \begin{bmatrix} \alpha^i_0, \alpha^i_1, \cdots, \alpha^i_{\mu_i-1}, 1 \end{bmatrix} \begin{bmatrix} c_i^{0T}(k) \\ c_i^{1T}(k) \\ \vdots \\ c_i^{\mu_i T}(k) \end{bmatrix} \\ \lambda_i^T(k) &= [0, \cdots, 0, 1, \gamma_{i(i+1)}, \cdots, \gamma_{im}] \end{aligned} \tag{21}$$

Let define $D(k)$ and $\Lambda(k)$ by the following.

$$D(k) = \begin{bmatrix} D_1^T(k) \\ D_2^T(k) \\ \vdots \\ D_m^T(k) \end{bmatrix}, \quad \Lambda(k) = \begin{bmatrix} \lambda_1^T(k) \\ \lambda_2^T(k) \\ \vdots \\ \lambda_m^T(k) \end{bmatrix} \tag{22}$$

Then, by applying the state feedback

$$u(k) = -\Lambda^{-1}(k)D(k)x(k) \tag{23}$$

to the system (1), the closed loop system from $u(k)$ to $\eta(k)$ becomes as follows.

$$\begin{bmatrix} q^1(z) & & \\ & \ddots & \\ & & q^m(z) \end{bmatrix} \eta(k) = 0 \tag{24}$$

This system is time-invariant and has the following state representation.

$$\omega(k+1) = A^*\omega(k) \tag{25}$$

The characteristic polynomial of A^* is

$$q(z) = \prod_{i=1}^{m} q^i(z) \tag{26}$$

and, the state variable, $\omega(k)$, is

$$\begin{aligned} \omega(k) := \left[\begin{array}{c} \eta_1(k) \\ \vdots \\ \eta_1(k+\mu_1-1) \\ \hline \vdots \\ \hline \eta_m(k) \\ \vdots \\ \eta_m(k+\mu_m-1) \end{array}\right] &= \left[\begin{array}{c} c_1^{0T}(k) \\ \vdots \\ c_1^{\mu_1-1T}(k) \\ \hline \vdots \\ \hline c_m^{0T}(k) \\ \vdots \\ c_m^{\mu_m-1T}(k) \end{array}\right] x(k) \\ &= P(k)x(k) \end{aligned} \tag{27}$$

On the other hand, from (1) and (23), the closed loop system becomes

$$x(k+1) = (A(k) - B(k)D^T(k))x(k). \tag{28}$$

Thus, the system (28) is equivalent to the system (25), under the transformation matrix $P(k)$. It is then obvious that the following equation holds.

$$P(k+1)(A(k) - B(k)D^T(k))P^{-1}(k) = A^* \tag{29}$$

This implies that the state feedback (23) makes the closed loop system equivalent to the system (25) that has an arbitrarily stable characteristic polynomial, $q(z)$.

Note that the transformation matrix $P(k)$ and $P^{-1}(k)$ must be bounded functions, in other words, $P(k)$ must be a Lyapunov transformation, to ensure the stability of the closed-loop system.

The procedures to obtain the state feedback gain is summarized below.

Pole Placement Design Procedure

STEP 1 Calculate the reachability matrix $U_R(k-n)$ and the reachability indices μ_i.

STEP 2 Calculate $C(k) = WR^{-1}(k-n)$ for the new output sigal, $\eta(k)$, using the trancated reachability matrix $R(k)$.

STEP 3 Determine the desired closed-loop characteristic polynomials, i.e., $q^i(z) = z^{\mu_i} + \alpha^i_{\mu_i-1} z^{\mu_i-1} + \cdots + \alpha^i_1 z + \alpha^i_0$ for $i = 1, \cdots, m$.

STEP 4 Using (21) and (22), calculate $D(k)$. Then the state feedback for the pole placement is

$$u(k) = -\Lambda^{-1}(k) D^T(k) x(k)$$

3.1 Example 1

Consider the following system.

$$x(k+1) = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 5+\cos(0.2k) \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ \sin(0.2k) & 1 \\ 1 & 0 \end{bmatrix} u(k) \tag{30}$$

This system is unstable and completely reachable in $n(=3)$ steps. The pole placement state feedback is calculated according to the following steps.

STEP 1 The reachability matrix is

$$U_R(k-3) = \begin{bmatrix} 1 & 0 & 3 & 0 & 5 & 0 \\ \sin(0.2(k-1)) & 1 & h_1(k-1) & 0 & h_2(k-1) & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{31}$$

where $h_1(k) = 6 + \cos(0.2k)$ and $h_2(k) = 8 + \cos(0.2k)$. From this, the reachability indices are obtained as $\mu_1 = 2$ and $\mu_2 = 1$.

The trancated reachability matrix $R(k-3)$ is

$$R(k-3) = \begin{bmatrix} 1 & 3 & 0 \\ \sin(0.2(k-1)) & h_1(k-1) & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{32}$$

STEP2 Using (16), calculate the new output matrix $C(k)$ as follows.

$$C(k) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -\frac{1}{2} & 0 & \frac{3}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ \delta_1(k) & 1 & \delta_1(k) \end{bmatrix} \tag{33}$$

$$= \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ \delta_1(k) & 1 & \delta_2(k) \end{bmatrix} \tag{34}$$

where

$$\delta_1(k) = \frac{1}{2}\sin(0.2(k-1)) - \frac{1}{2}\cos(0.2(k-2)) - 3$$

$$\delta_2(k) = -\frac{3}{2}\sin(0.2(k-1)) + \frac{1}{2}\cos(0.2(k-2)) + 3$$

STEP3 The desired stable characteristic polynomials of the closed-loop system are chosen as

$$q^1(z) = z^2 + \alpha^1_1 z + \alpha^1_0 = z^2 + 0.4z - 0.05 \tag{35}$$

$$q^2(z) = z + \alpha^2_0 = z + 0.1 \tag{36}$$

STEP4 From (21) and (24), we have

$$D(k) = \begin{bmatrix} D^T_1(k) \\ D^T_2(k) \end{bmatrix}, \quad \Lambda(k) = \begin{bmatrix} 1 & \frac{-2}{-2-h(k)+h(k+1)} \\ 0 & 1 \end{bmatrix} \tag{37}$$

where

$$D^T_1(k) = \begin{bmatrix} -0.05 & 0.4 & 1 \end{bmatrix} \begin{bmatrix} c^{0T}_1(k) \\ c^{1T}_1(k) \\ c^{2T}_1(k) \end{bmatrix} \tag{38}$$

$$D^T_2(k) = \begin{bmatrix} 0.1 & 1 \end{bmatrix} \begin{bmatrix} c^{0T}_2(k) \\ c^{1T}_2(k) \end{bmatrix} \tag{39}$$

and

$$c^{0T}_1(k) = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$$c^{1T}_1(k) = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & h_3(k) \\ 0 & 0 & 1 \end{bmatrix}$$

$$c^{2T}_1(k) = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & h_3(k+1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & h_3(k) \\ 0 & 0 & 1 \end{bmatrix}$$

$$c^{0T}_2(k) = \begin{bmatrix} \delta_1(k) & 1 & \delta_2(k) \end{bmatrix}$$

$$c^{1T}_2(k) = \begin{bmatrix} \delta_1(k+1) & 1 & \delta_2(k+1) \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & h_3(k) \\ 0 & 0 & 1 \end{bmatrix}$$

where $h_3(k) = 5 + cos(0.2k)$.

Thus, the pole placement state feedback is $u(k) = -\Lambda^{-1}(k) D^T(k) x(k)$.

The simulation results are shown in Fig.1.

4 Pole Placement of Non-Lexicographically-Fixed Systems

In the previous section, the reachability indices are supposed to be fixed. Such a system is called a lexicographically-fixed system. However, since the

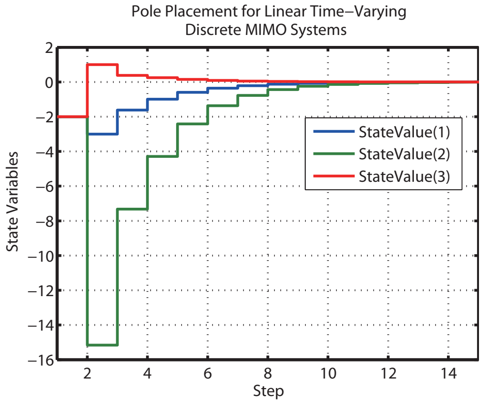

Figure 1. Simulation Result of the Pole Placement Control

system has time-varying parameters, there is a possibility that the reachability indices are also variable. In this section, we consider the pole placement control desing procedure for a non-lexicographically-fixed systems. Valasek et. al. proposed the pole placement design method for non-lexicographically-fixed multivariable continuous systems in [8]. Here, we apply this method to the discrete system together with the the new pole placement technique stated in the previous section. Assume that the system (1) is completely reachable in n steps, and is non-lexicographically-fixed. It is also assumed that the maximum value of each reachability index μ_i is known, i.e.,

$$v_i = \max_k \mu_i(k) \quad (i = 1, \cdots, m) \tag{40}$$

Next, we extend μ_i to v_i in $R(k)$ to obtain $R_v(k) \in R^{n \times n_g}$ as follows.

$$R_v(k) = \left[b_1^0(k), \cdots, b_1^{v_1-1}(k) | \cdots \right. \\ \left. \cdots | b_m^0(k), \cdots, b_m^{v_m-1} \right] \tag{41}$$

Define the augmented system by

$$x_g(k+1) = A_g(k)x_g(k) + B_g(k)u(k) \tag{42}$$

where $x_g(k) \in R^{n_g}$ and

$$A_g(k) = \begin{bmatrix} A(k) & 0 \\ A_2(k) & A_1(k) \end{bmatrix}, x_g(k) = \begin{bmatrix} x(k) \\ x_e(k) \end{bmatrix} \tag{43}$$

$$B_g(k) = \begin{bmatrix} B(k) \\ B_e(k) \end{bmatrix}. \tag{44}$$

Matrices $A_1(k)$, $A_2(k)$ and $B_e(k)$ should be chosen such that the reachability indices of the augmented system, (44), are v_i, and they are *lexicographically-fixed*.

The $n_g \times n_g$ trancated reachability matrix of (44) can be written as

$$R_g(k) = \left[b_{g_1}^0(k) \cdots b_{g_1}^{v_1-1}(k) | \cdots | b_{g_m}^0(k) \cdots b_{g_m}^{v_m-1}(k) \right] \tag{45}$$

where $b_{gi}^l(k)$ is calculated as follows.

$$b_{g_i}^0(k) = \begin{bmatrix} b_i(k+n-1) \\ b_{e_i}(k+n-1) \end{bmatrix}$$

$$b_{g_i}^1(k) = \begin{bmatrix} A(k+n-1)b_i^0(k-1) \\ [A_2(k+n-1) \; A_1(k+n-1)] \, b_{g_i}^0(k-1) \end{bmatrix}$$

$$\vdots$$

$$b_{g_i}^{v_m}(k) = \\ \begin{bmatrix} A(k+n-1)(b_i^{v_m-1}(k-1)) \\ [A_2(k+n-1) \; A_1(k+n-1)] \, b_{g_i}^{v_m-1}(k-1) \end{bmatrix} \tag{46}$$

where b_{g_i} and b_{e_i} are the i-th column of B_g and $B_e(k)$ respectively, for $i = 1, \cdots, m$. This implies that the upper block of $R_g(k)$ is the augmented trancated reachability matrix of (1), $R_v(k)$, i.e.,

$$R_g(k) = \begin{bmatrix} R_v(k) \\ R_e(k) \end{bmatrix} \tag{47}$$

The problem is then to find $A_1(k)$, $A_2(k)$ and $B_e(k)$ such that $R_g(k)$ is non-singular for all k. Since the rank of $n \times n_g$ matrix $R_v(k)$ is n, there exists a matrix $R_e(k)$ such that $R_g(k)$ is non-singular for all k. Let such a matrix $R_e(k)$ be denoted by

$$R_e(k) = \\ \left[r_{e_1}^0(k) \cdots r_{e_1}^{v_1-1}(k) | \cdots | r_{e_m}^0(k) \cdots r_{e_m}^{v_m-1}(k) \right] \tag{48}$$

In addition to this, we choose m vectors, $r_{e_1}^{v_1}(k), \cdots, r_{e_m}^{v_m}(k)$, arbitrarily. Then, from the lower block of (48) we have

$$\left[A_2(k+n-1) \; A_1(k+n-1) \right] R_g(k-1) \\ = R_e^+(k-1) \tag{49}$$

where $R_e^+(k)$ is defined by the following.

$$R_e^+(k) = \left[r_{e_1}^1(k) \cdots r_{e_1}^{v_1}(k) | \cdots | r_{e_m}^1(k) \cdots r_{e_m}^{v_m}(k) \right] \tag{50}$$

Since $R_g(k)$ is chosen to be non-singular for all k, $[A_2(k), \; A_1(k)]$ is obtained by

$$\left[A_2(k) \; A_1(k) \right] = R_e^+(k) \bar{R}_g^{-1}(k-n) \tag{51}$$

and,

$$B_e(k) = \left[r_{e_1}^0(k) \; r_{e_2}^0(k), \cdots, r_{e_m}^0(k) \right] \tag{52}$$

Thus, the augmented *lexicographically-fixed* system is obtained. Then, the pole placement state feedback for the original *non-lexicographically-fixed* system can be designed by applying the proposed method to this augmented system.

4.1 Example 2

Consider the following *non-lexicographically-fixed* system.

$$A(k) = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2}\cos(0.2k) & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{2}\sin(0.2k) & 0 \end{bmatrix} \quad (53)$$

The reachability matrix $U_R(k-3)$ is

$$U_R(k-3) = \begin{bmatrix} 1 & 0 & \frac{1}{2}\sin(0.2(k-2)) & 0 & * & * \\ 0 & 1 & \frac{1}{2}\cos(0.2(k-1)) & 0 & * & * \\ \frac{1}{2}\sin(0.2(k-1)) & 0 & 0 & 1 & * & * \end{bmatrix} \quad (54)$$

This system is completely reachable in n $(= 3)$ steps. However, the reachability indices should be regarded as $\mu_1 = 2$, $\mu_2 = 1$ or $\mu_1 = 1$, $\mu_2 = 2$ depending on the value of $\sin(0.2k)$ and $\cos(0.2k)$. We can choose

$$v_1 = 2, \quad v_2 = 2 \quad (55)$$

Then, $R_g(k)$ becomes

$$R_g(k-3) = \begin{bmatrix} 1 & \frac{1}{2}\sin(0.2(k-2)) & 0 & 0 \\ 0 & \frac{1}{2}\cos(0.2(k-1)) & 1 & 0 \\ \frac{1}{2}\sin(0.2(k-1)) & 0 & 0 & 1 \\ r^0_{e_1} & r^1_{e_1} & r^0_{e_2} & r^1_{e_2} \end{bmatrix} \quad (56)$$

We choose $r^0_{e_1} = r^1_{e_1} = r^0_{e_2} = 0$, $r^1_{e_2} = 3$ so that $Rg(k)$ is non-singular. From this, we have

$$[A_2(k)\ A_1(k)] = \left[3\ 0\ 0\ -\frac{1}{2}\sin(0.2(k-2))\right] \quad (57)$$

then, $A_g(k)$ and $B_g(k)$ for the augmented system are as follows.

$$A_g(k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{2}\cos(0.2k) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & -\frac{1}{2}\sin(0.2(k-2)) \end{bmatrix} \quad (58)$$

$$B_g(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{2}\sin(0.2k) & 0 \\ 0 & 0 \end{bmatrix} \quad (59)$$

For this augmented system, the new output matrix $C(k)$ so that the system has a relative degree, $n_g = 4$ is,

$$C(k) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{3} \\ -\frac{1}{2}\sin(0.2(k-1)) & 0 & 1 & \frac{1}{12}\sin^2(0.2(k-1)) \end{bmatrix} \quad (60)$$

Finaly, by the desing prodecure proposed in Section 2, the pole placement state feedback is obtained for the augmented system. The simulation result is shown in Fig.2.

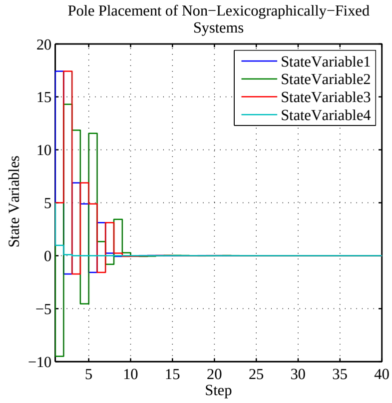

Figure 2. Simulation Result of the Pole Placement Control

5 Conclusions

In this paper, the Ackermann type of pole placement controller was proposed for linear time-varying multivariable discrete systems. By using the notion of the relative degree, the controller can be derived directly from the system parameters without transforming the system into any standard form. Further, this method was applied to the non-lexicographically-fixed systems.

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