THE ACCOUNT OF SECOND TERMS IN STEADY SOLUTION OF THE WING OSCILLATION PROBLEM IN SUPERSONIC GAS FLOW

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Abstract
Wing oscillation problem in supersonic gas flow is considered. Possibility of separation steady and unsteady parts of the problem at the same exact level is shown. Second approximation of steady part of the problem is obtained.

Key words
Thin wing oscillation, supersonic gas flow.

1 Introduction

Usually thin wing oscillation problem in supersonic gas flow is considered with the assumption, that the oscillation amplitude is small in comparison with the wing thickness [Barantsev, Radzevich, 1982]. On this basis the unsteady part of the problem is separated from its steady part. If the wing airfoil is given by

$$y = f(x,t,\varepsilon) = \varepsilon f^0(x) + \varepsilon^2 f'(x)e^{i\omega t}, \varepsilon << 1, \quad (1)$$

then velocity potential may be presented in form

$$\Phi = x + \varepsilon\varphi^0(x,y) + \varepsilon^2\varphi'(x,y)e^{i\omega t} \quad (2)$$

and problems for φ^0 and φ', every in its approximation are solved independently. However higher approximations φ^0 are become comparable with φ', and the question about its reciprocal influence is appeared. In any answer on this question it is naturally to solve both parts of the problem at the same exact level.

2 Statement of the problems

Inserting expression (2) for Φ in known equation

$$\begin{aligned}&(a^2-\Phi_x^2)\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy} + \\ &(a^2-\Phi_y^2)\Phi_{yy} - 2\Phi_x\Phi_{xt} - \\ &2\Phi_y\Phi_{yt} - \Phi_{tt} = 0,\end{aligned} \quad (3)$$

where $a^2 = M^{-2} + \frac{\varkappa-1}{2}(1-\Phi_x^2-\Phi_y^2-2\Phi_t)$, M is the Mach number, $\varkappa$ is the adiabatic exponent, we obtain

$$\begin{aligned}&\varepsilon(\varphi_{yy}^0 - k^2\varphi_{xx}^0) + \varepsilon^2\left\{-M^2[(\varkappa+1)\varphi_x^0\varphi_{xx}^0 + \right.\\ &2\varphi_y^0\varphi_{xy}^0 + (\varkappa-1)\varphi_x^0\varphi_{yy}^0] + [\varphi'_{yy} - k^2\varphi'_{xx} - \\ &\left.2i\omega M^2\varphi'_x + \omega^2M^2\varphi']e^{i\omega t}\right\} + O(\varepsilon^3) = 0,\end{aligned} \quad (4)$$

where $k^2 = M^2 - 1$. Let us assume, that

$$\varphi^0 = \varphi_0^0 + \varepsilon\varphi_1^0 + O(\varepsilon^2). \quad (5)$$

In that case we have for φ_0^0 the wave equation

$$\varphi_{0yy}^0 - k^2\varphi_{0xx}^0 = 0, \quad (6)$$

for φ_1^0 - the non-homogeneous wave equation

$$\begin{aligned}&\varphi_{1yy}^0 - k^2\varphi_{1xx}^0 = M^2((\varkappa+1)\varphi_{0x}^0\varphi_{0xx}^0 + \\ &2\varphi_{0y}^0\varphi_{0xy}^0 + (\varkappa-1)\varphi_{0x}^0\varphi_{0yy}^0),\end{aligned} \quad (7)$$

and for φ' - the equation

$$\varphi'_{yy} - k^2\varphi'_{xx} - 2i\omega M^2\varphi'_x + \omega^2M^2\varphi' = 0. \quad (8)$$

Let us consider boundary conditions. On wing airfoil must be

$$\begin{aligned}&\Phi_x n_x + \Phi_y n_y = f_t n_y,\\ &n_x = -f_x(1+f_x^2)^{-1/2},\\ &n_y = (1+f_x^2)^{-1/2}.\end{aligned} \quad (9)$$

Inserting here our expansions for Φ and f, we obtain (after transfer on $y = 0$)

$$\varphi_{0y}^0 = f_x^0, \quad (10)$$

$$\varphi^0_{1y} = \varphi^0_{0x} f^0_x, -\varphi^0_{0yy} f^0, \quad (11)$$

$$\varphi'_y = f'_x + i\omega f'. \quad (12)$$

On the shock wave $x = l(y,t)$ we have dynamic compatibility conditions

$$\begin{aligned} \Phi &= l, \\ \Phi_x &= 1 + \tfrac{2}{\varkappa+1}\tfrac{n_x}{\theta_-}(\theta_-^2 - M^{-2}), \\ \Phi_y &= \tfrac{2}{\varkappa+1}\tfrac{n_y}{\theta_-}(\theta_-^2 - M^{-2}), \\ n_x &= (1+l_y^2)^{-1/2}, \\ n_y &= -l_y(1+l_y^2)^{-1/2}, \\ \theta_- &= (l_t - 1)(1+l_y^2)^{-1/2}. \end{aligned} \quad (13)$$

Let us suppose

$$l(y,t) = ky + \varepsilon l^0_0(y) + \varepsilon^2 l^0_1(y) + \varepsilon^2 l'(y) e^{i\omega t} \quad (14)$$

and insert expansions (2), (5), (14) into expression (13). Using obtained expression

$$l^0_0(y) = -\tfrac{\varkappa+1}{4k^2} M^4 f^0_x(0) y, \quad (15)$$

we transfer the conditions for φ on characteristic $x = ky$. The result is

$$\varphi^0_0(ky, y) = 0, \quad (16)$$

$$\varphi^0_1(ky, y) = \varphi^0_{0x}(ky, y)\tfrac{\varkappa+1}{4k^2} M^4 f^0_x(0) y, \quad (17)$$

$$\varphi'(ky, y) = 0. \quad (18)$$

Thus, steady and unsteady problem separation is kept at the same exact level.

3 Solution of the problems

The first approximation of steady part of the problem (6), (10), (16) is well-known

$$\varphi^0_0(x,y) = -\tfrac{1}{k} f^0(x - ky). \quad (19)$$

The problem (8), (12), (18) for φ' is solved in [Arsent'ev, 2007]. Solution of the problem (7), (11), (17) we will find as a sum $\varphi^0_1 = \bar\varphi + \tilde\varphi$, where $\bar\varphi$ is the solution of non-homogeneous wave equation with null boundary conditions

$$\left.\begin{aligned} \tilde\varphi_{yy} - k^2\tilde\varphi_{xx} &= 0 \\ \tilde\varphi_y(x,0) &= \varphi^0_{0x}(x,0) f^0_x(x) - \\ &\quad \varphi^0_{0yy}(x,0) f^0(x) \\ \tilde\varphi(ky,y) &= \varphi^0_{0x}(ky,y)\tfrac{\varkappa+1}{4k^2} M^4 f^0_x(0) y - \\ &\quad \bar\varphi(ky,y) \end{aligned}\right\} \quad (20)$$

The system for $\bar\varphi$ [Smirnov, 1964] has the next solution

$$\begin{aligned} &\bar\varphi(x,y) = \\ &\tfrac{1}{2k}\int_0^y \left[\int_{x-k(y-\tau)}^{x+k(y-\tau)} M^2 \left((\varkappa+1)\varphi^0_{0x}(t,\tau)\varphi^0_{0xx}(t,\tau) + \right.\right. \\ &2\varphi^0_{0y}(t,\tau)\varphi^0_{0xy}(t,\tau) + \\ &\left.\left.(\varkappa-1)\varphi^0_{0x}(t,\tau)\varphi^0_{0yy}(t,\tau)\right) dt\right] d\tau. \end{aligned} \quad (21)$$

Let us simplify the expression (20) for $\bar\varphi$, using formula (19).

$$\bar\varphi = b\int_0^y \left[\int_{x-k(y-\tau)}^{x+k(y-\tau)} f^0_x(t-k\tau) f^0_{xx}(t-k\tau) dt\right] d\tau, \quad (22)$$

where $b = \frac{M^4(\varkappa+1)}{2k^3}$.

For finding $\tilde\varphi$ the general solution of homogeneous wave equation is applied.

$$\tilde\varphi(x,y) = \alpha(x-ky) + \beta(x+ky), \quad (23)$$

where α and β are free functions, which are found from the boundary condition. Then summing $\bar\varphi$ and $\tilde\varphi$ and applying formula (19), we obtain final solution of the problem (7), (11), (17)

$$\begin{aligned} &\varphi^0_1(x,y) = -\left(\tfrac{M^2 f^0_x(0)}{2k^2}\right)^2 (\varkappa+1)x - \\ &b\int_0^{\frac{x-ky}{2k}} \left[\int_{k\tau}^{x-ky-k\tau} f^0_x(t-k\tau) f^0_{xx}(t-k\tau) dt\right] d\tau - \\ &b\int_0^{\frac{x+ky}{2k}} \left[\int_{k\tau}^{x+ky-k\tau} f^0_x(t-k\tau) f^0_{xx}(t-k\tau) dt\right] d\tau - \\ &\int_0^{x-ky} f^0_{xx}(t) f^0(t) - \left(\tfrac{f^0_x(t)}{k}\right)^2 dt + \\ &b\int_0^y \left[\int_{x-k(y-\tau)}^{x+k(y-\tau)} f^0_x(t-k\tau) f^0_{xx}(t-k\tau) dt\right] d\tau. \end{aligned} \quad (24)$$

4 Conclusion

Steady and unsteady part of the problem are separated at the same exact level. Second approximation of steady part of the problem is obtained by formula (24) for φ^0_1.

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